The Technical Efficiency of Schools in Chile

Alejandra Mizala
Pilar Romaguera
Darío Farren

Center for Applied Economics
Department of Industrial Engineering
University of Chile
República 701, Santiago, Chile

Abstract

This paper assesses the technical efficiency of schools in Chile, which is defined as the capacity of schools to generate the maximum output (academic achievement) given the quantity of inputs they use. Two alternative methodological approaches for measuring efficiency are used: (i) estimation of a stochastic production frontier, and (ii) data envelopment analysis (DEA), which allows us to identify the efficient production frontier of the schools analyzed non-parametrically.

Each of these techniques has advantages and limitations, which are discussed in the paper; they lead, however, to the same conclusions when a sample of 2,000 schools is analyzed. The results obtained provide interesting points for educational policy discussion in Chile.

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I. INTRODUCTION

In the early 1980s Chile's education system underwent a far-reaching reform process, whose key feature was the transfer of public school administration to the local (municipal) level. The reforms also facilitated private sector participation in the market for education supply, through the introduction of a per-student subsidy mechanism (voucher scheme). This subsidy goes toward financing both privately owned subsidized schools and public schools. The per-student subsidy is supposed to cover the school's operating costs. This policy, along with allowing parents freedom to choose between the different schools, is also supposed to stimulate competition between schools to attract and retain students, which should bring with it improved efficiency and better quality educational services.

As a result of this policy, the private sector has set up many schools and three school types have emerged: (i) privately-owned subsidized schools, which are funded by the per-student subsidy paid by the state[1] and operated by the private sector; these currently serve 33.6% of primary school enrollment; (ii) public schools, also funded by the subsidy, but run by city governments,[2] currently serving 57.6% of total primary enrollment; and (iii) privately-owned fee-paying schools, financed solely by fees paid by parents, which are run by the private sector and represent 8.9% of primary school enrollment.

The 1990s also saw the introduction of reforms designed to achieve two basic objectives: improve the quality of education and distribute it more fairly. In pursuing these goals, the Ministry of Education set up special programs, including an education quality

[1] Some subsidized private schools also receive funding from parental fees (in what is known as co-financing).
[2] Some schools (those administered by local city governments) also receive funding from local city governments.

improvement program (known by its Spanish acronym, MECE) and support programs targeting the poorest schools (the P-900 program). Moreover, the Ministry strengthened school assistance programs and recently introduced another far-reaching reform to lengthen the school day and renew the curriculum, as well as implementing measures to strengthen teaching in line with these policies.

This paper aims to assess the technical efficiency of schools in Chile, i.e. determine whether or not they are maximizing output given the quantity of inputs they use. We define output as students' educational achievement, measured by results obtained on standardized tests applied nationwide. This is a major policy issue, because enormous public concern for educational quality must be brought into line with the need for public sector austerity, thus raising the question of whether education can be improved with existing resources. In other words: is there room for greater efficiency?

Given that this paper is concerned with comparing the efficiency of the three kinds of schools currently in existence, efficiency estimates are carried out for schools overall and then broken down to analyze and compare the three kinds of schools. Given different funding arrangements for the three types of schools, however, it is interesting to repeat the analysis, applying separate exercises for each kind of school. In this case, we ask a different question, given that we are comparing the efficiency of schools within a specific category. The results of this exercise are included in the Appendix.

This paper is organized into three sections, apart from this introduction. Section II reviews literature regarding the definition and measurement of technical efficiency, discussing two methods for assessing technical efficiency: estimation of a stochastic production frontier to derive efficiency coefficients, and *Data Envelopment Analysis* (DEA), which is a non-parametric way of identifying the efficient production frontier of a productive unit. Section III presents and compares the results of estimating schools' technical efficiency using each of the two methods, breaking schools down by type (i.e. private fee-paying, private subsidized, or public). Estimations are carried out using data for fourth grade students in 1996. Finally, section IV summarizes the main results and presents the conclusions of this study.

II. PRODUCTION FRONTIER AND TECHNICAL EFFICIENCY

Concept of Efficiency

The relevant literature distinguishes between two types of efficiency: (i) efficiency in resource allocation, that is, the capacity of decision-making units (DMUs) to adequately select input amounts in light of their relative prices, and (ii) technical efficiency, which is DMUs' capacity to maximize output given a certain level of inputs. This paper is concerned with the latter.

Many authors have pointed out the relevance of measuring technical efficiency. In his classic paper on the measurement of productive efficiency, Farrell (1957) argues that measuring technical efficiency is important because it allows us to determine whether outputs can be increased simply by raising efficiency, without needing to increase input quantities. Moreover, Lovell (1993) states that measuring efficiency makes it possible to rank and evaluate the DMUs analyzed, thus permitting the design of incentive mechanisms to reward the best DMUs, as well as policies to raise efficiency.

One of the first authors to define technical efficiency was Koopmans (1951), who defined it as a vector of inputs and outputs, where it is technologically impossible to increase any output (and/or reduce any input) without simultaneously reducing some other output (and/or increasing some other input). Debreu (1951) and Farrell (1957), for their part, developed technical efficiency indices. Debreu (1951) was the first to design a measure of productive efficiency, which he called the "coefficient of resource utilization". Farrell (1957) proposed measuring productive efficiency by comparing optimal and actual output. The production frontier predicts the optimum (or efficient) value of output, y^*, and given that for each DMU we have the observed value of its output y^0, we can obtain an efficiency coefficient expressed as:

$$\eta = \frac{y^o}{y^*} \tag{1}$$

The efficiency coefficient can also be calculated using the Jondrow (1982) method, which calculates expected inefficiency and hence the optimum value of output for each productive unit. Given this optimum value, the efficiency coefficient is calculated as follows:

$$\eta = \frac{y^o}{y^o + E(\upsilon)} \tag{2}$$

Where y^0 represents the observed level of output and $E(\quad)$ expected inefficiency.

To measure technical efficiency, production frontiers need to be estimated. Both economic and operations research literature offer basically two approaches to estimating production frontiers: parametric and non-parametric models. Parametric models are probably more common, with a prime example being the stochastic frontier, whose greatest disadvantage lies in having to assume an explicit functional form for technology, as well as a given distribution of inefficiency. Non-parametric methods, on the other hand, particularly DEA, do not require assuming any functional form, as they measure DMU efficiency relative to other DMUs in the sample. The two main disadvantages of DEA are: results may be sensitive to the selection of variables; and technical efficiency is measured in relative terms compared to the performance of the best productive unit in the sample, thus requiring the use of population data.

Stochastic Frontier and DEA Models

Stochastic Production Frontier

In estimating the production frontier it is assumed that the empirical production function can be represented linearly. However, the error structure is more complex when specifying a model with production inefficiencies than it is in the efficient production model. Error is assumed to be composed of two parts: υ and *e*.

$$y = \alpha_o + \sum_i \beta_i x_i - (\upsilon_i - e_i) \tag{3}$$

Where $\upsilon \geq 0$ may have different distributions[3] and *e* is the OLS random error. The term υ represents productive inefficiency, whereas the term *e* represents noise.

To determine whether inefficiency exists in the data, Aigner *et al.* (1977) suggest maximizing the following likelihood function:

$$L = L(\alpha_o, \beta_i, \lambda, \sigma | y, x_i) \tag{4}$$

[3] Possible distributions include Half Normal, Normal truncated at zero, and Exponential.

Where $\alpha_o, \beta, \lambda, y$ and x come from equation (3), $\lambda = \frac{\sigma_\upsilon}{\sigma_e}$ and $\sigma = \sigma_\upsilon + \sigma_e$, where σ_υ is the standard error of the term that captures inefficiency in the data, and σ_e is the standard error of the term capturing noise. The functional form of equation (4) depends on the assumptions made about the distribution of υ and *e*. Following the example of Jondrow *et al*. (1982) we assume a normal distribution for *e* and a half normal distribution for υ.

