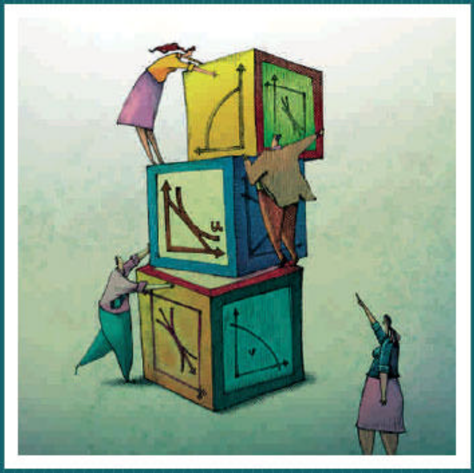
CENTRO DE ECONOMÍA APLICADA

INGENIERÍA INDUSTRIAL · UNIVERSIDAD DE CHILE

DOCUMENTOS DE TRABAJO
Serie Economía



Nº 267 COSTLY INFORMATION ACQUISITION. BETTER TO TOSS A COIN?

MATTEO TRIOSSI

FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS UNIVERSIDAD DE CHILE

Costly information acquisition.

Better to toss a coin?*

Matteo Triossi†

First Draft: September 2007.

This Draft: Christmas 2009.

Abstract

Citizens have little and uneven levels of political knowledge, consistently with the rational ignorance hypothesis. The paper presents a strategic model of common value elections with endogenous information acquisition accounting for these facts. It proves, that contrary to the most optimistic positions about direct democracy, majoritarian elections can fail to aggregate information, when voters have heterogeneous skills. Informational inefficiencies can be partially corrected by improving the skills of the electorate as the population increase or by limiting participation to most competent citizens. The first interpretation is consistent with Rousseau view that an educated citizenry is necessary for a well functioning democracy. The second interpretation provides rational foundations for an epistocratic form of government.

JEL Classification Numbers: C72, D72, D82.

Keywords: Costly Information Acquisition, Condorcet Jury Theorem.

*I am grateful to Sophie Bade, Felipe Balmaceda, Theo Diasakos, Juan Escobar, Paolo Ghirardato, Laurent Mathevet, Fausto Segala and Simone Scotti and Jorge Streb for insightful comments. Many thanks also to the audience of seminars at Collegio Carlo Alberto, Universidad de Valencia, Universidad de las Islas Baleares, Universidad de Chile, Universidad de San Luis, Universiad Diego Portales and participants to the 9th International Meeting of the Society for Social Choice and Welfare, to SECHI Meeting 2008, to IX JOLATE Meeting and XIII LAMES-LACEA Meeting in Rio de Janeiro for useful suggestions. Financement from Project Fondecyt Iniciación nr.11080132 and from SEJ2005-01481/ECON is aknowledged.

†CEA, Departamento de Ingenieria Industrial, Universida de Chile, Av. Republica 701, Santiago, Chile. E-mail: mtriossi@dii.uchile

1 Introduction

Electors have little and uneven knowledge about policies, economic conditions and the backgrounds of elected governmental officials (see, for instance, Delli Carpini and Keeter, 1996, Blendon et al. 1997, Nannestad and Paldam, 2000). This facts is consistent with the rational ignorance hypothesis as formulated by Schumpeter (1950) and Downs (1957): since each vote has little impact on the outcome of a large election and information acquisition is costly, individuals have little incentives to acquire information. Determining the consequences of this hypothesis has important implications about the quality of democratic deliberations.

There are two contrasting positions on this issue. According to the most optimistic view, the voting outcome will be able to reflect the public interest even when most individuals are poorly informed. Though people may lack knowledge and skills, this void will be filled by the self-regulating character of democracy. In the process of preference aggregation, the more or less random opinion of poorly informed voters would cancel out. If each voter is more likely to be right than wrong, then the probability that large elections will reach the best decision will approach one. This statement constitutes the so called Condorcet Jury Theorem (Condorcet 1785). Grofman and Feld (1988) present an interpretation of Rousseau theory of general will under the light of the Condorcet Jury Theorem (see also Young 1988): by aggregating the information dispersed among poorly informed voters, the elections would be able to discover the general will. The Condorcet Jury Theorem has fascinated political scientists, legal scholars (see Edelman 2002) and game theorists alike and contributed to provide epistemic basis to democratic theory (see Cohen 1986 and Coleman and Ferejohn 1986).

At the same time, scholars have long feared that democracy might not function if citizens are not informed enough. Rousseau, while lodging a strenuous defense of direct democracy, denied that elections would always yield results according with the common good, since those deliberations may reflect insufficient information.

> "It follows from what has gone before that the general will is always right and tends to the public advantage; but it does not follow that the deliberations of the people are always equally correct. [...] If, when the people, being furnished with adequate information, held its deliberations, the citizens had no communication one with another, the grand total of the small differences would

always give the general will, and the decision would always be good." [1]

Actually, many opposing egalitarian and unmediated democracy assert that citizens do not have enough knowledge or skill to govern. One of the most fierce objections to democracy is presented by Plato, in the sixth book of "The Republic", where he compares the state to a ship, which needs an experienced captain in order to accomplish a safe and successful journey. An ignorant and untrained person at the helm of a ship would endanger vessel, cargo, crew and passengers alike. Democratic government does not work because ordinary people have not learned how to run the ship of state and they are not inclined to acquire such knowledge. In a similar way Stuart Mill opined that the quality of democratic decisions might be endangered by uneducated voters:

> "It is not useful, but hurtful, that the constitution of the country should declare ignorance to be entitled to as much political power as knowledge."[2]

These arguments are used by Plato to defend the need of a government of philosophers and by Stuart Mill to sustain a form of democracy where the educated class is given more voting power. This is what Estlund (2008) calls "epistocracy of the educated thesis".[3] Brennan (2009) arguments that, while no citizen should be denied voting rights, people lacking the necessary competence should abstain not to "pollute" the outcome of democratic deliberation.

There is a vast empirical literature attempting to assess the extent to which political judgments and deliberations would differ if voters were well informed (see Bartels 1996, Althaus 1998, 2003, Gilens 2001, Blais et al. 2009, Nordin 2009). According to Althaus (2003) "Knowledge does matter, and the way it is distributed in society can cause collective preferences to reflect disproportionately the opinions of some groups more than others. Sometimes collective preferences seem to represent something like the will of the people, but frequently they do not". Most of this studies find systematic differences among the way people do vote and the way people would have voted were they informed. More importantly, they find that electoral outcomes would have been different were voters informed.

According to empirical evidence the degree of political knowledge is positively correlated with a commonly

[1] The Social Contract, book 2, chap. 3.
[2] Considerations on representative government, chap. 8.
[3] A form of government that he rejects.

accepted indicator of sophistication as education: the more educated are the citizens the more information they have (see, for instance, Blendon et al. 1997, Nannestad and Paldam, Lau and Redlawsk 2006, Nordin 2009). This evidence suggests that the observed heterogeneity in political knowledge reflects an heterogeneity in skills and that people possess limited political knowledge because they need to invest effort to process the available information (see Aidt 2000).

The objective of this paper is to analyze the problem from a rational choice perspective. We introduce a model of common value elections with two candidates, accepting Rousseau's theory of the general will in the interpretation of Grofman and Feld (1988). Citizens agree about which candidate is the best one at each state of the world but, at the moment of the vote, they do not know the state of the world. They do not have access to a free and reliable font of information and need to invest effort to process the information they are exposed to, so information acquisition is costly. Obtaining more precise information entails higher costs. Voters have different ability in processing information so so less skilled electors must invest more effort in order to extract the same amount of information, which reflects in higher costs of acquiring information. The quantity of information each voter acquires will be determined endogenously at equilibrium.

We begin the analysis by proving the validity of the rational ignorance hypothesis. Information is a public good and a large population amplifies the free riding problem by reducing the the probability any voter is decisive. Then, we pass to characterize optimal strategies and characterize them in an intuitive way: voters acquire information equating marginal gains to marginal costs. Although simple, these preliminary results prove that the model is able to account for three empirical relevant facts: (i) on average, every voter is poorly informed; (ii) only a small fraction of the electorate is informed; (iii) the less skilled are the electors the less information they acquire so the distribution of information among voters is uneven.

We pass investigate the existence of equilibria with information acquisition and their information aggregation properties. We prove that a bias in favor of one of the two candidates makes the free riding problem excruciating: an equilibrium with information acquisition exists if and only if the expected gains from electing the best candidate the two states of the world are equal. At equilibria with information acquisition, every elector is, on average, more likely to be right than wrong, but the probability a probability the best candidate is selected approaches one half. Elections perform not better than the toss of a fair coin.

A natural question is whether access to cheaper information can alleviate this informational failure.

Reducing the marginal costs of acquiring information of the most skilled electors should induce voters to acquire larger amounts of information. In the model, costs depend both on the quality of information and on the skills of citizens. We show that, in order to obtain more permissive results, it is necessary to reduce marginal costs in both directions. Under generic conditions, equilibria with information acquisition exist for any prior bias in favor of one of the two candidates. Furthermore, when there are many electors the probability the best candidate is elected is approximately one, when there are many electors. Thus the Condorcet Jury Theorem holds in its strongest form. In other cases equilibria with information acquisition exist if the bias in favor of one of the two candidates is not too large. The probability of electing the best candidate is boundedly above one half. So, the Condorcet Jury Theorem holds in a weak form.

We continue by considering the overall efficiency of the elections and prove that elections are efficient if and only if they perfectly aggregate information. Indeed, it is always possible to find a profile of strategy such that the probability of electing the best candidate approaches one and the aggregate costs approaches zero when there are many electors. However, such strategies are not equilibrium ones: in a strategic setup voters do not internalize the informational externality.

The claim of many opposing direct democracy is that the general electorate is not competent enough to vote. Thus, we consider a simple model of epistocracy where only wiser citizens are allowed to vote. Actually, it is worth to observe that many political systems have worked in an pseudo-epistocratic way before allowing universal suffrage. Indeed many democracies have excluded a consistent part of the citizenry from voting for long time on the basis of census and literacy. For instance, in US, Southern States employed literacy tests as part of the voter registration process since the late nineteenth century. Only in 1965, the Voting Rights Act banished such practices. In many Latin American and European Countries only literate citizens had voting rights. Italy introduced universal male suffrage only in 1919. Chile has allowed literate women to vote since 1949, but universal suffrage was introduced in 1970. In these cases expansions of the electoral basis monitored closely the expansion of literacy, which is as an imperfect measure of political competence and was highly correlated with census.

We prove that if the least skilled citizens who is allowed to vote is properly selected, the probability that the best candidate is selected surpasses one half when there are many electors. We characterize the conditions that allow perfect information aggregation under an epistocratic government. The results might seem pessimistic: the outcome of elections is improved by restricting voting rights. This interpretation is overly negative. A more genuine one is probably closer to Rousseau thought. Indeed, the same result can be obtained by improving voters' skills. Such a reading agrees with the idea that only an educated citizenry is vital for a well functioning democracy.

The key state-variable in this paper is the average probability that a voter votes for the best candidate. This is the notion of competence considered in the early literature about the Condorcet's Jury Theorem.[4] These authors assume that electors are naive: they always vote according to the signal they receive (see Berg 1993, Berend and Paroush 1998, Ladha 1992, 1993). While this approach has been successful in proving the Condorcet Jury Theorem under rather general conditions about the distribution of the signals, it has neglected that voting according to the signal, is not necessarily in the best interest of every player, which is it is not necessarily an equilibrium behavior. This issue have been raised more recently by Austen-Smith and Banks (1996). Thus, literature has shifted to study the problem under sophisticated voting. Remarkable examples are Austen-Smith and Banks (2006), Feeddersen and Pesendorfer (1997) and Myerson (1998). In this case, the Condorcet Jury Theorem holds under more restrictive conditions.

Most of these models take the informational structure as given independent on the size of the electorate and do not address the question on how citizens acquire the information they use to vote. However, consistently con the rational ignorance hypothesis, the larger is the electorate the less information voters should acquire. The issue is not at all irrelevant. Paroush (1997) and in Yariv (2004) considered models where the amount of information the electors own is still exogenous but varies with the size of the electorate. They show that, if the quality of information decreases too fast, elections fail to aggregate information, even when voters are more likely to receive the correct signal than the wrong one.[5]

Martinelli (2006) has been the first to endogeneize the informational structure of the problem. He considers a model with costly information acquisition, where voters can acquire information of different quality but they are all equally skilled, which is they all have the same cost function. In a companion paper (Martinelli 2007) he allows heterogeneity in information acquisition costs, but voters can acquire information of one given quality.[6] In Martinelli (2006) voters acquire information of decreasing quality but every elector acquires the

[4]In this literature competence is totally exogenous. See below.
[5]While Paroush (1997) consider a model with naives voters, Yariv (2004) assumes that voters are strategic.
[6]Which does not depend on the size of the electorate.

same quality of information (they have the same costs). In Martinelli (2007) a decreasing part of the electorate acquire information but the quality of information acquired is always the same. Both works conclude that (at least partial) information aggregation in large election is always possible, but cannot account simultaneously for uneven and positive levels of information in the electorate. While we focus on heterogeneity in skills to explain observed heterogeneity in political knowledge, Oliveros (2007, 2009) takes a complementary approach: he assumes that voters have different risk attitudes, but they are equally skilled and prove results similar to Martinelli (2006).