This likelihood function specification makes it possible to carry out a direct test to determine the presence of inefficiency in the productive process. If the coefficient is statistically different from zero, then there is evidence of inefficiency in the data.

Data Envelopment Analysis

DEA analysis is based on finding the best virtual producer corresponding to each real producer, where the virtual producer does not necessarily exist, but is imputed from a linear combination of the inputs and outputs of one or more efficient producers. If the corresponding virtual producer does better than the real producer by producing more output with the same level of inputs or the same output with fewer inputs, then the real producer is inefficient. The procedure for finding the best virtual producer can be formulated as a programming problem for each DMU.

We assume that there is *n* DMUs to be evaluated. Each DMU consumes *m* different inputs in producing *s* different outputs. Specifically, DMU_j consumes x_{ij} of input *i* and produces y_{jr} of output *r*. We further assume that $x_{ij} > 0$ and $y_{jr} > 0$, and also that each DMU has at least one input and one output.

A measure of efficiency is based on a virtual efficient unit, constructed as a weighted average of real efficient units, which is used as a unit of comparison for other DMUs. To determine the efficiency of any DMU *0* we have:

$$\max_{u,v} h_o(u,v) = \frac{\sum_r u_r y_{ro}}{\sum_i v_i x_{io}} \tag{5}$$

In mathematical programming terms, this ratio is the objective function to be maximized where, the *u* and *v* are output and input weights respectively.

In addition, there are a set of constraints, one for each DMU, which reflect the condition that the ratio of virtual output to virtual input must be less than or equal to one for all observed DMUs.

A nonlinear programming problem arises from the above ratio form of the model; Charnes, Cooper and Rhodes (CCR) (1978), however, have shown that it may be replaced by a linear programming problem that takes the form[4]:

$$\begin{aligned}
&\max_{u,v} h_o(u,v) = \sum_r u_r y_{ro} \\
&s.t. \\
&\sum_r u_r y_{rj} - \sum_i v_i x_{ij} \le 0, \quad \mathrm{j} = 1,\ldots,\mathrm{n} \\
&\sum_i v_i x_{i0} = 1 \\
&u_r, v_i \ge 0, \quad \mathrm{r} = 1,\ldots,\mathrm{s} \quad \mathrm{i} = 1,\ldots,\mathrm{m}
\end{aligned} \tag{6}$$

[4] DEA can also be motivated using the concept of ratios between weighted outputs and weighted inputs. The relationship between this form and the "virtual producer" form used in this paper is that each represents a primal-dual linear programming formulation of the same problem. We have used the virtual producer form because it is easier for non-DEA readers to grasp.

Solving this linear programming problem we obtain the efficient or virtual production for each DMU and the efficiency index.

The Charnes, Cooper and Rhodes (CCR) (1978) ratio allows for both technical and scale inefficiencies via the optimal value of the ratio form. Moreover, Banker, Charnes and Cooper (BCC) (1984) offer a variant that allows inefficiencies to be divided into scale and technical inefficiency measures, a variant that becomes useful when returns to scale are important. In this case, the linear programming problem is expressed as:

$$
\begin{aligned}
&\max_{\mathrm{u,v}} h_0(u,v) = \sum_r u_r y_{r0} + u_0 \\
&s.t. \\
&\sum_r u_r y_{rj} - \sum_{\mathrm{i}} v_i x_{ij} + u_0 \leq 0 \,, \quad j = 1,\ldots,n \\
&\sum v_i x_{i0} = 1 \\
&u_r, v_i \geq 0 \,, \quad r = 1,\ldots,s \quad i = 1,\ldots,m \\
&u_0 \ \text{free}
\end{aligned}
\qquad (7)
$$

Technical Efficiency in Educational Production

One area where the concept of technical efficiency has been applied is education. Applications of DEA to educational production can be found in Bessent and Bessent (1980), Bessent *et al.* (1982), Bessent *et al.* (1983), Bessent *et al.* (1984), Färe *et al.* (1989), Bonesronning and Rattso (1993), Johnes and Johnes (1995), Kirjavainen and Loikkanen (1998), among others.

Sengupta and Sfeir (1986) as well as Deller and Rudnicki (1993) analyze the efficiency of educational production using the stochastic production function approach.

Vinod (1968), Chizmar and Zak (1984) and Gyimah-Brempong and Gyapong (1991) apply a method based on canonical correlation analysis to estimate an educational production function with multiple outputs. Canonical correlation seeks to determine a linear combination of outputs and a linear combination of inputs whose correlation is maximized.

Other studies, Ruggiero (1996), McCarty and Yaisawarng (1993) and Subhash (1991), have used a mixed methodological approach incorporating both non-parametric techniques and regression analysis to evaluate the efficiency of educational production in different school districts in the U.S.A.

Furthermore, Johnes (1998) compares the efficiency measures that emerge from the stochastic frontier model with those obtained by data envelopment analysis for traditional universities in the United Kingdom.

III. ESTIMATION RESULTS

Stochastic Frontier Model Results

In the following we use the stochastic frontier model to estimate an education production frontier. The data come from the Ministry of Education (Mineduc), the Educational Quality Measurement System (Simce) and the National School Support and Scholarship Board (Junaeb). Simce test results are for 1996 fourth Grade students, using a random sample of 2,000 schools.[5]

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[5] As mentioned above, DEA analysis should be applied to the complete set of schools. Our DEA software, however, suffers from capacity limits that could not be resolved. We therefore worked with a random sample of schools, which represent the frontier of efficient schools well, given that the efficiency distribution is statistically equal for the three random samples used (the evidence supporting this claim is available upon request).

Output is measured in terms of academic achievement, i.e. each school's average score for the Simce's Spanish and Mathematics tests.[6] Inputs used are:[7]

a) Student characteristics:
   Socioeconomic level: (A, B, C, D)
   Vulnerability index

b) School characteristics:
   Type of school: (FPPS, SPS, Public)
   Geographical index: (A, B, C, D, E)
   School size: number of students
   Pupil-teacher ratio
   Whether or not pre-school education is provided
   Gender: (boys' schools, girls' schools, coeducational schools)

c) Teacher characteristics: Average experience

It is necessary to keep in mind that socioeconomic information is very sweeping. Moreover, it is reported by the school head principles who have incentives to underdeclare the real socioeconomic level, as this information is used to compare achievements between schools. Using data from Chile's national household survey (CASEN) and from the Junaeb, Carnoy and McEwan (1997) show that parental education levels vary considerably within the categories defined by Simce;[8] in particular, the average level of parental education is higher in private subsidized schools than in public schools within the educational strata defined by Simce. This means that even when socioeconomic level is controlled for, parental education continues to vary, a factor that

[6] Although Simce test results exist at the individual student level, socioeconomic data is not available for each family. The information available represents the average of all families in a given school. This is not ideal, but it should be remembered that the topic of this paper is school level achievement and efficiency.
[7] The Appendix provides details of variables and their descriptive statistics.
[8] The categories used by Simce are described in the Appendix under "Details of variables".

we would not be taking into account. For this reason we also include the vulnerability index as a measure of socioeconomic status.[9]

Table 1 shows the results of ordinary least square (OLS) and stochastic frontier estimations for 1996 fourth grade students.[10]

As mentioned above, if parameter is significantly different from zero, there is sufficient statistical evidence for the presence of inefficiencies in the data; in this case we find a statistically different from zero at the 1% level. The coefficients estimated from the stochastic frontier and OLS models are not significantly different from each other. The constant term, however, is greater in the case of the production frontier estimation, suggesting that this is the data envelope.