The structure of the article is the following. Section 2 introduces the model. Section 4 studies the validity of the rational ignorance hypothesis. Section 5 studies the existence of equilibria with information acquisition and their information aggregation properties. Section 5 studies the asymptotic efficiency of equilibria. Section 6 considers a simple model of epistocracy. Section 7 concludes. The proofs are in the appendix.

2 The Model

An odd number $N = 2n + 1$ of voters have to choose among two candidates A and B, by simple majority. There are two possible states of the world, a and b. The prior probability of state a is $q_a = q \in (0, 1)$ and the prior probability of state b is $q_b = 1 - q$. Voters have common preferences and agree that A is the best candidate when the state is a and B is the best candidate when the state is b. Let $C \in \{A, B\}$ and let $\omega \in \{a, b\}$. The utility a voter derives from the election of candidate C at state ω is denoted by $U(C, \omega)$. The gains from electing the best candidate at state a and at state b are denoted by $\Delta U_a = U(A, a) - U(B, a) > 0$ and by $\Delta U_b = U(B, b) - U(A, b)$, respectively. Thus, $q_\omega \Delta U_\omega$ measures the expected utility gain from taking the right decision at state ω for $\omega = a, b$.

Electors do not know the true state of the world and before voting they independently receive signal $s \in \{s_a, s_b\}$. Before receiving the signal they can acquire information of quality $x \in \left[0, \frac{1}{2}\right]$. When a voter receives a signal of quality x the likelihood of receiving the signal s_ω at state $\omega = a, b$ is $p(s_\omega \mid \omega, x) = \frac{1}{2} + x$. Voters have different acquisition costs, that reflects their different skills in acquiring information. Types are indicized by a nonnegative real number $\alpha \in [0, 1]$. They are distributed independently across the electorate according to the same distribution F, with continuous density function $f : [0, 1] \to \mathbf{R}$, where $f(0) > 0$.

An elector of type α faces a cost $C(x,\alpha)$ to purchase information of quality x. Thus, if candidate C has been elected at state ω, the utility of a voter of type α who has acquired information of quality x is $U(C,\omega) - C(x,\alpha)$.

The cost function C is of class $C^2\left(\left[0,\frac{1}{2}\right) \times [0,1]\right)$ and $C_x(0,0) = 0$. Acquiring a positive amount of information has a strictly positive cost, while acquiring no information entails no costs: $C(0,\alpha) = 0$ and $C(x,\alpha) > 0$ for all $x \in \left(0,\frac{1}{2}\right]$ and for all $\alpha \in [0,1]$. For every type, gathering information of higher better quality has increasingly higher costs, $C_x(\alpha,x) > 0$ and $C_{xx}(\alpha,x) > 0$ for all $\alpha > 0$ and for all $x > 0$.[7,8] Higher types have increasingly higher costs, which is $C_\alpha(\alpha,x) > 0$, and $C_{\alpha x}(\alpha,x) > 0$ for all $\alpha > 0$ and for all $x > 0$. Finally we assume, that there are k and t such that $C_{x^{(t)}}(0,0) > 0$ and $C_{x\alpha^{(t)}}(0,0) > 0$.

A strategy specifies how much information a voter of a given type acquires and for which candidate she votes, conditional on the signal received.

Definition 1 *An individual strategy consists of a information acquisition strategy* $x : [0,1] \to \left[0,\frac{1}{2}\right]$ *and of a voting strategy* $v : [0,1] \times \{s_a,s_b\} \to \{A,B\}$ *such that* x *is measurable and* $v(\cdot,s)$ *is measurable for* $s \in \{s_a,s_b\}$.[9]

A strategy of player i is denoted by (x_i,v_i), a strategy profile $(x_i,v_i)_{i=1,\dots,2n+1}$ is denoted by (X,V) and $(X,V)_{-i}$ is the coalitional strategy of all voter but i. Consider a voter of type α who has acquired information of quality x, receives signal $s \in \{s_a,s_b\}$ and votes for candidate $v \in \{A,B\}$. We denote by $U\left(v \mid x,s,(X,V)_{-i}\right) - C(\alpha,x)$ the expected utility of her decision at the moment of the vote. Similarly, consider a voter of type α who acquires information of quality x, votes for candidate v_a, when she receives signal s_a, votes for candidate v_b, when she receives signal s_b. Let $v = (v_a,v_b)$ We denote $U\left(x,v \mid (X,V)_{-i}\right) - C(\alpha,x)$, the expected utility of her decision.

The equilibrium concept we employ is symmetric Bayesian equilibrium. At a symmetric Bayesian equilibrium every player employs the same strategy. Individuals vote optimally conditional on the signal received, on the

[7] We use the following notation for partial derivatives: $C_{xx}(\alpha,x) = \frac{\partial^2 C(x,\alpha)}{\partial x^2}$ and $C_{x^{(t)}\alpha^{(k)}}(x,\alpha) = \frac{\partial^{k+t} C(x,\alpha)}{\partial x^{(t)}\partial\alpha^{(k)}}$ for every non-negative integers k and t let.

[8] The conditions imposed on C imply that $C_x(0,x) > 0$ for all $x > 0$. Assume by contradiction $C_x(0,x) = 0$ for some $x > 0$. Notice that $C_{xx}(0,\xi) \geq 0$ for all $\xi \geq 0$.Thus $C(0,\xi) = 0$ for all $\xi \leq x$. From the intermediate value theorem there exists $\xi \in (0,x)$ such that $C(0,x) = C(0,x) - C(0,0) = C_x(0,\xi)\,x = 0$, which yields a contradiction because, by assumption $C(0,x) > 0$.

[9] A more rigorous definition would make the voting strategy contingent on the investment $v : [0,1] \times \left[0,\frac{1}{2}\right] \times \{s_a,s_b\} \to \{A,B\}$. However, only the signal is disclosed before the voting decision is taken so the definition presented is without loss of generality.

investment and on the strategy of the other voters. Information acquisition and voting strategies are ex ante optimal, given the strategy of the other voters.

Definition 2 *A symmetric Bayesian equilibrium (SBE from now on) is given by a strategy* $(\hat{x}, \hat{v})$ *such that the profile* $\left(\hat{X}, \hat{V}\right) = (\hat{x}, \hat{v})_{i=1,...2n+1}$ *satisfies:*

1. $U\left(v\left(\hat{x}\left(\alpha\right), s\right) \mid \hat{x}\left(\alpha\right), s, \left(\hat{X}, \hat{V}\right)_{-i}\right) \geq U\left(v \mid \hat{x}\left(\alpha\right), s, \left(\hat{X}, \hat{V}\right)_{-i}\right)$ *for all* $\alpha \in [0,1]$, *for all* $v \in \{A, B\}$ *and for all* $s \in \{s_a, s_b\}$.
2. $U\left(\hat{x}\left(\alpha\right), \hat{v} \mid \left(\hat{X}, \hat{V}\right)_{-i}\right) - C\left(\hat{x}\left(\alpha\right), \alpha\right) \geq U\left(x, v \mid \left(\hat{X}, \hat{V}\right)_{-i}\right) - C\left(x, \alpha\right)$ *for all* $\alpha \in [0,1]$ *for all voting rules* v.

We will consider equilibria in pure strategy only. Notice that a SBE always exists: no voter acquires information and everybody votes for the same alternative independently on the type and on the signal received. The concern of the paper are equilibria with information acquisition, where a non-zero measure of types acquire information.

Definition 3 *A SBE* $(\hat{x}, \hat{v})$ *exhibits information acquisition* $F\left(\{\alpha : x\left(\alpha\right) > 0\}\right) > 0$.

When no ambiguity is possible, we omit any reference to $(X, V)_{-i}$ so, for instance, we will write $U\left(x, v\right)$ for $U\left(x, v \mid (X, V)_{-i}\right)$.

3 Optimal Strategies and Equilibria: Characterization

A voter can affect the outcome of the election only when she is pivotal, which is when there are exactly n electors voting for candidate A and n electors voting for candidate B. For $i = 1, ..., 2n+1$, we denote by $p_\omega = p\left(piv \mid \omega, (X, V)_{-i}\right)$, the probability voter i is pivotal at state $\omega \in \{a, b\}$. The utility that she derives from a voting strategy (v_a, v_b), net of the information acquisition costs is

$$\sum_{\omega \in \{a,b\}} p_\omega q_\omega \left[U\left(v_a, \omega\right) p\left(s_a|\omega\right) + U\left(v_b, \omega\right) p\left(s_b \mid \omega\right)\right] + U_{-i}.$$

where $U_{-i} = U_{-i}\left((X,V)_{-i}\right)$ depends only on the strategy of the other voters. A voter decision affect her own utility only when she is pivotal which accrues $\sum_{\omega\in\{a,b\}} p_\omega q_\omega \left[U(v_a,\omega)\, p(s_a|\omega) + U(v_b,\omega)\, p(s_b|\omega)\right]$ to her. Notice that the term $p(s_\omega|\omega')$ depends on the information acquisition strategy of the the voter: $p(s_\omega|\omega) = \frac{1}{2} + x$, where x is the quantity of information acquired by the voter. If the probabilities of being pivotal are very small almost her utility will be almost independent on her strategy. Thus, she will have little incentives in acquiring information.

Let us study the optimal strategy which depends only on pivotal probabilities.

Let's start from behavior of electors who acquire information, if any. The reader can easily check that any voter who finds it optimal to acquire information strictly prefers to follow the signal rather than to vote against it. A voter who ignores the signal is strictly better off by not acquiring information. The benefit from acquiring x units of information and following the signal $U(x, A, B)$ can be written as

$$U(x,A,B) = (p_a q_a \Delta U_a + p_b q_b \Delta U_b)\left(\frac{1}{2} + x\right) + p_a q_a U(B,a) + p_b q_b U(A,b) - C(\alpha, x) + U_{-i}.$$

Notice that the marginal gain of acquiring information is the expected gain from voting for the best alternative at each state:

$$p_a q_a \Delta U_a + p_b q_b \Delta U_b.$$

If a voter of type α finds it optimal to acquire a positive amount x of information, she equates the marginal gains from acquiring information to its marginal costs or she acquires the information of the highest quality $(x = \frac{1}{2})$ if its marginal costs is lower than marginal gains. Thus, $x > 0$ must solve:

$$(p_a q_a \Delta U_a + p_b q_b \Delta U_b) = C_x(\alpha, x) \tag{1}$$

if $C_x\left(\alpha, \frac{1}{2}\right) > q_a \Delta U_a + p_b q_b \Delta U_b$ and $x = \frac{1}{2}$, if $C_x\left(\alpha, \frac{1}{2}\right) \leq q_a \Delta U_a + p_b q_b \Delta U_b$. Let $\alpha_1 = \alpha_1(p_a, p_b)$ be the highest type such that Equation 1 has a solution, α_1 solves

$$(p_a q_a \Delta U_a + p_b q \Delta U_b) - C_x(\alpha_1, 0) = 0 \tag{2}$$

if any such α_1 exists and set $\alpha_1 = \alpha_1(p_a, p_b) = 1$ otherwise. If $\alpha_1 < 1$, for a voter of type α_1 the marginal utility of acquiring information is the same that the marginal costs of acquiring no information. Thus, for types $\alpha \geq \alpha_1$ it is never optimal to acquire information.

Define $x(\alpha, p_a, p_b)$ as the solution to equation 1 for $C_x\left(\alpha, \frac{1}{2}\right) \geq q_a \Delta U_a + p_b q_b \Delta U_b \geq C_x(\alpha, 0)$. Set $x(\alpha, p_a, p_b) = \frac{1}{2}$ for $C_x\left(\alpha, \frac{1}{2}\right) < q_a \Delta U_a + p_b q_b \Delta U_b$ and set $x(\alpha, p_a, p_b) = 0$ for $C_x(\alpha, 0) > q_a \Delta U_a + p_b q_b \Delta U_b$ (which is for $\alpha > \alpha_1(p_a, p_b)$). Let $(p_a, p_b) \neq (0,0)$ and $0 \leq \alpha < \alpha_1(p_a, p_b)$. A straightforward application of the implicit function theorem yields:

$$x_\alpha(\alpha, p_a, p_b) = -\frac{C_{x\alpha}(\alpha, x)}{C_{xx}(\alpha, x)}$$

$$x_{p_\omega}(\alpha, p_a, p_b) = \frac{q_\omega \Delta U_\omega}{C_{xx}(\alpha, x(\alpha, p_a, p_b))} \text{ for } \omega = a, b$$

$$x_{\Delta U_\omega}(\alpha, p_a, p_b) = \frac{p_\omega q_\omega}{C_{xx}(\alpha, x(\alpha, p_a, p_b))} \text{ for } \omega = a, b$$

and $\lim_{(p_a, p_b) \to (0,0)} x(\alpha, p_a, p_b) = 0$ for every α. It follows that types with lower costs acquire better information and that an higher probability of being decisive provides higher incentives to acquire more precise information. Finally, the larger are the stakes the better is the information acquired by electors.

The function $\alpha_1(p_a, p_b)$ is differentiable in the interior of the set where 2 is satisfied. Its partial derivatives are

$$\alpha_{1p_\omega}(p_a, p_b) = \frac{q_\omega \Delta U_\omega}{C_{x\alpha}(\alpha(p_a, p_b), 0)}$$

$$\alpha_{1\Delta U_\omega}(p_a, p_b) = \frac{q_\omega \Delta U_\omega}{C_{x\alpha}(\alpha(p_a, p_b), 0)}$$

for for $\omega = a, b$. Higher pivotal probabilities and larger stakes guarantee that a larger fraction of the electorate acquires information. Notice that $\lim_{(p_a, p_b) \to (0,0)} \alpha_1(p_a, p_b) = 0$ f

Up to now we have characterized, optimal information acquisition strategies, for players who do acquire information. We will see that in every equilibrium with information acquisition all voters of type $\alpha < \alpha_1$, will find it optimal to acquire a positive amount of information, but in general it is not the case. Voters with better skills than α_1 might find it optimal to ignore the signal and not buying information at all. Thus we have to determine which types prefer to acquire information to vote uninformed. The payoff of an uninformed elector who votes for alternative A can be written as:

$$\frac{p_a q_a \Delta U_a + p_b q \Delta U_b}{2} + \frac{p_a q_a \Delta U_a - p_b q_b \Delta U_b}{2} + p_a q_a U(B,a) + p_b q_b U(A,b) + U_{-i}.$$

The payoffs of an uninformed elector who votes for alternative A is:

$$\frac{p_a q_a \Delta U_a + p_b q \Delta U_b}{2} + \frac{p_b q_b \Delta U_b - p_a q_a \Delta U_a}{2} + p_a q_a U(B,a) + p_b q_b U(A,b) + U_{-i}.$$

An uninformed elector prefers to vote for candidate A if and only if the expected gain from voting for the best candidate at state a exceeds the expected gain from voting for the best candidate at state b which is if and only if

$$p_a q_a \Delta U_a \geq p_b q_b \Delta U_b. \tag{3}$$

Thus, an elector of type α finds optimal to acquire information if and only if they payoff from informed voting is higher than the maximal payoff from uninformed voting which is if and only if:

$$(p_a q_a \Delta U_a + p_b q \Delta U_b)\, x(\alpha, p_a, p_b) - C(\alpha, x(\alpha, p_a, p_b)) \geq \frac{|p_b q_b \Delta U_b - p_a q_a \Delta U_a|}{2}. \tag{4}$$

Let $\alpha_2(p_a, p_b) \leq \alpha_1(p_a, p_b)$ be the type who is indifferent between optimally acquiring information and staying uninformed, formally $\alpha_2 = \alpha_2(p_a, p_b)$ is defined as the solution of:

$$(p_a q_a \Delta U_a + p_b q \Delta U_b)\, x(\alpha, p_a, p_b) - C(\alpha, x(\alpha, p_a, p_b)) = \frac{|p_b q_b \Delta U_b - p_a q_a \Delta U_a|}{2} \tag{5}$$

if any exists and $\alpha_2(p_a, p_b) = 1$ otherwise. Set $\alpha(p_a, p_b) = \min\{\alpha_1(p_a, p_b), \alpha_2(p_a, p_b)\}$. Type $\alpha(p_a, p_b)$ is the cutoff type, the supremum of the set of types who find it optimal to acquire information. With abuse of language, but with a more intuitive flavour, one could say that $\alpha(p_a, p_b)$ is the least skilled voter who is willing to acquire information. Given (p_a, p_b), every voter of type $0 \leq \alpha \leq \alpha(p_a, p_b)$ finds it optimal to acquire the amount of information determined by $x(\alpha, p_a, p_b)$. Every voter of type $\alpha > \alpha(p_a, p_b)$ finds it optimal not to acquire information. If $p_b q_b \Delta U_b = p_a q_a \Delta U_a$, electors who do not acquire information are indifferent among the two candidates. Otherwise, they find optimal voting for candidate A if $p_b q_b \Delta U_b < p_a q_a \Delta U_a$ and for candidate B if $p_b q_b \Delta U_b > p_a q_a \Delta U_a$. Every type $\alpha \in [0, \alpha(p_a, p_b)]$ votes for the correct candidate with probability $\frac{1}{2} + x(\alpha, p_a, p_b)$.

It is easy to show that that $\alpha(p_a, p_b) = \alpha_1(p_a, p_b)$ if and only if $p_b q_b \Delta U_b = p_a q_a \Delta U_a$. Thus, the optimal information acquisition strategy is continuous if and only if $p_b q_b \Delta U_b = p_a q_a \Delta U_a$.

Define $\tilde{x}(p_a, p_b)$ as the the expected amount of information acquired by a voter of unknown type, when the probability of being pivotal at states a and b are p_a and p_b, respectively:

$$\tilde{x}(p_a, p_b) = \int_0^{\alpha(p_a, p_b)} x(\alpha, p_a, p_b) f(\alpha) \, d\alpha.$$

Let $\lambda(\alpha, p_a, p_b)$ be the probability a voter of type $\alpha > \alpha(p_a, p_b)$ votes for candidate A.[10] When $\alpha(p_a, p_b) < 1$ define $\lambda(p_a, p_b)$ as the conditional probability a voter of unknown type votes for A, given that she does not acquire information: $\lambda(p_a, p_b)$ solves $\int_{\alpha(p_a,p_b)}^{1} \lambda(\alpha, p_a, p_b) f(\alpha) \, d\alpha = \lambda(p_a, p_b)(1 - F(\alpha(p_a, p_b)))$.[11] Set $\mu(p_a, p_b) = \lambda(p_a, p_b) - \frac{1}{2} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. If all uninformed voters vote for alternative A we have $\lambda(p_a, p_b) = 1$ and $\mu(p_a, p_b) = \frac{1}{2}$. If all uninformed voters vote for alternative B we have $\lambda(p_a, p_b) = 0$ and $\mu(p_a, p_b) = -\frac{1}{2}$.

The probability a voter of unknown type votes for the correct alternative at state a and at state b are:

$$\frac{F(\alpha(p_a, p_b))}{2} + \tilde{x}(p_a, p_b) + \lambda(p_a, p_b) = \frac{1}{2} + \tilde{x}(p_a, p_b) + \mu(p_a, p_b)(1 - F(\alpha'(p_a, p_b)))$$

and

$$\frac{F(\alpha(p_a, p_b))}{2} - \tilde{x}(p_a, p_b) + \lambda(p_a, p_b) = \frac{1}{2} - \tilde{x}(p_a, p_b) - \mu(p_a, p_b)(1 - F(\alpha'(p_a, p_b)))$$

respectively.

The probability a voter is pivotal at state a is

$$P_{a\mu}(p_a, p_b) = \binom{2n}{n} \left\{ \frac{1}{4} - \left[\tilde{x}(p_a, p_b) + \mu(p_a, p_b)(1 - F(\alpha(p_a, p_b)))\right]^2 \right\}^n. \tag{6}$$

[10] As we consider pure strategy only $\lambda(\alpha, p_a, p_b) \in \{0, 1\}$.

[11] $\lambda(p_a, p_b)$ is well defined because strategies are assumed to be measurable.

The probability a voter is pivotal at state b is

$$P_{b\mu}(p_a,p_b)=\binom{2n}{n}\left\{\frac{1}{4}-\left[\tilde{x}(p_a,p_b)-\mu(p_a,p_b)(1-F(\alpha(p_a,p_b)))\right]^2\right\}^n. \tag{7}$$

Given any optimal strategy for uninformed types and corresponding λ, let $\bar{\alpha}=\bar{\alpha}(p_a,p_b)$ be such that be such that $F(\bar{a})-F(\alpha(p_a,p_b))=\lambda(p_a,p_b)(1-F(\alpha(p_a,p_b)))$.

This formulation reduces the problem of finding an equilibrium in finding pivotal probabilities and a "wedge" μ such that (p_a,p_b) are a fixed point of the map $(p_a,p_b)\mapsto(P_{a\mu}(p_a,p_b),P_{b\mu}(p_a,p_b))$.

A SBE equilibrium without information acquisition corresponds to the case $(p_a,p_b)=(0,0)$ and $\mu\in\left\{-\frac{1}{2},\frac{1}{2}\right\}$.

We can thus derive the following characterization.

Proposition 1 *(i) A SBE with information acquisition equilibrium exists if and only if there are* $(p_a,p_b)\in[0,1]^2\setminus\{(0,0)\}$ *and* $\mu\in\left(-\frac{1}{2},\frac{1}{2}\right)$ *such that* $(P_{a\mu}(p_a,p_b),P_{b\mu}(p_a,p_b))=(p_a,p_b)$.

Equilibrium strategies (x,v)*, satisfy:*

1. $(x,v)(\alpha)=(x(\alpha,p_a,p_b,A,B))$ *for* $\alpha\leq\alpha(p_a,p_b)$*,*

2. $(x,v)(\alpha)=(0,A,A)$ *if* $\alpha>\alpha(p_a,p_b)$ *and* $p_aq_a\Delta U_a<p_bq_b\Delta U_b$*,*

3. $(x,v)(\alpha)=(0,A,A)$ *if* $\alpha>\alpha(p_a,p_b)$ *and* $p_aq_a\Delta U_a<p_bq_b\Delta U_b$*,*

4. $(x,v)(\alpha)\in\{(0,A,A),(0,B,B)\}$ *if* $p_bq_b\Delta U_b=p_aq_a\Delta U_a$*,*

5. $\left(\mu+\frac{1}{2}\right)(1-F(\alpha(p_a,p_b)))=F(\bar{\alpha}(p_a,p_b))-F(\alpha(p_a,p_b))$.

Notice that for any $\alpha^*\leq 1$ and for any measurable function $\rho(\alpha)$ with values in $[0,1]$ one there exists $\bar{\alpha}\geq\alpha^*$ satisfying $\int_{\alpha^*}^{\bar{\alpha}}f(\alpha)\,d\alpha=\int_{\alpha^*}^{1}\rho(\alpha)f(\alpha)\,d\alpha$. It follows that there is no loss of generality in considering pure strategy only.

Let (x_n,v_n) be a SBE for an election with $2n+1$ electors, let (p_{an},p_{bn},μ_n) be the corresponding pivotal probabilities and equilibrium wedge, respectively. We denote by $\alpha_n=\alpha(p_{an},p_{bn})$ the cutoff type and by $\tilde{x}_n=\tilde{x}(p_{an},p_{bn})$ the average quality of information acquired by informed voters.

3.1 The rational Ignorance hypothesis

Here, we investigate whether voters have incentives in acquiring information in large electorates, which is we analyze the validity of the rational ignorance hypothesis. Due to heterogeneity in skills, we investigate whether information acquired by individuals voters declines and whether the fraction of agents acquiring information declines, when the electorate grows large. In order to address this questions it suffices to consider equilibrium pivotal probabilities. Notice that

$$P_{a\mu}\left(p_a, p_b\right) \leq \binom{2n}{n} 2^{-2n}$$

for every μ and for every $\left(p_a, p_b\right) \in [0,1]^2$. From Stirling's Formula we have

$$\binom{2n}{n} \approx \frac{2^{2n}}{\sqrt{\pi n}}$$

as $n \to \infty$, so that for $\omega = a, b$

$$P_{\omega\mu}\left(p_a, p_b\right) = O\left(\frac{1}{\sqrt{\pi n}}\right),$$

uniformly in $\left(p_a, p_b, \mu\right)$.[12,13] Thus, as $n \to \infty$ $\alpha_n \to 0$ and $x_n\left(\alpha\right) \to 0$ at equilibrium. Furthermore, $\tilde{x}_n = \int_0^{\alpha_n} x_n\left(\alpha\right) f\left(\alpha\right) d\alpha \leq \frac{1}{2}\int_0^{\alpha_n} f\left(\alpha\right) d\alpha \to 0$ as $n \to \infty$.

In every sequence of SBE the probability a voter is pivotal approaches zero when the population grows large. So even if the every equilibrium might exhibits information acquisition, only vanishing fraction of electors, the ones with the smallest costs, acquire information and this information is itself of vanishing quality.

Proposition 2 *For any sequence of SBE, $\left(x_n, v_n\right)$:*

1. *$lim_{n\to\infty} p_{\omega n} = 0$, for $\omega = a, b$.*

2. *$lim_{n\to\infty} \alpha_n = 0$.*

[12]We use the following notation. Let $f, g : X \to \mathbb{R}$ where X is a metric space. Let $z \in X$. We write $f \approx g$ for $x \to z$ if $lim_{x\to z}\frac{f(x)}{g(x)} = 1$, $f = o\left(g\right)$ for $x \to z$ if $lim_{x\to z}\frac{f(x)}{g(x)} = 0$ and $f = O\left(g\right)$ for $x \to z$ if there exists $C > 0$ such that $\left|f\left(x\right)\right| \leq C\left|g\left(x\right)\right|$ in a neighborhood of z. Let $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$ two sequences of real numbers. We write $a_n \approx b_n$ if $lim_{n\to\infty}\frac{a_n}{b_n} = 1$, $a_n = o\left(b_n\right)$ if $lim_{n\to\infty}\frac{a_n}{b_n} = 0$ and $a_n = O\left(b_n\right)$ if there exists $C > 0$ such that$\left|a_n\right| \leq C\left|b_n\right|$ for n large enough.

[13]Stirling Formula provides an approximation for the factorial function $n!$ as $n! \approx \sqrt{2\pi n} n^n e^{-n}$. See Chow and Teicher (1997).