The estimated coefficients suggest that, after we control for school, student and family characteristics, private fee-paying schools perform better than private subsidized schools and public schools (six points more in the Simce tests). There are, however, no statistically significant differences between the two types of subsidized schools (private subsidized and public).[11]

The results show that socioeconomic variables (socioeconomic level and vulnerability index) are very important in explaining student achievement. In general, students from lower socioeconomic levels perform more poorly, on average, than students from

[9] The Junaeb calculates the Vulnerability Index to assign food rations provided by schools. This index is built using a logistic regression and takes into account students' weight, height, need for medical and dental attention, and mother's education. The index ranges from 0 to 100. See appendix for a more in-depth explanation of this index.

[10] As mentioned, we assume that the error term capturing inefficiency in the productive process is subject to a half-normal distribution, whereas random disturbances are normally distributed. The results do not vary significantly if one assumes other distributions, such as exponential or normal truncated at 0.

[11] For more on this issue see Mizala and Romaguera (2000)

families with higher income and education levels. This is a very well known finding in educational production function estimations for different countries.[12]

In recent years many studies have examined the main factors, aside from students' socioeconomic characteristics, which influence school performance in developed and developing countries. These studies emphasize that the length of the school day, availability of texts and basic infrastructure show a high correlation with academic performance and confirm the importance of pre-school education's contribution to students' elementary school results. Other factors that show a positive correlation with performance are initial teacher training, teacher experience and frequency of homework. However, some factors do not consistently show a correlation with performance, among them class size (Heyneman and Loley (1983), Hanushek (1995), Fuller and Clarke (1994), Wolff, Shiefelbein and Valenzuela (1993)).

Chile's experience indicates that when school size is measured by the number of students, bigger schools do better on Simce tests. The number of students squared has also been included as a variable in specifying SFA. In this case, however, for both the ordinary least squares (OLS) and the frontier estimation, the result is not statistically significant, although it is negative. This result is mainly due to the performance of public schools, which are the majority. For these schools, both large and small institutions perform well, making size statistically insignificant when it comes to explaining educational achievement. This is not the case for fee-paying private and subsidized private schools, for which the term squared is negative and statistically significant, thus indicating that

[12] This result was first identified by Coleman et. al.'s pioneering study (1966) and was confirmed in later studies for both developed and developing countries (See Summers and Wolfe (1977), Hanushek (1986), Hanushek and Taylor (1990), Deller and Rudnicki (1993), Berger and Toma (1994), Goldhaber and Brewer (1997), Mizala and Romaguera (2000), among others)

there is an optimum school size.[13] Moreover, girls' schools do better on standardized tests than boys' and coed schools.

The student-teacher ratio is significant and negative, so in this case the larger the class the more poorly students perform.[14] The geographical index variable, which reflects the type of city and school accessibility due to location, behaves unexpectedly, partly because much of its effect is captured by the vulnerability index, which measures poverty.

Using the conditional expected value of the stochastic frontier, a Farrell-type efficiency measure (1957) was calculated for each observation.[15] The average school in the sample has an efficiency level of 0.9318; the most efficient school has a coefficient of 0.9819 and the least efficient 0.7304.

Figure 1 compares efficiency and educational achievement as measured by Simce tests at the national level.

The graph is divided into four quadrants. Quadrant I includes 975 schools (49%), which show above average achievement for the sample (68.7) and above average technical efficiency (0.9318) as calculated using the stochastic frontier model. These are the most efficient and effective schools according to the model, as they obtain more output with less input (efficient), and the output obtained is higher than average (effective).

Quadrant II includes 266 schools (13%), which are efficient but show low achievement levels. Quadrant III includes 708 schools (35%) with below average achievement and below average efficiency. These schools can be classified as both inefficient and ineffective. Finally, Quadrant IV includes just 51 schools (3%), which achieve above

[13] These results are available on request.
[14] This topic has been the subject of considerable debate in the literature; in fact much of the research on the relationship between class size and achievement is inconclusive, see Hanushek's (1996) survey.
[15] The measure used corresponds to Equation 2.

average results, but given the inputs available to them, could do better if they made full use of their potential. This is an exceptional situation, so there are very few schools in this quadrant. These schools should be encouraged to become more efficient.

Most schools fall within the first and third quadrants; i.e. there is a positive relation between achievement and efficiency. No school achieves an efficiency level of 100, due to the fact that expected inefficiency is always non-zero.

The school type variable (i.e. whether a school is fee-paying, private subsidized or public) is highly relevant for analyzing schools in Chile, so any assessment of efficiency should take it into consideration. Table 2 shows the same information as Figure 1, but breaks schools down by type. In the most efficient quadrant (I) we find most private fee-paying schools (90%); these schools have better inputs and high achievement levels. Slightly over half of all private subsidized schools (59%) also fall within this Quadrant. Of public schools, 39% are located in Quadrant I and 42% in Quadrant III, these last typically scoring below average for both Simce tests and efficiency. Nevertheless, 18% of public schools fall within Quadrant II, indicating below average results but high efficiency, given that these schools have lower inputs than others with similar scores.

For each category of school, Figure 2 shows the cumulative percentage of schools up to any given efficiency coefficient, thus indicating how schools are distributed over the efficiency range.

Figure 2 shows that most private fee-paying schools have high (above 0.90) efficiency coefficients, unlike public schools and private subsidized schools, where some schools have lower coefficients. One can also see a difference, albeit smaller, between private subsidized and public schools. The stochastic production frontier model shows that private fee-paying schools have more resources than other types of school and they make the most of this. Private subsidized schools behave more like public than private fee-paying schools, although a larger percentage of private subsidized schools have efficiency coefficients above those of public schools.[16]

Figure 2 shows that the distribution of efficiency coefficients among private fee-paying schools displays stochastic dominance compared to the distribution of coefficients among the other school types.

We carried out two non-parametric tests to find out if the distribution of efficiency coefficients for one school type is stochastically higher than those for the other types. The Wilcoxon-Mann-Whitney and Kolmogorov-Smirnov tests indicate that if we choose one school of each type at random, the probability of the private fee-paying school having a higher efficiency coefficient than the other two types of school is greater than one half. Similarly, the probability of a private subsidized school, chosen at random, having an efficiency coefficient above that of a public school is also greater than one half.[17]

Table 3 provides another way of looking at these results in a distribution table showing efficiency coefficients above and below the median of all schools. Here it can be seen that efficiency coefficients for over 80% of private fee-paying schools are above the median, with just 19.5% below it. In contrast, 55% of private subsidized schools show coefficients over the median, as do 44% of public schools. Forty five percent of private subsidized schools and 56% of all public schools have efficiency coefficients below the median.

[16] Efficiency coefficients by type of school average: 0.9473 for fee-paying private schools, 0.9332 for subsidized private schools and 0.9292 for public schools.

[17] The values obtained for the tests are shown in Table A5 in the Appendix.

DEA model results

In this section we approach the issue of school efficiency using the DEA method, to make a comparison with the stochastic production frontier estimates. We use the same inputs from the stochastic production frontier model and the same output measurement, i.e. the average result per school on standardized Spanish and Mathematics tests (Simce).