3. $lim_{n\to\infty} x_n(\alpha) = 0$ *for all* $\alpha \in [0,1]$.

4. $lim_{n\to\infty} \tilde{x}_n = 0$.

Thanks to Proposition 2 we can prove that the condition $C_x(0,0) = 0$ is necessary for a SBE with information acquisition to exist for large electorates. By contradiction assume $C_x(0,0) > 0$ and that a SBE with information acquisition exists. For large enough n, in an equilibrium with information acquisition type $\alpha = 0$ acquires a positive amount of information which satisfies: $(p_{an} q_a \Delta U_a + p_{bn} q_b \Delta U_b) = C_x(0, x_n(0))$. From Proposition 2 the left hand side of the the equation converges to 0 as $n \to \infty$ from Proposition 2, but the right hand side converges to $C_x(0,0) > 0$, a contradiction.

Corollary 1 *For arbitrarily large* n*, a* SBE *with information acquisition exists only if* $C_x(0,0) = 0$.

From now on we will consider cost functions C that satisfy $C_x(0,0) = 0$.

4 Equilibria with information acquisition: existence and aggregation

In this section we tackle two issues the existence of equilibria with information acquisition and its informational efficiency. In particular we will verify whether the Condorcet Jury Theorem holds at least in a weak form. We say that the Condorcet Jury Theorem holds in a weak form if the probability the best candidate is selected in large elections at is bounded above one half at both states. We say that the Condorcet Jury Theorem holds in a strong form if the probability the best candidate is selected in large elections at is about one.

Let's start by stating the conditions under which large elections aggregates information. Consider a SBE (x_n, v_n) with $2n+1$ voters and set $\theta_{na} = \tilde{x}_n + \mu_n(1 - F(\alpha_n))$ and set $\theta_{nb} = \tilde{x}_n - \mu_n(1 - F(\alpha_n))$. The random variable taking value 1 when a voter vote for the best candidate at state ω follows a Bernoulli distribution with probability of success $\frac{1}{2} + \theta_{n\omega}$. Thus, the probability the best candidate is elected at state ω is:

$$P_{\omega n} = \sum_{m=n+1}^{2n+1} \binom{2n+1}{m} \left(\frac{1}{2} + \theta_{n\omega}\right)^m \left(\frac{1}{2} - \theta_{n\omega}\right)^{2n+1-m}.$$

for $\omega = a, b$. Even if $\lim_{n\to\infty} \theta_{n\omega} 0$, the sum does not necessarily go to $\frac{1}{2}$ because the number of the term grows. Actually, if the the De Moivre-Laplace central limit theorem applied, one would conclude that the probability of taking the right decision when n is large can be approximated by:

$$\Phi\left(-\frac{n-(2n+1)\left(\frac{1}{2}+\theta_{n\omega}\right)}{\sqrt{(2n+1)\left(\frac{1}{4}-\theta_{n\omega}^2\right)}}\right) \approx \Phi\left(2\sqrt{2}\sqrt{n}\theta_{n\omega}\right) \tag{8}$$

where Φ denotes the standard normal distribution: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$.

The classic version of the De Moivre-Laplace central limit theorem do not apply here because the parameter of the Bernoulli distribution varies with the number of the agents. However, we can prove that the approximation provided by 8 holds, using the Berry-Esseen Theorem.

Proposition 3 *Let $(x_{n_k}, v_{n_k})_{k\in\mathbb{N}}$ be a sub-sequence of SBE strategies such that*

$$t_\omega = \lim_{k\to\infty} \sqrt{n_k}\left(\theta_{n_k\omega}\right)$$

exist for $\omega = a, b$. Let $P_{\omega k}$ be the probability the right decision is taken at state ω, at the corresponding SBE. Then

$$\lim_{k\to\infty} P_{\omega k} \to \Phi\left(2\sqrt{2}t_\omega\right)$$

for $\omega = a, b$. The Condorcet Jury Theorem holds in a weak form if and only if $t_\omega \in (0, +\infty)$ for $\omega = a, b$. The Condorcet Jury Theorem holds in a strong form if and only if $t_\omega = \infty$ for $\omega = a, b$.

In principle, there are two classes of equilibria with information acquisition. In the first class, uninformed voters are indifferent among the two candidates which is equilibria where $p_a q_a \Delta U_a = p_b q_b \Delta U_b$. In the second class, uninformed voters strictly prefer to vote for one candidate, for candidate A if $p_a q_a \Delta U_a > p_b q_b \Delta U_b$ for candidate B if $p_a q_a \Delta U_a < p_b q_b \Delta U_b$. The next result shows that, in sequences of SBE with information acquisition, uninformed voters are indifferent among the two candidates in large elections.

Lemma 1 *For every sub-sequence $(x_{n_k}, v_{n_k})_{k\in\mathbb{N}}$ of SBE with information acquisition we have:*

$$p_{an_k} q_a \Delta U_a = p_{bn_k} q_b \Delta U_b \tag{9}$$

for large k.

Lemma 1 implies that the cutoff point α coincides with α_1 when there are many voters. Furthermore, it excludes the existence of sequences SBE with information acquisition where the probability that one candidate is elected, independently on the state of the world, approaches one.

From Proposition 3 follows that large elections aggregates information only if the average information owned by voters converges to zero no faster than $\frac{1}{\sqrt{n}}$. Equation1 and Equation 2 imply that the speed of convergence of $\tilde{x}_n$ to zero depends, on how fast marginal costs increase. So the larger is the curvature of the cost function, the faster the information acquired by voters vanishes and the lower is the cut-off point. It will suffice to consider the curvature of C at $(0, 0)$: according to Proposition 2 only the types with the lowest costs acquire information and this is of low quality according . For generic cost functions, either $C_{xx}(0, 0) > 0$ or $C_{x\alpha}(0, 0) > 0$.

In order to understand the main ideas and techniques used, consider the simplest case where $C_{xx}(0, 0) > 0$ and $C_{x\alpha}(0, 0) > 0$. When $q_a \Delta U_a = q_b \Delta U_b$, Lemma 1 implies that $p_{an} = p_{bn} = p_n$ for large n along any sequence of SBE with information acquisition. If $p_{an} = p_{bn} = p_n$ in a SBE with information acquisition, then $\mu_n = 0$ because Equation 6 and Equation 7 hold. This is, half of the voters who do not acquire information vote for A and and half vote for B.[14] Brower's fixed point implies that $P_{a0} = P_{b0}$ has a fixed point such that $(p_n, p_n) \neq (0, 0)$ (the detailed proof is in the appendix), so a SBE with information acquisition exists. For small values of α and x, the marginal cost function is approximately linear in x and α if $C_{xx}(0, 0) > 0$ and $C_{x\alpha}(0, 0) > 0$. Thus, linearizations of Equations1 and 2 provide good linear approximations of $x(\alpha, p_n, p_n)$ and $\alpha(p_n, p_n)$, respectively. Integrating $x(\alpha, p_a, p_b)$ one reaches the conclusion that $\tilde{x}_n = \tilde{x}(p_n, p_n)$ is approximately quadratic in p_n: there exists $C > 0$ such that $\tilde{x}_n \approx C p_n^2$ for large values of n. Since $\lim_{n\to\infty} \tilde{x}_n = 0$, then $\left(1 - 4\tilde{x}_n^2\right)^n \approx e^{-4n\tilde{x}_n^2}$. From Equations 6 we obtain $p_n \approx \frac{e^{-4n\tilde{x}_n^2}}{\sqrt{\pi n}}$. Taking the squares of both sides of the equivalence and replacing p_n^2 by $\frac{\tilde{x}_n}{C}$, we have

$$\frac{\sqrt{n}\tilde{x}_n}{C} \approx \frac{e^{-8n\tilde{x}_n^2}}{\pi\sqrt{n}}$$

which implies $\lim_{n\to\infty} \sqrt{n}\tilde{x}_n = 0$ as $n \to \infty$.

[14] Equivalently, voters who do not acquire information equally randomize between A and B.

In such equilibria, every elector is always more likely to vote for the best candidate than for the worst one. It is the same situation as in earlier works on the Condorcet Jury Theorem (like, for instance, in Grofman at al. 1987, Young 1988, Austen Smith and Banks 1996). There are two important differences. In those works the competence of electors, which is the precision of the signal was exogenou and independent on the dimension of the electorate. Here, the the average precision of the signal is determined endogenously at equilibrium and depends on the number of the voters. In Martinelli (2006 and 2007) average competence is endogenous and depends on the size of the electorate, too. However, in Martinelli 2006 every voters acquires the same amount of information with probability one, because there is no heterogeneity among voters. In Martinelli 2007, voters have different information acquisition costs but there is a unique quality information available, so all informed electors acquire information of the same positive quality which is independent on the size of the electorate. Here the heterogeneity in skills and the possibility of acquiring information of different quality, implies that informed voters present different level of political knowledge, which explains the different results..

In general, Lemma 1 simplifies the problem of finding SBE of large election by reducing the dimension of the problem. It implies that $p_{an} = \frac{\Delta U_b}{\Delta U_a} p_{bn}$ in sequences of equilibria with information acquisition, for large n. However, when $q_a \Delta U_a \neq q_b \Delta U_b$ a "natural guess" for μ is not readily available. It is possible to show that in this case, SBE with information acquisition do not exist for large electorates. The result extends to the case when only one among $C_{xx}(0,0)$ and $C_{x\alpha}(0,0)$ is zero.

Theorem 1 *Assume that $C_{xx}(0,0) > 0$ or $C_{x\alpha}(0,0) > 0$. For n large, a SBE with information acquisition exists if and only if $q_a \Delta U_a = q_b \Delta U_b$. As the number of electors goes to infinity, the probability of taking the right decision converges to $\frac{1}{2}$ along every sequence of SBE with information acquisition.*

Thus, equilibria with information acquisition exist only when the expected gains from taking the right decision are the same at state a and at state b. Even in this case large elections are as likely to reach the the right decision as the wrong one. Notice when $q_a \Delta U_a = q_b \Delta U_b$ SBE with information acquisition and without information acquisition provide voters with approximately the same utility, $q_a U(B,a) + q_b U(A,b)$. Finally consider the two SBE without information acquisition for the case $q_a \Delta U_a \neq q_b \Delta U_b$. Both equilibria elect one of the two candidate with probability one. However, if $q_a \Delta U_a > q_b \Delta U_b$ (resp. if $q_a \Delta U_a < q_b \Delta U_b$), then voting for B (resp. for A) is a weakly dominated strategy. The only SBE without information acquisition which survives to the elimination of weakly dominated strategy is the one where electors vote for the candidate which provide larger expected gains.

4.1 Possibility results

Next, we investigate conditions guaranteeing at least partial information aggregation. To derive more permissive results we need to assume that marginal costs grows slower both when the quality of information increases and when the skills decrease. Formally, we have to assume that $C_{xx}(0,0) = C_{x\alpha}(0,0) = 0$. Let us begin by considering a simple example where the cost function is a polynomial and where electors are ex-ante indifferent among the two candidates. It will provide useful intuition for the main result of the section.

Example 1 *Assume that the cost function is a polynomial of degree three and that* $C_{xx}(0,0) = C_{x\alpha}(0,0) = 0$. *Then* C *is of the form* $C(\alpha, x) = \frac{A}{3}x^3 + B\alpha x^2 + C\alpha^2 x + D\alpha^3 + E\alpha^2 + F\alpha$ *where* A, B, C *are strictly positive numbers such that* $B^2 - AC \geq 0$, $E \geq 0$ *and* $F \geq 0$. *Notice that marginal costs can be written as* $C_x(\alpha, x) = A\left(x + \frac{B\alpha}{A}\right)^2 - \frac{B^2 - AC}{A}\alpha^2$. *So larger values of* $B^2 - AC$ *imply smaller marginal costs. Assume that* $q_a \Delta U_a = q_b \Delta U_b$. *In this case* SBE *with information acquisition exist (see above) for every* n. *In* SBE *with information acquisition* $p_a = p_b = p$. *For* $p \geq 0$, *the cutoff type is* $\alpha(p) = \sqrt{\frac{2pq_a\Delta U_a}{C}}$ *and the optimal quality of information acquired by every voter solves:* $Ax^2 + 2B\alpha x + C\alpha^2 - 2q_a\Delta U_a p = 0$, *which is* $x(\alpha, p) = \frac{-B\alpha + \sqrt{(B^2 - AC)\alpha^2 + 2Ap}}{A}$, *for all* $\alpha \leq \alpha(p)$. *Consider the two extreme cases:* $B^2 - AC = 0$ *and* $B^2 - AC > 0$.

(i) $B^2 - AC = 0$. *Integrating, one obtains* $\tilde{x}(p) = \frac{q_a \Delta U_a}{B}p$. *From equation 6 we have* $B\sqrt{\pi n}\tilde{x}_n \approx q_a \Delta U_a e^{-4n\tilde{x}_n^2}$ *as* $n \to \infty$. *So* $l = \lim_{n\to\infty} \sqrt{n}\tilde{x}_n$ *exists, is positive, finite and satisfies* $B\sqrt{\pi}l \approx e^{-4l^2}$. *So, according to Proposition 3 the probability of taking the right decision at any of the two state approaches* $\Phi\left(2\sqrt{2}l\right) > \frac{1}{2}$ *for large* n.

(ii) $B^2 - AC > 0$. *We have* $\tilde{x}(p) = \frac{-B + 2\sqrt{2AC}}{AC} q_a \Delta U_a + \int_0^{\alpha(p)} \frac{\sqrt{(B^2 - AC)\alpha^2 + 4Aq_a\Delta U_a p}}{A} d\alpha$. *After elementary calculations one obtains* $\tilde{x}(p) = A_1 p - A_2 p \log p$, *where* A_1 *and* A_2 *are constant and* A_2 *is strictly positive. Then* $\tilde{x}(p) \approx -A_2 p \log p$ *as* $p \to 0$ *and* $\lim_{p\to 0} \frac{\tilde{x}(p)}{p} = \infty$. *From Equation 6 we have* $\sqrt{\pi}p_n \approx \frac{e^{-4n\tilde{x}_n^2}}{\sqrt{n}}$. *Dividing both sides by* $\tilde{x}_n$, *we have* $\frac{p_n}{\tilde{x}_n} \approx \frac{e^{-4n\tilde{x}_n^2}}{\sqrt{n}\tilde{x}_n}$ *as* $n \to \infty$. *The left-hand side of the equivalence converges to* 0 *so*

$\lim_{n\to\infty}\sqrt{n}\tilde{x}_n=\infty$*. From Proposition 3 it follows that the probability the elections select the best candidate approaches one when the number of electors is large.*

In order to connect the example with the general case, notice that in Example 1 $A=\frac{C_{x^{(3)}}(0,0)}{2}$, $B=\frac{C_{x^{(2)}\alpha}(0,0)}{2}$ and $C=\frac{C_{x\alpha^{(2)}}(0,0)}{2}$. Marginal costs are minimized at $(x,\alpha)=(0,0)$. The second order necessary condition is $C^2_{x^{(2)}\alpha}(0,0)-C_{x\alpha^{(2)}}(0,0)\,C_{x^{(3)}}(0,0)\geq 0$. When α and x are small and $C_{\alpha x}(0,0)=C_{xx}(0,0)=0$ we have $C_x(\alpha,x)\approx C_{x^{(3)}}(0,0)\left(x+\frac{C_{x^{(2)}\alpha}(0,0)}{C_{x^{(3)}}(0,0)}\alpha\right)^2-\frac{\left(C_{x^{(2)}\alpha}(0,0)\right)^2-C_{x^{(3)}}(0,0)C_{x\alpha^{(2)}}(0,0)}{C_{x^{(3)}}(0,0)}\alpha^2$. Thus, marginal costs are "approximately maximized" when $\left(C_{x^{(2)}\alpha}(0,0)\right)^2-C_{x^{(3)}}(0,0)\,C_{x\alpha^{(2)}}(0,0)=0$ which corresponds to case (i) in Example 1.

C1 $C_{xx}(0,0)=C_{x\alpha}(0,0)=0$, $C^2_{x^{(2)}\alpha}(0,0)\neq 0$ and $C^2_{x^{(2)}\alpha}(0,0)-C_{x\alpha^{(2)}}(0,0)\,C_{x^{(3)}}(0,0)=0$.[15]

Assume that Condition C1 holds and priors are not too biased in favor of one of the two candidates. (i.e. the expected gains from taking the right decision are the two states are not too far from each other). In this case equilibria with information acquisition exist and the Condorcet Jury Theorem holds in a weak form. In a large electorate the probability of electing the best candidates stays boundedly above $\frac{1}{2}$. The decision is more likely to be correct in the state to which agents attach more importance which is in the state with the highest $q_\omega\Delta U_\omega$. If the bias in favor of one candidate is too large no SBE with information acquisition exists.

Theorem 2 *Assume that C satisfies Condition C1.*

(i) There exist positive real numbers t_1,t_2, $0<t_1<1<t_2$ such that if $t_1<\frac{q_a\Delta U_a}{q_b\Delta U_b}<t_2$ an equilibrium with information acquisition exists for every n large enough.

Along any sequence of SBE with information acquisition probability of electing the best candidate at state ω converges to $l_\omega>\frac{1}{2}$ as $n\to\infty$, where $l_\omega>l_{\omega'}$ if and only if $q_\omega\Delta U_\omega>q_{\omega'}\Delta U_{l_{\omega'}}$, for $\omega,\omega'\in\{a,b\}$.

(ii) There exist positive real numbers, t'_1,t'_2 $0<t'_1<t_1<1<t_2<t'_2$ such that a SBE with information acquisition does not exists for n large if either $\frac{q_a\Delta U_a}{q_b\Delta U_b}<t'_1$ or $\frac{q_a\Delta U_a}{q_b\Delta U_b}>t'_2$.

In Theorem 2 we prove that along sequences of equilibria with information acquisition $\sqrt{n}\tilde{x}_n\to l$, $\sqrt{n}\mu_n\to k$, where l and k satisfy the following system of equations

[15] Notice that if Condition C1 holds then $C_{x^{(3)}}(0,0)>0$ and $C_{x\alpha^{(2)}}(0,0)>0$.

$$2f(0)\,\Delta U_a q_a e^{-4(l+k)^2} = C_{x^{(2)}\alpha}(0,0)\,l \tag{10}$$

$$2f(0)\,\Delta U_b q_b e^{-4(l-k)^2} = C_{x^{(2)}\alpha}(0,0)\,l \tag{11}$$

and that from Proposition 3 follows that $l_a = \Phi\left(2\sqrt{2}\,(k+l)\right)$ and $l_b = \Phi\left(2\sqrt{2}\,(k-l)\right)$. If $\frac{q_a \Delta U_a}{q_b \Delta U_b} \geq 1$, then $k \geq 0$. the expected gain from taking the right decision at state a is larger than the expected gain from taking the right decision at state b. From Equations 10and 11 follows that $k \geq 0$, $l \geq 0$. Also, the greater $\frac{q_a \Delta U_a}{q_b \Delta U_b}$ the greater the probability the best candidate is elected at state a and the lower the probability the best candidates is elected at state b. Notice however that the bias in favor of candidate a derives from the behavior introduced

When $C^2_{x^{(2)}\alpha}(0,0) - C_{x^{(3)}}(0,0)\,C_{x\alpha^{(2)}}(0,0) > 0$ or when $C_{x^{(2)}\alpha} = 0$, information is "cheaper" for the most skilled voters. Consider the following condition which includes case (ii) in Example1.

C2 $C_{xx}(0,0) = C_{x\alpha}(0,0) = 0$ and Condition C1 does not hold.

Theorem 3 *Assume that C satisfies Condition C2. If $q_a > 0$ and if n is large a SBE with information acquisition exists. Along every sequence of SBE with information acquisition, the probability of electing the best candidate decision converges to* 1*, when .*

The proofs of Theorem 2 and of Theorem 3 rely closely on the intuition provided in Example 1, through quadratic approximations of the marginal cost function.

5 Welfare analysis

In this sections we study the asymptotic efficiency of equilibria of large elections. The expected utility accrued to an individual voter at the best uninformative equilibrium is

$$U_{\min} = |q_a \Delta U_a - q_b \Delta U_b| + q_a U(B,a) + q_b U(A,b)\,.$$

Along every sequence of *SBE* with information acquisition, the expected cost paid by every voter approaches zero when the number of electors converge to infinity. It follows that $U_{\min}$ is also the (approximate) value of the individual utility at large elections at any SBE with information acquisition when either $C_{xx}(0,0) > 0$ or $C_{x\alpha}(0,0) > 0$. More precisely, the approximate value of expected utility is

$$U_{\min} = q_a U(B,a) + q_b U(A,b)$$

because, in this case, equilibria with information acquisition exist only when $q_a \Delta U_a = q_b \Delta U_b$.

When Condition C1 holds, the expected utility along a sequence of equilibria with information acquisition is approximately

$$U = q_a \Delta U_a \Phi\left(2\sqrt{2}(l+k)\right) + q_b \Delta U_b (B \mid b)\, \Phi\left(2\sqrt{2}(l-k)\right) + q_a U(B,a) + q_b U(A,b)$$

where l and k are defined as solutions of equations 10 and 11. The reader can easily check that $U > U_{\min}$.

The expected utility from voting correctly at every state is

$$U_{\max} = q_a U(B \mid a) + q_b U(A \mid b).$$

Notice that $U_{\max}$ approximate the expected individual utility of SBE with information acquisition when Condition C2 holds. Equilibria with information acquisition are asymptotically efficient in this case. Actually, this is the minimal assumption conditions which assures that election are asymptotically efficient.

Proposition 4 *Large elections are asymptotically efficient if and only if Condition C2 holds.*

In order to prove Proposition 4, it suffices to find a sequence of symmetric strategies such that, when all agents employ them the probability of electing the best candidate approach one and the cost of using them approach zero in large electorates. The idea to construct them is basic. The individual gains from acquiring information of quality x are $(p_a q_a \Delta U_a + p_b q_b \Delta U_b)$ and the marginal costs are $C(\alpha, x)$. However the "social gains" from such a a strategy are much higher: they are $(2n+1)(p_a q_a \Delta U_a + p_b q_b \Delta U_b)$. The intuition is to consider strategies that equate marginal social gains and individual marginal costs, which is strategies such that $(2n+1)(p_a q_a \Delta U_a + p_b q_b \Delta U_b) = C_x(\alpha, x)$ for every voter who acquires information. Using a fixed point argument we show that if all agents use such strategies the the probability of electing the best candidate approaches one and costs approach zero proving the claim of Proposition 4.

6 The Epistocracy of the educated thesis: a rational foundation.

Plato proposed to put the saviours in charge of the state, because he did not trust citizens to have the skills to run public affairs. Similar in spirit was the concern of Stuart Mill, who believed that more educated people should have greater voting power. Both form of government are unified by Estlund with the etiquette of "epistocracy of the educated thesis". The objective of this section is not to present a philosophical discussion of these assertions. We have shown that the Condorcet Jury Theorem is no robust to the introduction of an electorate with heterogeneous information processing abilities. We want to understand whether these epistocratic proposals are rationally founded. We assume that less skilled types can be identified and excluded from voting. We investigate whether an appropriate choice of the least skilled citizen who is allowed to vote can lead to positive information aggregation. We allow the cutoff type to depend on the number of player, otherwise the results of Section 4 would apply. In order to simplify the exposition we consider only the case $q_a \Delta U_a = q_b \Delta U_b$ and assume that $C_{x\alpha}(0,0) \neq 0$ and $C_{xx}(0,0) \neq 0$. In this case the existence result of Theorem 1 applies and SBE with information acquisition exist whatever is the number of the voters. If citizens of any competence are allowed to vote, elections are as likely to elect the best candidate as the wost one. One of the reasons that leads to this negative result of is the noise introduce by the random vote of uninformed electors.

We study a a game with a population of $2n+1$ citizens having voting rights. They have been selected to have type at most β_n, where $0 < \beta_n < 1$. Thus voters competence is distributed as $g_n(\alpha) = \frac{f(\alpha)}{F(\beta_n)}$ on $[0, \beta_n]$. The game defined by β_n will be called β_n-epistocratic game. Our objective is to determine whether an appropriate choice of β_n can lead to positive information aggregation.

The model has an alternative and lecture. Assume that society starts with a given number of agents with skills distributed according to f. Assume that it is possible to improve the skills of the electors when their number grows, by improving the competence of the least skilled voters from 1 to β_n. In this setup, our previous question is equivalent to ask if an appropriately educated electorate can solve the informational inefficiency revealed in the previous sections.

We prove that it is indeed the case. An appropriate choice of the cutoff β_n improves the informational efficiency of the elections. However, it does not lead to perfect information aggregation if $C_{xx}(0,0)$ and

$C_{x\alpha} = (0,0)$.

Proposition 5 *Assume that* $q_a \Delta U_a = q_b \Delta U_b$.

(i) Let $\{\beta_n\}_{n\geq 0}$ *be a sequence strictly positive real numbers,* $\beta_n \in (0,1)$*, for all* n*. Consider any sub-sequence of SBE with information acquisition of the corresponding* β_n*-epistocratic games* (x_n, v_n)*. Let* Π_n *be the probability the right decision is taken at such SBE. Then* $\sup\lim_{n\to\infty} \Pi_n \leq \Phi\left(2\sqrt{2}t\right) = \bar{l} > \frac{1}{2}$*, where* $t \in (0, +\infty]$ *solves* $e^{-4t^2} = \sqrt{\pi n}\frac{C_{xx}(0,0)}{3q_a \Delta U_a} t$.

(ii) There exists a sequence $\{\beta_n\}_{n\geq 0}$ *and a sequence of SBE with information acquisition of the* β_n*-epistocratic game such that the probability of electing the best candidate converges to* $\bar{l}$ *along this sequence of SBE.*

In particular, there exists a sequence $\{\beta_n\}_{n\in\mathbf{N}}$ *of positive real numbers and a sequence of equilibria of the corresponding* (x_n, v_n) *a SBE equilibrium of the corresponding* β_n*-epistocratic games such that along this sub-sequence the probability of electing the best candidate is boundedly above the probability of electing the worst candidate. This probability can approach one if and only if* $C_{xx}(0,0) = 0$.

The proof of Proposition 5, determines a cutoff β_n such that every agents type β_n is indifferent between acquiring information and not acquiring information and proves (ii) through a simple limit argument. Then it proves that no sequence of β_n-epistocratic games can reach a better information efficiency.