It is important to point out that unlike the stochastic production frontier method, DEA compares each school against a virtual school constructed from a linear combination of the inputs and outputs of the most efficient ones and provides a relative ranking for the most efficient units. As in the production frontier estimation, each school is compared with others of the same type, so if schools of a particular type are more heterogeneous in their behavior the comparison will be more demanding and they will appear more inefficient, to the extent that there are some high achieving schools. In other words, unlike the stochastic frontier method, which is a matter of averages, the DEA method is more sensitive to extremes.[18]

Using the DEA methodological approach, efficiency coefficients were estimated for each of the 2,000 schools included in the random sample.[19] This random sample of schools allowed us to suitably identify their efficiency distribution. Proof of this is the fact that using other random samples of schools, which when combined cover the total population of schools, provides statistically similarly results. Thus, we are correctly representing the efficient frontier for schools.[20]

[18] This is an important point because schools differ widely in Chile today (see table A1 in the appendix).
[19] This is the same random sample used for the stochastic production frontier estimation.
[20] These results are available upon request

Because school size as measured by the number of students[21] is a significant variable, we have opted for using the Banker, Charnes and Cooper (BCC) (1984) variant of DEA, which allows inefficiencies to be sorted according to scale and technical measures. As mentioned above, this variant is useful when returns to scale are important. Nonetheless, because the main purpose of this paper is to compare the technical efficiency of Chilean schools using two different methodologies we will not differentiate between inefficiencies of scale and technical inefficiencies.

The results indicate that a typical school in the sample is 95.39% efficient; according to DEA analysis, 58% of schools are fully efficient, in that they display a degree of efficiency of 100%, while efficiency ranges from 53% to 100%.[22] [23]

Figure 3 shows the relative efficiency and average achievement measured by Simce tests for fourth grade students at each school. As with the stochastic frontier model estimates, we found a positive correlation between crude performance (Simce test) and efficiency.

Furthermore, schools can be classified according to whether they are above or below the average score (achievement) or average efficiency. In Figure 3 we see that there are schools that can be considered efficient, even though their test scores are low. Closer scrutiny reveals that all are located in isolated rural areas, with very high vulnerability indices, and serve families of a similar socio-economic level. They are efficient, performing slightly better than other schools with the same characteristics.

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[21] Results are similar when the model is estimated using the number of teachers as a size indicator.

[22] Note that these efficiency coefficients, obtained through DEA, are not numerically comparable with the coefficients obtained by estimating the stochastic production frontier.

[23] Studies carried out for the USA, with smaller samples of schools from the Houston school district, found that 53% were fully efficient, while efficiency ranged from 80% to 100% (Bessent *et al*., 1982). Ruggiero (1996), meanwhile, found that 32% of schools in the New York district were efficient. However, these results cannot be compared with our results because we are not studying the same production processes: the models underlying these measurement of efficiency are different.

Figure 3 is divided into four quadrants. In Quadrant I there are 904 schools (45%), which achieved above average scores in the Simce tests, and have efficiency levels higher than the sample average. These schools are effective because they obtain relatively high scores in the Simce. They are also efficient, as they achieve high scores by making the most of the inputs available to them. In Quadrant II, meanwhile, there are 489 schools that are efficient (25%); these schools are doing well compared to similar schools in terms of inputs; however they do not manage high scores in the tests, perhaps due to a lack of resources. In Quadrant III there are 485 schools that are not efficient (24%): given their resources they are not maximizing academic achievement, as they obtain lower than average scores. Finally, in Quadrant IV there are 122 schools (6%) which perform well, but which could do even better with the same level of inputs.

The fact that there is a significant variance in the scores among “efficient” schools (100% efficiency), shows that there is significant room for educational policies to have an impact: efficient schools with low scores on the Simce tests would probably improve substantially if the inputs available to them were increased.

It is also possible to analyze efficiency and academic achievement by type of school. Table 4 shows the results of this.

In Table 4, the efficiency-achievement matrix shows that nearly all (86%) private fee-paying schools belong to the quadrant representing the most efficiency and the highest test scores. Private subsidized schools are also to be found mostly in the first quadrant (56%), whereas 35% of all public schools are in this quadrant. Public schools, on the other hand, are almost evenly (around one-third of schools in each one) distributed among the first, second and third quadrants.

A cumulative efficiency graph was constructed by school type. Figure 4 shows that public schools are concentrated in the efficiency range 53% to 100%, whereas private subsidized schools are in the 74% to 100% range. Finally, efficiency at private fee-paying schools ranges from 75% to 100%, but far fewer show efficiency coefficients below 90%, as compared with the other two types of schools.[24]

From Figure 4 it can be concluded that the distribution of efficiency coefficients among private fee-paying schools displays first-order stochastic dominance in relation to the distribution of coefficients among the other school types. Likewise, the distribution of efficiency coefficients for private subsidized schools shows first-order stochastic dominance in relation to public schools.

The Wilcoxon-Mann-Whitney and Kolmogorov-Smirnov tests indicate that if we choose one school at random from each type, the probability that the private fee-paying school has a higher efficiency coefficient than that of the other types of school is greater than half. In turn, a private subsidized school chosen at random has a probability greater than half of having a higher efficiency coefficient than a public school.[25]

Table 5 shows the distribution of efficiency, confirming the results mentioned above: 74% of private fee-paying schools have efficiency coefficients above the median of the school sample as a whole; 63% of private subsidized schools have a efficiency coefficient above the median, whereas 55% of public schools have efficiency coefficients that are above the median.

[24] Efficiency coefficients by type of school average: 98.55% for fee-paying private schools, 96.35% for subsidized private schools and 94.55% for public schools.

[25] The values obtained for each of the tests are shown in Table A.6 in the Appendix.

Results for Each School Type

Given the fact that three types of school exist in Chile (according to funding arrangement) we repeated the stochastic frontier and DEA analyses, carrying out separate exercises for each category of school. The results are presented in Part IV of the Appendix.

In this case, we are comparing the technical efficiency of schools within a category. By doing this, we reach different results from those for schools overall. In particular, a much smaller percentage of fee-paying private schools appear in the quadrant characterized by above average efficiency and academic achievement for the group. In fact, in this case the results obtained using a stochastic frontier analysis are very similar for all three kinds of schools (see Tables A7 and A8 in the Appendix).

One interesting result is that, as with the overall school analysis, both methodologies (DEA and stochastic frontier) find a positive correlation between the results of students' achievement tests and efficiency for each school type. This is a very robust result of this study.

In this case, however, some of the results differ according to the stochastic production frontier or DEA methodologies used.[26] Specifically, with the DEA methodology fee-paying private schools continue to be more efficient (82% show 100% efficiency), followed by subsidized private schools (60% show 100% efficiency), and finally public schools (55% show 100% efficiency). Using the stochastic frontier method, no one type of school shows stochastic dominance over the others (see Figures A4 and A8 in the appendix).

IV. CONCLUSIONS

This paper has attempted to assess the technical efficiency of Chilean schools. To this end, we analyzed a random sample of 2,000 schools, classified by type (i.e. private fee-paying, private subsidized and public schools).

To ensure the robustness of the results, two methods were used to analyze schools' efficiency: estimation of a stochastic production frontier from which efficiency coefficients were obtained and the non-parametric DEA (BCC) method. Results allow us to conclude that the two methodologies give a similar ranking of schools in efficiency terms.

The results obtained allow the conclusion that schools in Chile display an average technical efficiency of 0.93, as measured by the stochastic production frontier method, ranging from 0.73 to 0.98. According to the DEA results, a typical school has an efficiency of 95%, while the range is from 53% to 100%. The larger range for efficiency coefficients obtained using the DEA methodology can be explained by the fact that in Chile there is an important variance in academic achievement among schools with similar characteristics.

When technical efficiency is estimated for schools as a whole and then analyzed for the three types of school, it can be concluded that private fee-paying schools are the most efficient, followed by private subsidized and public schools. Both methodologies show a significant difference in technical efficiency between private fee-paying schools and the other school types. For its part, the stochastic production frontier method displays a smaller difference in technical efficiency between private subsidized schools and public

[26] We should note that the adjustment to the stochastic production frontier estimation worsens upon estimating a regression for each kind of school.

schools than that shown by DEA, which is explained by the differences between the two methodologies. However, in both cases private subsidized schools have higher efficiency indices.