While we study information aggregation in a model of compulsory voting the Proposition 5 seems to suggest that a model of voluntary voting might improve the information aggregation properties of election (see also Börgers (2004) or Krasa and Polborn (2009)). Observe that the choice of the cutoff point β_n makes every voter better off. Thus, every voter would be willing willingly to abstain. The point is that a positive level of abstention increases the probability any voter is pivotal and thus increases the incentives to acquire information.

7 Conclusions

When voters can acquire information of different qualities and have heterogeneous skills in large elections fail to aggregate information. This is consistent with the most pessimistic view of the rational ignorance hypothesis. Information aggregation is possible only under quite restrictive assumptions. The results have its efficiency counterpart. Large elections are efficient only when they aggregate perfectly information. Limiting the voting rights to most skilled citizens or improving the competence of the electorate citizens can solve the informational inefficiency. The result is compatible with the idea an educated citizenry is vital to a properly functioning democracy, but also with an elitist view of public decision making.

This last result suggests that allowing for abstention might lead better results improve informational efficiency. If less informed voters abstain the probability an informed voter is decisive increases and so the incentive to acquire information for skilled voters. Less competent citizens would abstain and prefer to delegate their decision to better informed ones (see also Feddersen and Pesendorfer 1996).

However, there are aspects that are not reflected in our model and could have important implications. It is not clear the impact of communication or correlation among different sources of information as it would introduce new strategic considerations.[16] Furthermore, here and in every model investigating the Condorcet the technology producing information is not specified. This way, it is not possible to compare the effects of different technologies and market structure on information aggregation.

There are at least other two aspects that deserve deserve investigation and might help to solve the paradox of the rational ignorance. The first one is the use made by voters of informational shortcuts. Empirical and experimental investigation do not have reached an agreement whether the use of this shortcuts facilitate information aggregation in election. (see, for instance Lau and Redlawsk (2006) and Nordin (2009)). The second argument concerns the intrinsic value of information. If the information has a consumption value in itself, voter would acquire more information. It is note clear whther they would acquire the information sufficient to improve the efficiency of the elections.

Appendix: Proofs

Proof of Proposition 3. Without loss of generality assume that $t_a = \lim_{n\to\infty} \sqrt{n}\,(\theta_{na})$. The event that a given voter votes for A at state a is a Bernoulli trial with probability of success $\frac{1}{2} + \theta_{na}$. For $i = 1, ..., 2n+1$,

[16] Gerardi and Yariv (2007) study a model of pre-voting communication communication, without information acquisition.

set

$$V_i^n = \begin{cases} \frac{1}{2} - \theta_{na} \text{ if voter } i \text{ votes for} A \\ -\frac{1}{2} - \theta_{na} \text{ if voter } i \text{ votes for } B \end{cases}$$

The V_i^n are i.i.d. Furthermore, $E\left(V_i^n\right) = 0$, $\sigma_n^2 = E\left(\left(V_i^n\right)^2\right) = \frac{1}{4} - \theta_{na}^2$ and $\gamma_n^3 = E\left(\left|V_i^n\right|^3\right) = \frac{1}{8} - 2\theta_{na}^4$.

.

Note that the alternative A wins the elections if and only if it gets at least $n+1$ votes which is if and only if

$$\sum_{i=1}^{2n+1} V_i^n > -\frac{1}{2} - (2n+1)\,\theta_{na}.$$

Let W^n be the normalized sum of the V_i^n.

$$W^n = \frac{\sum_{i=1}^{2n+1} V_i^n}{\sqrt{(2n+1)\,E\left(\left(V_i^n\right)^2\right)}}.$$

If $|t_a| = \infty$, the claim follows from applying the Chernoff bound to W^n. Thus assume that $t_a < \infty$, thus $\theta_{na} \to 0$.

Let F_n be the p.d.f. of W^n. The probability of choosing A at state a is $1 - F_n\left(J_n\right)$ where

$$J_n = \frac{-\frac{1}{2} - (2n+1)\,\theta_{na}}{\sqrt{(2n+1)\left(\frac{1}{4} - \theta_{na}^2\right)}} \approx -2\sqrt{2}\sqrt{n}\,(2n+1)\,\theta_{na}$$

for $n \to \infty$. From the Berry-Esseen Theorem (see Chow and Teicher 1997, p 322), there exists $C > 0$ such that

$$\left|F_n\left(J_n\right) - \Phi\left(J_n\right)\right| \leq \frac{C\gamma_n}{\left(\sigma_n^3 n\right)^{\frac{1}{2}}}$$

As $\theta_{na} \to 0$, σ_n is bounded away from zero, thus $\frac{C\gamma_n}{\left(\sigma_n^3\right)^{\frac{1}{2}}}$ is bounded above and

$$lim_{n\to\infty} P_{an} = 1 - \Phi\left(-2\sqrt{2}t_a\right) = \Phi\left(2\sqrt{2}t_a\right)$$

whether t_a is finite or infinite. This completes the proof for the case $\omega = a$. The proof of the case $\omega = b$ is identical and thus omitted. ■

Prof of Lemma 1. The proof is by contradiction. Assume that there is a sub-sequence of SBE with information acquisition such that $p_{an_k} q_a \Delta U_a > p_{bn_k} q_b \Delta U_b$ for infinitely many k. Without loss of generality assume that $n_k = k$. for every k. All uninformed voters must vote for candidate A. The probability that a voter is pivotal at state a and at state b are

$$p_{an} = \binom{2n}{n} \left(1 + \tilde{x}_n - \frac{F(\alpha_n)}{2}\right)^n (F(\alpha_n) - \tilde{x}_n)^n$$

and

$$p_{bn} = \binom{2n}{n} \left(1 - \tilde{x}_n - \frac{F(\alpha_n)}{2}\right)^n (F(\alpha_n) + \tilde{x}_n)^n$$

respectively. Observe that, in this case $p_{an} < p_{bn}$, for every n so $q_a \Delta U_a > q_b \Delta U_b$. Let $x_n = x_n(0) > 0$ be the information acquired in equilibrium by the lowest type. Type 0 prefers to acquire the information thus x_n satisfies inequality 4 for $\alpha = 0$, which is $C_x(0, x_n) x_n - C(0, x_n) \geq \frac{p_{an} q_a \Delta U_a - p_{bn} q \Delta U_b}{2}$ for every n. $l = \frac{q_a \Delta U_b}{q_b \Delta U_b} > 1$. As $C_x(0, x_n) = p_a q_a \Delta U_a + p_b q \Delta U_b$ and $C(0, x) > 0$,

$$2x_n \geq \frac{p_{an} q_a \Delta U_a - p_{bn} q \Delta U_b}{p_{an} q_a \Delta U_a - p_{bn} q \Delta U_b}. \tag{12}$$

for every n. Set $l = \frac{q_a \Delta U_b}{q_b \Delta U_b}$. Rearranging in 12 we obtain

$$2x_n \geq \frac{l - \frac{p_{bn}}{p_{an}}}{l + \frac{p_{bn}}{p_{an}}}$$

for every n, which implies $\lim_{n \to \infty} \frac{p_{bn}}{p_{an}} = l$, because $x_n \to 0$.

As $n \to \infty$

$$p_{an} \approx \binom{2n}{n} e^{n\left(\tilde{x}_n - \frac{F(\alpha_n)}{2}\right)} \left(\frac{F(\alpha_n)}{2}\right)^n e^{-2n \frac{\tilde{x}_n}{F(\alpha_n)}} \tag{13}$$

$$p_{bn} \approx \binom{2n}{n} e^{-n\left(\tilde{x}_n + \frac{F(\alpha_n)}{2}\right)} \left(\frac{F(\alpha_n)}{2}\right)^n e^{2n \frac{\tilde{x}_n}{F(\alpha_n)}} \tag{14}$$

Dividing 13 by 14 we obtain

$$\frac{p_{an}}{p_{bn}} \approx e^{2n\left(\tilde{x}_n - 2\frac{\tilde{x}_n}{F(\alpha_n)}\right)}$$

as $n \to \infty$. As $\lim_{n\to\infty} \frac{p_{bn}}{p_{an}} = l > 1$, then $\lim_{n\to\infty} \frac{n\tilde{x}_n}{F(\alpha_n)} = \frac{1}{4}\log l$ and $\lim_{n\to\infty} n\tilde{x}_n = 0$. From equivalence 13 and from Stirling Formula we obtain

$$p_{an} \approx \frac{1}{l^2\sqrt{\pi n}} e^{n\left(\log F(\alpha_n) - \log 2 - \frac{F(\alpha_n)}{2}\right)} \tag{15}$$

.

From equation 1 $2p_{an}q_b\Delta U_b \approx C_x(0, x_n)$, because $\lim_{n\to\infty} \frac{p_{bn}}{p_{an}} = l$. Furthermore, $C_x(0, x_n) \approx \frac{C_{x^{(t+1)}}(0,0)}{t!} x_n^t$ where t is the lowest integer such that $C_{x^{(t+1)}}(0, 0) \neq 0$. So equivalence 15 implies that

$$\frac{2p_{an}q_b\Delta U_b}{l^2\sqrt{\pi n}} e^{n\left(\log F(\alpha_n) - \log 2 - \frac{F(\alpha_n)}{2}\right)} \approx \frac{C_{x^{(t+1)}}(0,0)}{t!} x_n^t \tag{16}$$

as $n \to \infty$. Multiplying both sides of equivalence by n^t we have:

$$\frac{e^{2l}}{\sqrt{\pi}} e^{n\left(\log F(\alpha_n) - \log 2 - \frac{F(\alpha_n)}{2} + \frac{\log n}{n\left(t-\frac{1}{2}\right)} + \log 2\right)} e^{2l} \approx \frac{C_{x^{(t+1)}}(0,0)}{t!} (nx_n)^t \tag{17}$$

as $n \to \infty$. As $\lim_{n\to\infty} \alpha_n = 0$, then $\lim_{n\to\infty} \log F(\alpha_n) - \log 2 - \frac{F(\alpha_n)}{2} + \frac{\log n}{n\left(t-\frac{1}{2}\right)} + \log 2 = -\infty$ so the LHS of equivalence 17 converges to 0. It follows that $\lim_{n\to\infty} nx_n = 0$ as well. However, $\frac{\tilde{x}_n}{F(\alpha_n)} = \frac{1}{F(\alpha_n)} \int_0^{\alpha_n} x_n(\alpha) f(\alpha) d\alpha \leq \frac{x_n}{F(\alpha_n)} \int_0^{\alpha_n} f(\alpha) d\alpha = x_n$ for every n so $\lim_{n\to\infty} n\frac{\tilde{x}_n}{F(\alpha_n)} = 0$, which yields a contradiction. ■

Lemma 2 *Assume $C_{xx}(0,0) \neq 0$ or $C_{x\alpha}(0,0) \neq 0$. For every sub-sequence of SBE with information acquisition $(x_{n_k}, v_{n_k})_{k\in\mathbf{N}}$, $\lim_{k\to\infty} \sqrt{n_k}\theta_{\omega n_k} = 0$ for $\omega = a, b$. Then the probability of taking the right decision approaches $\frac{1}{2}$ when $n \to \infty$ along the sub-sequence.*

Proof. Assume that there are $2n+1$ agents. Without loss of generality we assume that for every $n \in \mathbf{N}$ there is an equilibrium with information acquisition (x_n, v_n). For n large enough, Equation 9 holds so $p_{an}q_a\Delta U_a = p_{bn}q_b\Delta U_b$. From Lemma 1, for n large enough p_{an}, p_{bn} satisfy $p_{bn} = p_{an}\frac{q_a\Delta U_a}{q_b\Delta U_b}$. Set $p = p_a$ set

$x(\alpha,p) = x\left(\alpha, p, \frac{q_a \Delta U_a}{q_b \Delta U_b} p\right)$ and set $\alpha(p) = \alpha\left(p, \frac{q_b \Delta U_b}{q_a \Delta U_a} p\right)$. Set $\tilde{x}(p) = \tilde{x}\left(p, \frac{q_a \Delta U_a}{q_a \Delta U_a} p\right)$. For n large enough an equilibrium is characterized by $(p_{an}, \mu_n) = (p, \mu)$ satisfying

$$p = \binom{2n}{n} \left\{ \frac{1}{4} - \left[\tilde{x}(p) + \mu\left(1 - F\left(\alpha(p)\right)\right)\right]^2 \right\}^n$$

$$\frac{q_b \Delta U_b}{q_a \Delta U_a} p = \binom{2n}{n} \left\{ \frac{1}{4} - \left[\tilde{x}(p) - \mu\left(1 - F\left(\alpha(p)\right)\right)\right]^2 \right\}^n$$

For n large enough the cutoff type $\alpha(p)$ is characterized by $2pq_a\Delta U_a = C_x(\alpha(p), 0)$. For $\alpha < \alpha(p)$, x(α, p) satisfies $2pq_a\Delta U_a = C_x(\alpha, x(\alpha, p))$. Next, we obtain estimations of the rate of convergence of $\tilde{x}(p)$ to 0. Let $t \geq 1$ be the lowest integer such that $C_{x\alpha^t}(0,0) \neq 0$. Using Taylor approximation $C_x(\alpha, 0) \approx \frac{C_{x\alpha^t}(0,0)}{t!}\alpha^t$ as $\alpha \to 0$. Thus $\alpha(p) \approx \left(\frac{t! 2 p_a q_a \Delta U_a}{C_{x\alpha^t}(0,0)}\right)^{\frac{1}{t}} p^{\frac{1}{t}}$ as $p \to 0$. Let $k \geq 1$ be the lowest integer such that $C_{x^{k+1}}(0,0) \neq 0$. Using Taylor approximation $C_x(0, x) \approx \frac{C_{x^{k+1}}(0,0)}{k!} x^k$ as $x \to 0$. Thus $x(0,p) \approx \left(\frac{k! 2 p_a q_a \Delta U_a}{C_{x^{k+1}}(0,0)}\right)^{\frac{1}{t}} p^{\frac{1}{k}}$ as $p \to 0$. Notice that using a Taylor approximation one gets $C_x(\alpha, x) = C_{x^{(k+1)}}(0,0)\frac{x^k}{k!} + C_{x\alpha^{(t)}}(0,0)\frac{\alpha^t}{t!} + F(x,\alpha)x^2 + G(x,\alpha)x\alpha + H(x,\alpha)\alpha^2$, where $F(x,\alpha)$ and $G(x,\alpha)$ and $H(x,\alpha)$ are bounded in a neighborhood of $(0,0)$ and either $k = 1$ or $t = 1$.