It is important to stress that the technical efficiency indices are calculated by comparing schools of the same type. If, within a school type, there is a high variance in achievement and there are schools with similar characteristics that perform very well on the tests, the comparison is more demanding, especially in the DEA method. This explains the differences between the DEA results and those of the stochastic frontier as regards public and private subsidized schools.

The comparison of technical efficiency by school type is subject to certain limitations. In particular, private subsidized schools, unlike their public counterparts, can select students for admission and eliminate low achievers. Indeed, Rounds Parry (1996) when interviewing a random sample of schools in Santiago, found that private subsidized schools make greater use of exams, minimum mark requirements and parental interviews in their student selection process. This implies that these schools accept students who achieve better results (those with more ability), a characteristic not controlled for in the available variables. Unfortunately, the information available in Chile does not permit correction for selection bias.

When each school type is analyzed separately the efficiency analysis confirms the positive correlation between crude performance and efficiency within each of the three school types. This result is very robust, for both the methodology used and upon consideration of a set of schools or each category on its own. Upon analyzing each category of school on its own, DEA confirms that fee-paying private schools are more efficient, followed by subsidized private schools and public schools. Stochastic frontier analysis, however, does not allow conclusions as to which type of school is more efficient than the others.

In summary, the results of this paper show that schools with similar characteristics and inputs display quite different results, so studying the reasons for these differences will help to design more effective educational policies. It is also interesting to point out that there are schools that, despite being efficient, do not achieve good results in the standardized Simce tests. These schools probably require more inputs to improve their performance. There are also inefficient schools that get bad results in the tests; in this case, before providing more inputs, the causes of their inefficiency need to be understood. More inputs provided to a technically inefficient school will not lead to better results.

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Table 1
Stochastic Production Frontier; Fourth Grade 1996
(dependent variable: Simce test fourth grade, 1996)

Variable	Coefficients	
	OLS	Frontier
Constant	68.409	73.369
	(48.413)**	(39.169)**
Socioeconomic Level A	8.850	8.752
	(6.799)**	(6.034)**
Socioeconomic Level B	6.044	5.902
	(6.356)**	(6.092)**
Socioeconomic Level C	1.316	1.269
	(1.794)	(1.929)
Vulnerability Index 96	-0.118	-0.119
	(-10.054)**	(-11.584)**
Dummy Fee-paying private school	5.962	5.946
	(5.530)**	(4.271)**
Dummy Subsidized private school	0.487	0.548
	(0.853)	(0.965)
Boys' schools	3.097	3.091
	(1.915)	(1.524)
Girls' schools	5.138	5.051
	(4.692)**	(3.134)**
Geographical Index B	1.630	1.605
	(2.501)*	(1.950)
Geographical Index C	1.137	1.135
	(1.725)	(1.463)
Geographical Index D	4.340	4.451
	(5.832)**	(5.858)**
Geographical Index E	1.815	2.063
	(1.769)	(2.317)*
Pupil/Teacher ratio	-0.100	-0.098
	(-2.961)**	(-2.976)**
Teacher experience	0.056	0.054
	(1.785)	(1.701)
Number of students	0.003	0.003
	(4.559)**	(3.351)**
Number of students squared	-0.38E-06	-0.36E-06
	(-1.075)	(-0.543)
Preschool Level	0.776	0.672
	(1.371)	(1.182)
λ	-	0.806
	-	(3.649)**
F	87.68**	
Adjusted R^2.	0.424	-
n	2000	2000

Note: Reference dummy variables are: Public Schools, Socioeconomic Level D, Coeducational schools, Geographical Index A. T statistics in parenthesis, ** statistically significant at 1%, * statistically significant at 5%

Table 2
Efficiency-Achievement Matrix. Stochastic Frontier Model; Fourth Grade 1996
(number of schools and percentage in each quadrant)

IV FPPS 11 (7%) SPS 24 (4%) Public 16 (1%) TOTAL 51 (3%)	I FPPS 143 (90.%) SPS 348 (59%) Public 484 (39%) TOTAL 975 (49%)
III FPPS 5 (3%) SPS 176 (30%) Public 527 (42%) TOTAL 708 (35%)	II FPPS 0 (0%) SPS 39 (7%) Public 227 (18%) TOTAL 266 (13%)

Note: The figures for "Total" correspond to the total number of schools in the respective quadrant, and figures for FPPS, SPS and Public relate to the number of schools of each type in each quadrant. The percentages indicate the fraction of the total number of schools of each type located in each quadrant.

Table 3
Efficiency Groups By Type of School. Stochastic Frontier Model.

	1 <0.9412	2 >0.9412	TOTAL
FPPS	31 (19.5%)	128 (80.5%)	159 (100%)
SPS	263 (44.8%)	324 (55.2%)	587 (100%)
Public	705 (56.2%)	549 (43.8%)	1254 (100%)
TOTAL	999 (50.0%)	1001(50.0%)	2000 (100%)

Note: Group 1 efficiency coefficient above the median
Group 2 efficiency coefficient below the median

Table 4
Efficiency-Achievement Matrix. DEA (BCC). Fourth Grade 1996

IV FPPS 18 (11%) SPS 44 (7.5%) Public 60 (4.8%) TOTAL 122 (6.1%)	I FPPS 136 (86%) SPS 328 (56%) Public 440 (35%) TOTAL 904 (45%)
III FPPS 2 (1%) SPS 102 (17%) Public 381 (30%) TOTAL 485 (24%)	II FPPS 3 (2%) SPS 113 (19%) Public 373 (30%) TOTAL 489 (25%)

Note: See note in table 2.

Table 5
Efficiency Groups By Type of School. DEA (BCC)

	1 <100	2 =100	TOTAL
FPPS	41 (25.8%)	118 (74.2%)	159 (100%)
SPS	217 (37.0%)	370 (63.0%)	587 (100%)
Public	565 (45.1%)	689 (54.9%)	1254 (100%)
TOTAL	823 (41.2%)	1117 (58.9%)	2000 (100%)

Note: see note in table 3

Figure 1
Efficiency-Achievement Matrix; Stochastic Frontier Model; Fourth Grade 1996