First assume that $C_{xx}(0,0) > 0$. Then $k = 1$ and $t \geq 1$. Integrating and eliminating infinitesimal of higher order we obtain: $\tilde{x}(p) \approx \frac{6(q_a \Delta U_a)^2}{C_{xx}(0,0) C_{x\alpha}(0,0)} f(0) p^2$ for $p \to 0$ if $t = 1$ and $\tilde{x}(p) \approx \frac{pF(\alpha(p))}{C_{xx}(0,0)} \approx \frac{pF(\alpha(p))}{C_{xx}(0,0)} \approx \frac{1}{C_{xx}(0,0)}\left(\frac{t! 2 p q_a \Delta U_a}{C_{x\alpha^t}(0,0)}\right)^{\frac{1}{t}} p^{1+\frac{1}{t}}$ as $p \to 0$ if $t > 1$. Now assume that $C_{xx}(0,0) = 0$. Then $k > 1$ and $t = 1$. Integrating and eliminating infinitesimal of higher order we obtain $\tilde{x}(p) \approx \left(\frac{k!}{C_{x^{(k+1)}}(0,0)}\right)^{\frac{1}{k}} \frac{k}{(1+k) C_{x\alpha}(0,0)} p^{1+\frac{1}{k}}$ as $p \to 0$. So, in each case there exists $C > 0$ and $k \geq 1$ such that $p \approx C(\tilde{x}(p))^{\frac{k}{k+1}}$.

Assume $\lim_{n\to\infty}\mu_n$ exists and is non negative.[17] Set $M = \lim_{n\to\infty}\mu_n$. Then $M \in \left[0, \frac{1}{2}\right]$. First assume $M = 0$. For $n \to \infty$

$$p_{an} \approx \frac{e^{-4\left\{\sqrt{n}\left[\tilde{x}_n + \mu_n\left(1 - F(\alpha_n)\right)\right]\right\}^2}}{\sqrt{\pi n}}$$

and

$$p_{bn} \approx \frac{e^{-4\left\{\sqrt{n}\left[\tilde{x}_n - \mu_n\left(1 - F(\alpha_n)\right)\right]\right\}^2}}{\sqrt{\pi n}}.$$

[17] In the case it does not exist it suffices to take a convergent sub-sequence.

Replacing $p_{an} = p$ with $C\left(\tilde{x}\left(p\right)\right)^{\frac{k}{k+1}}$ and p_{bn} by $\frac{q_a \Delta U_a}{q_b \Delta U_b} p_{an}$ and elevating both members of the equivalence to the power $\frac{k+1}{k}$ in the equivalence we have:

$$\left(\sqrt{n}\tilde{x}_n\right) e^{\frac{4(k+1)}{k}\left\{\sqrt{n}\left[\tilde{x}_n + \mu_n\left(1 - F\left(\alpha_n\right)\right)\right]\right\}^2} \approx \left(C\sqrt{\pi}\right)^{-\frac{k+1}{k}} n^{-\frac{1}{2k}}$$

$$\left(\sqrt{n}\tilde{x}_n\right) e^{8\left\{\sqrt{n}\left[\tilde{x}_n - \mu_n\left(1 - F\left(\alpha_n\right)\right)\right]\right\}^2} \approx \left(\frac{q_a \Delta U_a}{q_b \Delta U_b} C\right)^{-\frac{k+1}{k}} n^{-\frac{1}{2k}}$$

for $n \to \infty$. Combining the two equivalence we obtain: $\lim_{n\to\infty} \sqrt{n}\left[\tilde{x}_n + \mu_n\left(1 - F\left(\alpha_n\right)\right)\right] = 0$ and $\lim_{n\to\infty} \sqrt{n}\left[\tilde{x}_n - \mu_n\left(1 - F\right.\right.$ 0.

Next, we prove by contradiction that it cannot be the case that $M > 0$. Let $0 < M < \frac{1}{2}$. Then $q_a \Delta U_a > q_b \Delta U_b$. Set $\delta = \sqrt{\frac{1}{4} - M^2}$ and set $y_n = \left[\tilde{x}_n - \mu_n\left(1 - F\left(\alpha_n\right)\right)\right]^2 - M^2 = o\left(1\right)$ and set $z_n = \left[\tilde{x}_n - \mu_n\left(1 - F\left(\alpha_n\right)\right)\right]^2 - M^2 = o\left(1\right)$. We have

$$p_{an} \approx \frac{\left(2\delta\right)^{2n}}{\sqrt{\pi n}} e^{-n\frac{y_n}{\delta^2}}$$

$$p_{bn} \approx \frac{\left(2\delta\right)^{2n}}{\sqrt{\pi n}} e^{-n\frac{z_n}{\delta^2}}.$$

Furthermore, from Equation 9 $\lim_{n\to\infty} e^{-n\frac{y_n - z_n}{\delta^2}} = \frac{q_b \Delta U_b}{q_a \Delta U_a}$ so $\lim_{n\to\infty} -n\frac{y_n - z_n}{\delta^2} = ln\frac{q_b \Delta U_b}{q_a \Delta U_a}$. From $p \approx C\left(\tilde{x}\left(p\right)\right)^{\frac{k}{k+1}}$ we have $\widetilde{x}_n \approx O\left(\left(2\delta\right)^{\frac{k+1}{k}n} n^{-\frac{1}{2k}}\right)$. Then $\lim_{n\to\infty} n\left(y_n - z_n\right) = -4n\mu_n\left(1 - F\left(\alpha_n\right)\right)\widetilde{x}_n = 0$, because $\delta < \frac{1}{2}$, which yields a contradiction. The proof for the case$M = \frac{1}{2}$ is similar. In order to conclude and obtain a contradiction one has to observe that now $\widetilde{x}_n \approx o\left(\left(2\delta\right)^{\frac{k+1}{k}n} n^{-\frac{1}{2k}}\right)$ for every $\delta > 0$. The proof of the case $M < 0$ is the same, exchanging the roles of a and b. The last claim follows from Proposition 3.

■

Proof of Theorem 1. From Lemma 2 through Proposition we know that in any sequence of equilibria with information acquisition the probability of reaching the right decision approaches $\frac{1}{2}$ when $n \to \infty$. In order to complete the proof of the claim we have to show that for large values of n, a SBE with information acquisition exist if $q_a \Delta U_a = q_b \Delta U_b$ and that a SBE with information acquisition does not exist if $q_a \Delta U_a \neq q_b \Delta U_b$.

Let us start by considering the case $q_a \Delta U_a = q_b \Delta U_b$. In any equilibrium with information acquisition

$p_a = p_b = p$ (see Equation 9). So from Equation 6 and equation 7 follows that $\mu = 0$. Set $r = q_a \Delta U_a = q_b \Delta U_b$. For $p \in [0,1]$, let $\alpha(p)$ satisfying $2rp = C_x(\alpha, 0)$ if any such α exists and $\alpha(p) = 1$ otherwise. Let the function $x(\alpha, p)$ be defined on satisfying $2rp = C_x(\alpha, x)$ for every $\alpha \in [0, \alpha(p)]$ and $x(\alpha, p) = 0$ for $\alpha \in [\alpha(p), 1]$. Define:

$$T(p) = \binom{2n}{n} \left[\frac{1}{4} - \left(\int_0^{\alpha(p)} x(\alpha, p) f(\alpha) d\alpha \right)^2 \right]^n$$

The function $T : [0,1] \to [0,1]$ is well defined and continuous so it has a fixed point. Let p^* be a fixed point of T. Note that $p^* \neq 0$ Because $T(0) = \binom{2n}{n} 2^{-2n}$. Next, define $\tilde{\alpha}$ as the conditional median of the types who do not acquire information. Formally, $F(\tilde{\alpha}) - F(\hat{\alpha}(p^*)) = \int_{\alpha(p)}^{\tilde{\alpha}} f(\alpha) d\alpha = \frac{1 - F(\hat{\alpha}(p^*))}{2}$. Consider the strategy (x, v), where $(x, v)(\alpha) = (x(\alpha, p^*), A, B)$ for $\alpha \leq \hat{\alpha}(p^*)$, $(x, v)(\alpha) = (0, A, A)$ for $\alpha(p^*) \leq \alpha < \tilde{\alpha}$ and $(x, v)(\alpha) = (0, B, B)$ for $\tilde{\alpha} < \alpha \leq 1$. It is easily seen that $(x, v)_i = (x, v)$ for $i = 1, ...2n+1$ is a SBE. As $p^* \neq 0$, (x, v), is an equilibrium with information acquisition.

Next consider the case $q_a \Delta U_a \neq q_b \Delta U_b$. We prove by contradiction that if n is large no SBE with information acquisition exists. By contradiction assume that there is a sub-sequence of equilibria with information acquisition $(x_{n_k}, v_{n_k})_{k \in \mathbf{N}}$. Then for $n = n_k$, there are $\mu \in \left(-\frac{1}{2}.\frac{1}{2}\right)$ and (p_a, p_b) such that:

$$2\Delta U_a q_a \binom{2n}{n} \left[\frac{1}{4} - \left(\widetilde{x}(p_a, p_b) + \mu \left(1 - F(\alpha(p_a, p_b))\right) \right)^2 \right]^n = C_x(\alpha(p_a, p_b), 0)$$

$$2\Delta U_b q_b \binom{2n}{n} \left[\frac{1}{4} - \left(\widetilde{x}(p_a, p_b) - \mu \left(1 - F(\alpha(p_a, p_b))\right) \right)^2 \right]^n = C_x(\alpha(p_a, p_b), 0)$$

Fro Lemma 2 we have

$$lim_{k \to \infty} \frac{p_{an_k}}{p_{bn_k}} = lim_{k \to \infty} \frac{\binom{2n_k}{n_k} \left\{ \frac{1}{4} - \left[\tilde{x}_{n_k} + \mu_{n_k} \left(1 - F(\alpha_{n_k})\right) \right]^2 \right\}^{n_k}}{\binom{2n_k}{n_k} \left\{ \frac{1}{4} - \left[\tilde{x}_{n_k} - \mu_{n_k} \left(1 - F(\alpha_{n_k})\right) \right]^2 \right\}^{n_k}} = lim_{k \to \infty} \frac{e^{-4n_k \vartheta_{an_k}^2}}{e^{-4n_k \vartheta_{bn_k}^2}} = 1.$$

Equilibrium pivotal probabilities p_{an_k} and p_{b_k} satisfy Equation 9 for every k, which yields a contradiction because $q_a \Delta U_a \neq q_b \Delta U_b$. ■

Proof of Theorem 2. At every SBE $p_{an} = \frac{q_b \Delta U_b}{q_a \Delta U_a} p_{bn}$. Set $p_{nb} = p_n$. If $p \geq 0$ set $\alpha(p) = \alpha\left(\frac{q_b \Delta U_b}{q_a \Delta U_a} p, p\right)$, set $x(\alpha, p) = x\left(\alpha, \left(\frac{q_b \Delta U_b}{q_a \Delta U_a} p, p\right)\right)$ and set $\tilde{x}(p) = \tilde{x}\left(\frac{q_b \Delta U_b}{q_a \Delta U_a} p, p\right)$. Notice that $C_{xxx}(0,0) > 0$, $C_{x\alpha\alpha}(0,0) > 0$.

Set $B = C_{x\alpha\alpha}(0,0)$, $A = C_{xxx}(0)$ and $C = C_{xx\alpha}(0,0)$. We have $AB = C^2$.

For $p \to 0$

$$\alpha(p) \approx \left(\frac{2pq_b\Delta U_b}{B}\right)^{\frac{1}{2}}.$$

Furthermore for $p \to 0$,

$$x(0,p) \approx \left(\frac{2pq_b\Delta U_b}{A}\right)^{\frac{1}{2}}$$

and $x(\alpha,p) \le x(0,p)$ for every α and for every p, so $x(\alpha,p) = O\left(p^{\frac{1}{2}}\right)$, uniformly in α.