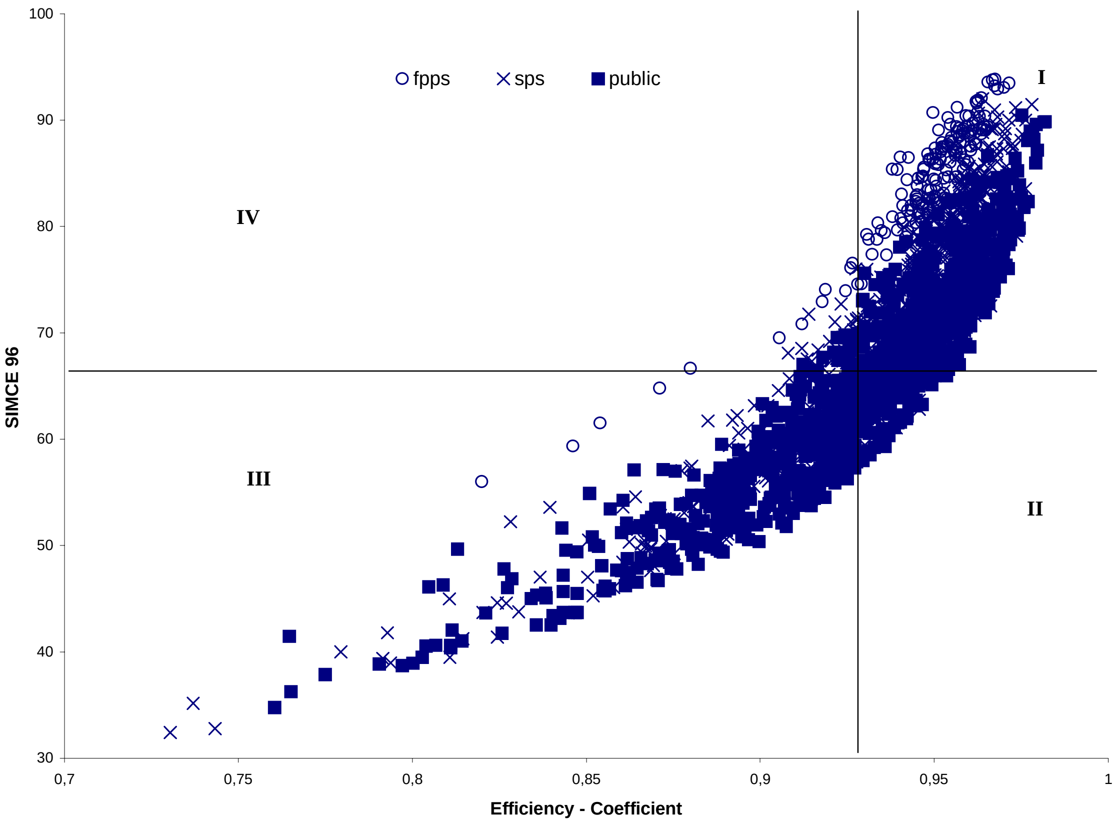


Figure 2
Efficiency By Type of School, Stochastic Frontier Model; Fourth Grade 1996

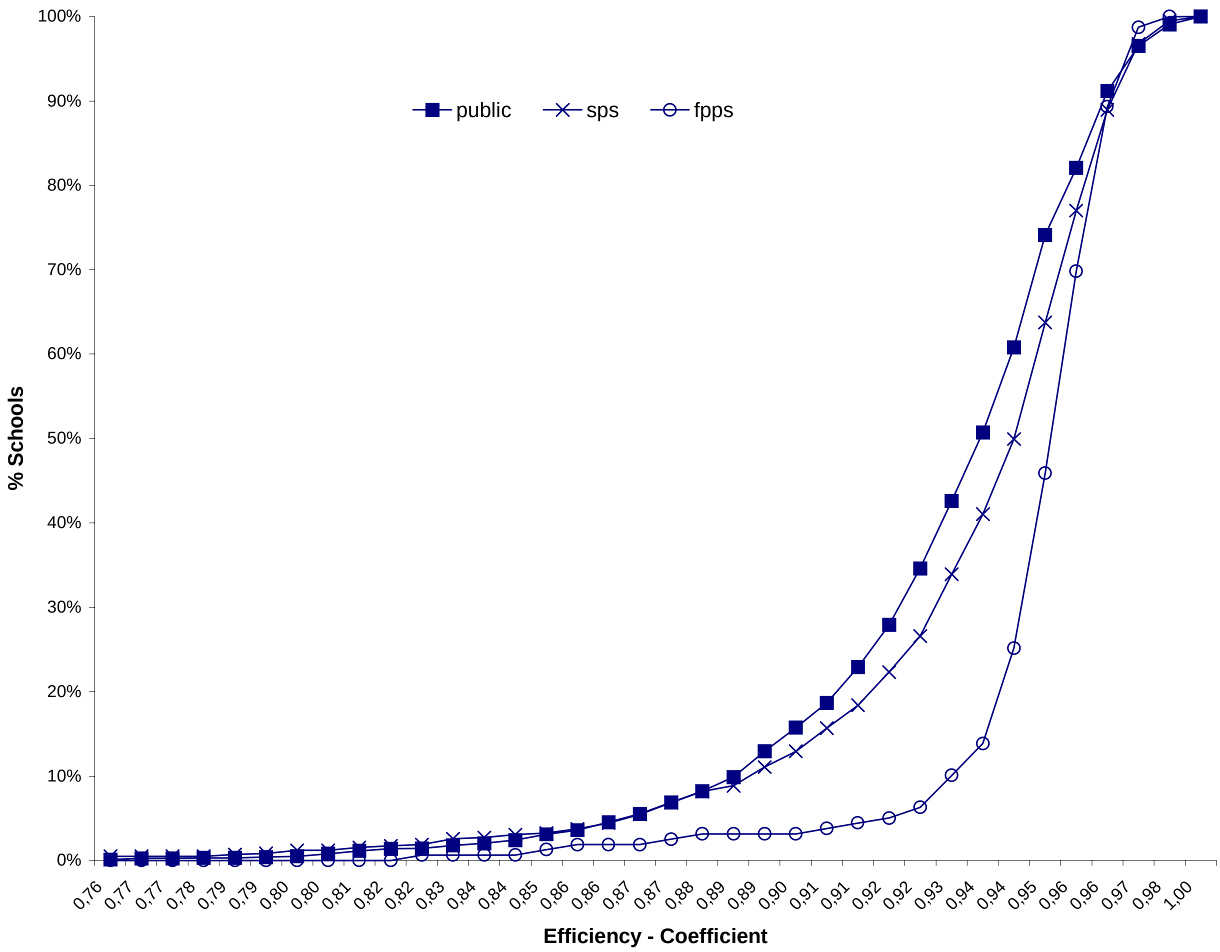


Figure 3
Efficiency-Achievement Matrix. DEA (BCC). Fourth Grade 1996.

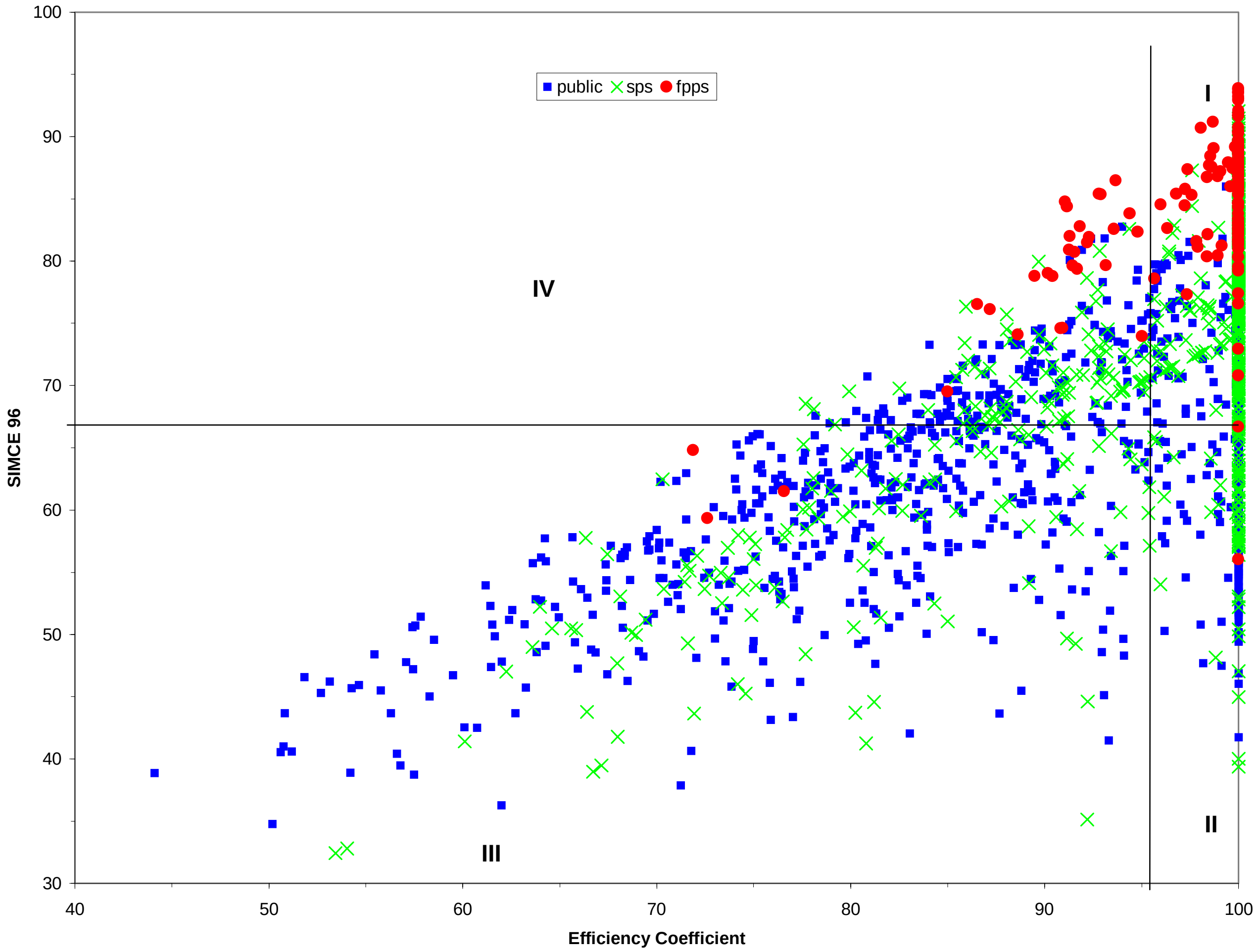
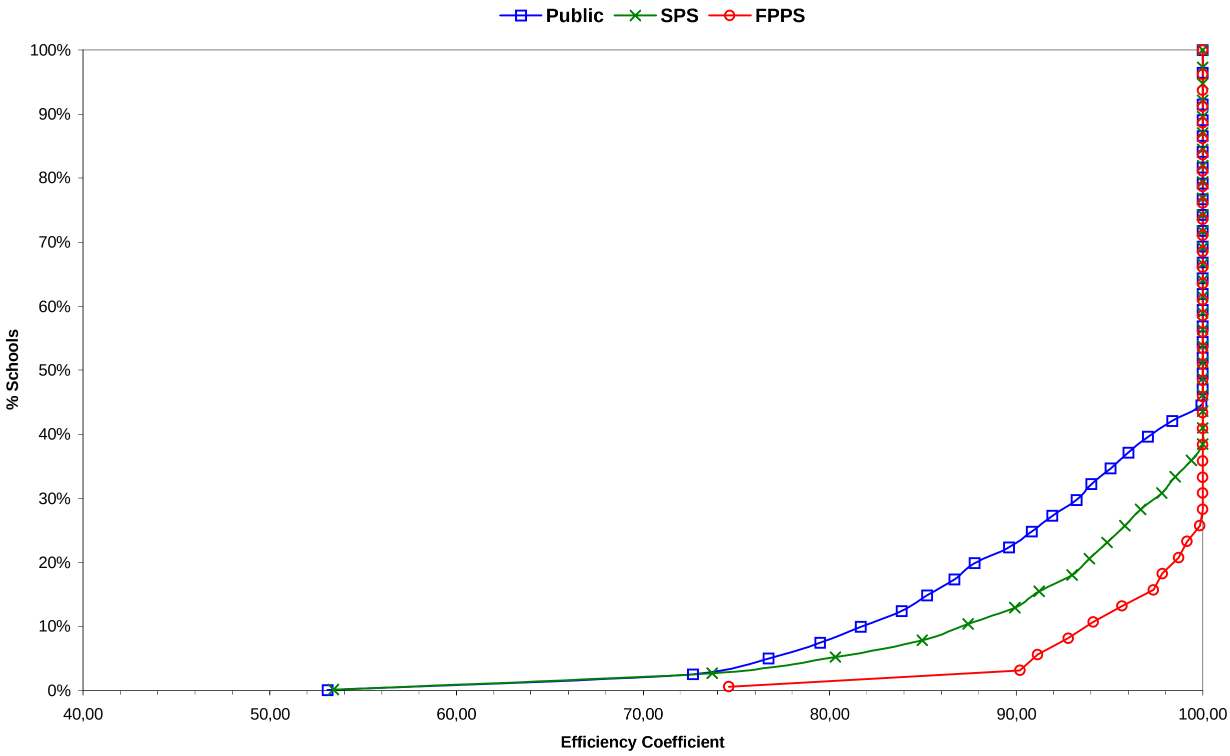


Figure 4
Cumulative Efficiency by Type of School. DEA (BCC). Fourth Grade 1996.



APPENDIX

I. Details of the Variables

Socioeconomic Level A:	Schools in which most parents have completed secondary education, or have gone on to higher education (complete or incomplete); monthly educational spending greater than 25,082 pesos (US$54.53 of 1996).
Socioeconomic Level B:	Schools in which most parents have higher, secondary or primary education, complete or incomplete, and whose monthly educational expenses are from 13,210 (US$28.72) to 25,081 pesos.
Socioeconomic Level C:	Schools where parents have incomplete secondary education, or complete primary education, or less, and whose educational expenses are between 5,284 (US$11.49) and 13,209 pesos.
Socioeconomic Level D:	Schools where parents have incomplete primary education, or less, and whose educational expenses are less than 5,283 pesos.
Vulnerability Index:	Index calculated by Junaeb for every school, which includes anthropometric measures such as weight, height, medical and dental needs of the students, as well as measures of mothers' education levels. The index is calculated using a logistic regression that provides the weighting factors for each of the indicators under consideration. The dependent variable is the number of children who need school meals, according to their teachers and the independent variables are the percentage of children who are underweight, undersize, with unsatisfied dental and medical needs and the mothers' educational level. The index ranges from 0 to 100: a school with a vulnerability index of 0 does not serve a vulnerable population, while an index of 100 means that all students at the school have unmet needs.
Geographical Index A:	Large cities with good accessibility.
Geographical Index B:	Medium and small cities with good accessibility.
Geographical Index C:	Medium and small cities with poor or fair accessibility, and marginal urban areas with poor, fair or good accessibility.
Geographical Index D:	Semi-rural areas with poor, fair or good accessibility, and rural areas with fair or good accessibility.
Geographical Index E:	Rural areas with minimal accessibility and areas with minimal or poor accessibility.
FPPS	Fee-Paying Private Schools
SPS	Subsidized Private Schools
Public	Public Schools

II. Descriptive statistics, 4th Grade 1996

Table A.1
Total Sample

Variable	Mean	Std.Dev.	Minimum	Maximum	Cases
Y96	68.7166950	11.0317657	32.4200000	93.8500000	2000
NSEA	.700000000E-01	.255210827	.000000000	1.00000000	2000
NSEB	.204000000	.403069761	.000000000	1.00000000	2000
NSEC	.523000000	.499595634	.000000000	1.00000000	2000
NSED	.203000000	.402333116	.000000000	1.00000000	2000
EXPER	16.3488050	6.86829721	.540000000	49.6400000	2000
TAP	21.9052800	7.33770090	2.17000000	63.2300000	2000
NUMPROF	20.9725000	18.0144943	1.00000000	168.000000	2000
IGA	.409500000	.491864529	.000000000	1.00000000	2000
IGB	.105500000	.307273426	.000000000	1.00000000	2000
IGC	.112000000	.315445325	.000000000	1.00000000	2000
IGD	.276500000	.447378837	.000000000	1.00000000	2000
IGE	.965000000E-01	.295349565	.000000000	1.00000000	2000
EDPA	.703000000	.457050815	.000000000	1.00000000	2000
ESH	.140000000E-01	.117519809	.000000000	1.00000000	2000
ESM	.320000000E-01	.176044017	.000000000	1.00000000	2000
EMIX	.948500000	.221070546	.000000000	1.00000000	2000
MAT96	479.633000	471.116491	9.00000000	4939.00000	2000
URB	.636500000	.481127313	.000000000	1.00000000	2000
VULNE96	45.1339400	33.9435643	.000000000	100.000000	2000

Note: All results based on non missing observations

Table A.2
Sample of Public Schools

Variable	**Mean**	**Std.Dev.**	**Minimum**	**Maximum**	**Cases**
Y96	65.6102100	9.62485017	33.6400000	93.0400000	2000
NSEB	.920000000E-01	.289098234	.000000000	1.00000000	2000
NSEC	.630000000	.482925055	.000000000	1.00000000	2000
NSED	.278000000	.448125438	.000000000	1.00000000	2000
EXPER	18.5598150	5.55245511	1.60000000	49.6400000	2000
TAP	21.1971600	6.04756922	2.17000000	51.8800000	2000
MAT2	351300.112	767720.878	64.0000000	19307236.0	2000
IGA	.266000000	.441974740	.000000000	1.00000000	2000
IGB	.835000000E-01	.276705679	.000000000	1.00000000	2000
IGC	.125000000	.330801625	.000000000	1.00000000	2000
IGD	.415500000	.492931274	.000000000	1.00000000	2000
IGE	.110000000	.312968009	.000000000	1.00000000	2000
EDPA	.674500000	.468678546	.000000000	1.00000000	2000
ESH	.550000000E-02	.739762546E-01	.000000000	1.00000000	2000
ESM	.125000000E-01	.111130216	.000000000	1.00000000	2000
EMIX	.981000000	.136558867	.000000000	1.00000000	2000
MAT96	411.644500	426.544121	8.00000000	4394.00000	2000
VULNE96	59.4749850	28.5205691	.000000000	100.000000	2000

Note: All results based on non missing observations

Table A.3
Private Subsidized Schools

Variable	Mean	Std.Dev.	Minimum	Maximum	Cases
Y96	70.2039973	11.3255878	30.4900000	92.0200000	1481
NSEA	.600945307E-01	.237742176	.000000000	1.00000000	1481
NSEB	.361242404	.480522881	.000000000	1.00000000	1481
NSEC	.461174882	.498658712	.000000000	1.00000000	1481
NSED	.117488184	.322109869	.000000000	1.00000000	1481
EXPER	12.4089332	6.85466624	.330000000	46.7500000	1481
TAP	25.4181972	8.27852719	3.77000000	63.2300000	1481
NUMPROF	20.9777178	15.7855618	1.00000000	135.000000	1481
IGA	.575286968	.494466340	.000000000	1.00000000	1481
IGB	.140445645	.347566157	.000000000	1.00000000	1481
IGC	.126941256	.333019604	.000000000	1.00000000	1481
IGD	.877785280E-01	.283068122	.000000000	1.00000000	1481
IGE	.695476030E-01	.254468971	.000000000	1.00000000	1481
EDPA	.724510466	.446911526	.000000000	1.00000000	1481
ESH	.209318028E-01	.143204433	.000000000	1.00000000	1481
ESM	.546927752E-01	.227456389	.000000000	1.00000000	1481
EMIX	.914922350	.279091451	.000000000	1.00000000	1481
MAT96	548.280891	496.942946	10.0000000	5814.00000	1481
URB	.846725186	.360373884	.000000000	1.00000000	1481
VULNE96	28.3597974	29.2233798	.000000000	100.000000	1481

Note: All results based on non missing observations

Table A.4
Private Fee-Paying Schools

Variable	Mean	Std.Dev.	Minimum	Maximum	Cases
Y96	84.3871259	6.24002496	49.3200000	95.7700000	421
NSEA	.603325416	.489789421	.000000000	1.00000000	421
NSEB	.391923990	.488760681	.000000000	1.00000000	421
NSEC	.475059382E-02	.688424497E-01	.000000000	1.00000000	421
NSED	.000000000	.000000000	.000000000	.000000000	421
EXPER	11.4313777	6.37093923	.630000000	47.4000000	421
TAP	13.4230404	5.09707598	2.96000000	35.2800000	421
NUMPROF	38.3681710	26.0351380	3.00000000	194.000000	421
IGA	.807600950	.394653787	.000000000	1.00000000	421
IGB	.149643705	.357146250	.000000000	1.00000000	421
IGC	.403800475E-01	.197083130	.000000000	1.00000000	421
IGD	.237529691E-02	.487370179E-01	.000000000	1.00000000	421
IGE	.000000000	.000000000	.000000000	.000000000	421
EDPA	.847980998	.359466482	.000000000	1.00000000	421
ESH	.475059382E-01	.212971501	.000000000	1.00000000	421
ESM	.736342043E-01	.261485407	.000000000	1.00000000	421
EMIX	.869358670	.337408636	.000000000	1.00000000	421
MAT96	532.275534	426.147521	40.0000000	2577.00000	421
URB	.988123515	.108459095	.000000000	1.00000000	421
VULNE96	.367102138	5.08848501	.000000000	100.000000	421

Note: All results based on non missing observations

III. Non parametric tests

Table A.5
Stochastic Frontier Model; Values of Wilcoxon-Mann-Whitney and Kolmogorov-Smirnov tests

Test	W-M-W[27]	K-S[28]
SPS-Public	-4.16	2.75
FPPS-SPS	-3.39	1.97
FPPS-Public	-5.85	3.67

Table A.6
DEA (BCC) Model; Values of Wilcoxon-Mann-Whitney and Kolmogorov-Smirnov tests

Test	W-M-W[1]	K-S[2]
SPS-Public	-4.38	2.41
FPPS-SPS	-3.20	1.61
FPPS-Public	-5.70	2.96

[27] The W-M-W test for large samples uses critical values obtained from a N(0,1).

[28] The K-S test for large samples uses critical values obtained from a χ^2

IV. Stochastic frontier and data envelopment analyses conducting separate exercises for each of the three categories of school

Table A7
Efficiency-Achievement matrix. Stochastic Frontier Model; differentiating by School-Type 4th Grade 1996
(number of schools and percentage in each quadrant)

IV FPPS 17 (4%) SPS 53 (4%) Public 26 (1%)	I FPPS 229 (54%) SPS 782 (53%) Public 1055 (53%)
III FPPS 140 (33%) SPS 471 (32%) Public 783 (39%)	II FPPS 35 (8%) SPS 175 (12%) Public 136 (7%)

Note: Figures for FPPS, SPS and Public relate to the number of schools of each type in each quadrant. The percentages indicate the fraction of the total number of schools of each type located in each quadrant.

Table A8
Efficiency-Achievement Matrix by differentiating by School-Type.
DEA (BCC). 4th Grade 1996
(number of schools and percentage in each quadrant)

IV	I
FPPS 17 (4%) SPS 94 (6%) Public 192 (10%)	FPPS 229 (54%) SPS 736 (50%) Public 896 (45%)
III	II
FPPS 47 (11%) SPS 326 (22%) Public 472 (24%)	FPPS 128 (30%) SPS 325 (22%) Public 440 (22%)

Note: Figures for FPPS, SPS and Public relate to the number of schools of each type in each quadrant. The percentages indicate the fraction of the total number of schools of each type located in each quadrant.

Table A9
Efficiency groups differentiating by School-Type. DEA (BCC)
(number of schools and percentage in each quadrant)

	1 <100	2 =100	TOTAL
FPPS	77 (18.3%)	344 (81.7%)	421 (100%)
SPS	586 (39.6%)	895 (60.4%)	1481 (100%)
Public	898 (44.9%)	1102 (55.1%)	2000 (100%)

Note: group 1 efficiency coefficient higher than the median
group 2 efficiency coefficient lower than the median