For $x = x(\alpha,p)$. we have:

$$4pq_b\Delta U_b = 2C_x(\alpha,x).$$

Then:

$$2C_x(\alpha,x) = C_{x\alpha^{(2)}}(0,0)\,\alpha^2 + C_{x^{(3)}}(0,0)\,x^2 + 2C_{x^{(2)}\alpha}(0,0)\,\alpha x + o\left(\|(\alpha,x)\|^2\right)$$

uniformly in (α,x). Set $B = C_{x\alpha\alpha}(0,0)$, $A = C_{xxx}(0)$ and $C = C_{xx\alpha}(0,0)$. Since $AB = C^2$, then $Ax^2 + B\alpha^2 + 2C\alpha x = A\left(x + \frac{C}{A}\alpha\right)^2$.

So, for $\alpha \le \bar{\alpha}(p) = min\left\{\alpha(p), \left(\frac{2pq_b\Delta U_b}{C_{x^{(2)}}(0,0)}\right)^{\frac{1}{2}}\right\}$ and since x

$$x(\alpha,p) = -\frac{C}{A}\alpha + 2\left(\frac{pq_b\Delta U_b}{A}\right)^{\frac{1}{2}} + o\left(\|\alpha,\sqrt{p}\|\right)$$

uniformly in $\alpha \le \bar{\alpha}(p)$, because $x(\alpha,p) = O\left(p^{\frac{1}{2}}\right)$, uniformly in α and $\bar{\alpha}(p) = O\left(p^{\frac{1}{2}}\right)$.

Notice that $\bar{\alpha}(p) - \alpha(p) = o\left(p^{\frac{1}{2}}\right)$. Integrating,

$$\tilde{x}(p) = \int_0^{\bar{\alpha}(p)} -\frac{C}{A}\alpha + 2\left(\frac{pq_b\Delta U_b}{A}\right)^{\frac{1}{2}} f(\alpha)\,d\alpha + o(p)$$

then

$$\tilde{x}(p) = \left[-\frac{C}{2A}\bar{\alpha}^2(p) + 2\bar{\alpha}(p)\left(\frac{pq_b\Delta U_b}{A}\right)^{\frac{1}{2}}\right] f(0) + o(p).$$

Recall that $\alpha(p) \approx \left(\frac{2pq_b\Delta U_b}{B}\right)^{\frac{1}{2}}$. So

$$\tilde{x}(p) = \left[-\frac{C}{2A} \left(\frac{2pq_b\Delta U_b}{B} \right) + 2 \left(\frac{2pq_b\Delta U_b}{B} \right)^{\frac{1}{2}} \left(\frac{pq_b\Delta U_b}{A} \right)^{\frac{1}{2}} \right] f(0) + o(p).$$

Since $AB = C^2$.

$$\tilde{x}(p) \approx \frac{pq_b\Delta U_b}{C} f(0).$$

Thus, in any sequence of SBE with information acquisition we have $p_n \approx \frac{C\tilde{x}_n}{f(0)q_b\Delta U_b} = D\tilde{x}_n$ for n large. The proof of the existence of a SBE with information acquisition in the case $q_b\Delta U_b = q_a\Delta U_a$ the same as in Theorem 1. Let's consider informational efficiency:

For $n \to \infty$, $p_{an} \approx \frac{1}{\sqrt{\pi n}} e^{-4\left(\sqrt{n}\tilde{x}_n\right)^2}$ so $\lim_{n\to\infty} \sqrt{n}\tilde{x}_n = l$, where l is the unique solution of the equation $Dl = \frac{1}{\sqrt{\pi}} e^{-4l^2}$. We can conclude with Proposition 3.

Now assume $q_a\Delta U_a > q_b\Delta U_b$(the proof of the other case is specular). For every $\alpha, \gamma \in [0,1]$ let $x'(\alpha,\gamma)$ be the amount of information acquired by a voter when the cutoff is γ thus $x'(\alpha,\gamma)$ is the solution of $C_x(\alpha, x'(\alpha,\gamma)) = C_x(\gamma, 0)$. $x(\alpha,\gamma)$. Whenever the cutoff is γ, the expected amount of information acquired by a voter is $\widetilde{x'}(\gamma) = \int_0^\gamma x'(\alpha,\gamma) f(\alpha)\, d\alpha$. It is easily seen that a SBE exists if and only if there exists a cutoff type γ and a wedge μ, $(\gamma,\mu) \in [0,1] \times \left[-\frac{1}{2}, \frac{1}{2}\right]$ satisfying:

$$2\Delta U_a q_a \binom{2n}{n} 2^{-2n} \left\{ 1 - 4\left[\widetilde{x'}(\gamma) + \mu(1 - F(\gamma))\right]^2 \right\}^n = C_x(\gamma, 0) \quad (18)$$

$$2\Delta U_b q_b \binom{2n}{n} 2^{-2n} \left\{ 1 - 4\left[\widetilde{x'}(\gamma) - \mu(1 - F(\gamma))\right]^2 \right\}^n = C_x(\gamma, 0) \quad (19)$$

The SBE has information acquisition if and only if $-\frac{1}{2} < \mu < \frac{1}{2}$ so that $\gamma > 0$. For every $\mu \in \left(-\frac{1}{2}, \frac{1}{2}\right)$. Let $\gamma_n^I(\mu)$ the solution of equation 18 and let $\gamma_n^{II}(\mu)$ the solution of equation 19. The function $\gamma_n^I(\mu)$ has a maximum γ_n^I which satisfies

$$2\Delta U_a q_a 2^{-2n} \binom{2n}{n} = C_x\left(\gamma_n^I, 0\right)$$

and it is reached for $\mu_n = \mu_n^I$ where

$$\mu_n^I = \frac{-\widetilde{x}\left(\gamma_n^I\right)}{\left(1 - F\left(\gamma_n^I\right)\right)}$$

The function $\gamma_n^{II}(\mu)$ has a maximum γ_n^{II} which satisfies

$$2\Delta U_b q_b 2^{-2n}\binom{2n}{n} = C_x\left(\gamma_n^{II}, 0\right)$$

and it is reached for $\mu_n = \mu_n^{II}$ where

$$\mu_n^{II} = \frac{\widetilde{x'}\left(\gamma_n^{II}\right)}{\left(1 - F\left(\gamma_n^{II}\right)\right)}$$

Observe that $-\frac{1}{2} < \mu_n^I < 0 < \mu_n^{II} < \frac{1}{2}$. Notice that $0 < \gamma_n^{II} < \gamma_n^I$ because $q_b\Delta U_b < q_a\Delta U_a$ and C_x is increasing in γ. All sequences converge to 0 as $n \to \infty$ because $2^{-2n}\binom{2n}{n} \approx \frac{1}{\sqrt{\pi n}}$. Furthermore, $\gamma_n^{II}\left(\mu_n^I\right) < \gamma_n^I$ so that in order to prove that a SBE with information acquisition exists for n large enough it suffices to prove that $\gamma_n^{II} \geq \gamma_n^I\left(\mu_n^{II}\right)$ for n large enough . The left hand side of equation 18 is decreasing in γ for $\mu > \mu_n^I$. So this is equivalent to check that

$$\left\{1 - 4\left[\widetilde{x'}\left(\gamma_n^{II}\right) + \mu_n^{II}\left(1 - F\left(\gamma_n^{II}\right)\right)\right]^2\right\}^n \leq \frac{\Delta U_b q_b}{\Delta U_a q_a}$$

for n large enough. For $n \to \infty$:

$$\left\{1 - 4\left[\widetilde{x'}\left(\gamma_n^{II}\right) + \mu_n^{II}\left(1 - F\left(\gamma_n^{II}\right)\right)\right]^2\right\}^n \approx e^{-4\left\{\sqrt{n}\left[\widetilde{x'}\left(\gamma_n^{II}\right) + \mu_n^{II}\left(1 - F\left(\gamma_n^{II}\right)\right)\right]\right\}^2}$$

Set $\tau_n = 2^{-2n}\binom{2n}{n} \approx \frac{1}{\sqrt{\pi n}}$. From the first part of the proof follows that $\tilde{x}\left(\gamma_n^{II}\right) \approx D\tau_n$ for some positive D.[18] Then $\mu_n^{II} = \frac{\widetilde{x}\left(\gamma_n^{II}\right)}{\left(1 - F\left(\gamma_n^{II}\right)\right)} \approx D\tau_n$ and

$$-4\left\{\sqrt{n}\left[\widetilde{x'}\left(\gamma_n^{II}\right) + \mu_n^{II}\left(1 - F\left(\gamma_n^{II}\right)\right)\right]\right\}^2 \to -8\frac{D}{\sqrt{\pi}}.$$

In particular an equilibrium exists if $\frac{\Delta U_a q_a}{\Delta U_b q_b} < t_2 = e^{\frac{\sqrt{\pi}}{8D}}$.

Let us prove the second part of the claim. If a SBE with information acquisition exists for n large enough we have

$$\sqrt{\pi} 4\Delta U_a q_a 2e^{-4n\left[\widetilde{x}_n + \mu_n\right]^2} \approx \frac{2C}{f(0)B}\sqrt{n}\tilde{x}_n$$

[18]The proof is exactly as above.

$$\sqrt{\pi}4\Delta U_b q_b 2e^{-4n[\tilde{x}_n-\mu_n]^2} \approx \frac{2C}{f(0)\,B}\sqrt{n}\tilde{x}_n$$

because from the first part of the proof we have $\tilde{x}_n \approx \frac{f(0)q_b\Delta U_b}{C}p_n$, where $p_n = p_{an}$. In particular there are converging sub-sequences $\sqrt{n}\tilde{x}_n \to l$, $\sqrt{n}\mu_n \to k$, where

$$\sqrt{\pi}4\Delta U_a q_a e^{-4(l+k)^2} = \frac{C}{f(0)\,B}l \tag{20}$$

$$\sqrt{\pi}4\Delta U_b q_b e^{-4(l-k)^2} = \frac{C}{f(0)\,B}l. \tag{21}$$

So the first part of the claim follows from Proposition 3.

In order to complete we show that if $z = \frac{q_a\Delta U_a}{q_b\Delta U_b}$ is large then no SBE with information acquisition exists. It suffices to show that the system constituted by equations 20 and 21 does not have a solution if z is too large. Notice that, if the system has solution (k,l), then $0 < l \leq \min\left\{\frac{\sqrt{\pi}4\Delta U_b q_b Bf(0)}{C}, \frac{\sqrt{\pi}4\Delta U_a q_a Bf(0)}{C}\right\} = \frac{\sqrt{\pi}4\Delta U_b q_b Bf(0)}{C} = \bar{l}$. The system can be written as

$$\bar{l}ze^{-4(l+k)^2} = l$$

$$\bar{l}e^{-4(l-k)^2} = l.$$

It follows that if (l,k) is a solution, then $16lk = \log z$. Substituting into equation 21 we obtain that the system has a solution if and only if there exists $l \in \left(0,\bar{l}\right]$ satisfying $H(l,z) = 0$, where

$$H(l,z) = 4l^2 + \frac{4\,(logz)^2}{16^2l^2} - 32\log z + \log l - \log \bar{l}.$$

Notice that $H_l(l,z) = \frac{\left(8l^2+2\right)^2 - 4 - (\log z)^2}{32l^3}$. Thus, $H_l(l,0) < 0$ on $\left(0,\bar{l}\right)$ if $z \geq e^{8\bar{l}^2+2}$. In this case, for every z, H has a minimum at $l = \bar{l}$, where it reaches the value $H\left(\bar{l},z\right) = 4\bar{l}^2 + \frac{4(logz)^2}{16^2\bar{l}^2} - 32\log z$. Since $\lim_{z\to\infty} H\left(\bar{l},z\right) = \infty$, the system does not have a solution if z is too large. ■

Proof of Theorem 3. Assume that $\frac{q_b\Delta U_b}{q_a\Delta U_a} \leq 1$ (the proof of the other case is specular). First assume that $C^2_{x^{(2)}\alpha}(0,0) - C_{x\alpha^{(2)}}(0,0)\,C_{x^{(3)}}(0,0) > 0$ with $C_{x^{(3)}}(0,0) > 0$ and $C_{x\alpha^{(2)}}(0,0) > 0$. At every SBE

$p_{an} = \frac{q_b \Delta U_b}{q_a \Delta U_a} p_{bn}$. Set $p_{nb} = p_n$. Set $\alpha(p) = \alpha\left(\frac{q_b \Delta U_b}{q_a \Delta U_a} p, p\right)$, set $x(\alpha, p) = x\left(\alpha, \left(\frac{q_b \Delta U_b}{q_a \Delta U_a} p, p\right)\right)$ and set $\tilde{x}(p) = \tilde{x}\left(\frac{q_b \Delta U_b}{q_a \Delta U_a} p, p\right)$. We start by obtaining an approximation of $\tilde{x}(p)$ as $p \to 0$.

First observe that $\alpha(p) \approx \left(\frac{2pq_b \Delta U_a}{C_{x\alpha^{(2)}}(0,0)}\right)^{\frac{1}{2}}$ as $p \to 0$. For $x = x(\alpha, p)$, we have:

$$4pq_b \Delta U_b \Delta = 2C_x(\alpha, x) = C_{x\alpha\alpha}(0,0)\alpha^2 + C_{xxx}(0,0)x^2 + 2C_{xx\alpha}(0,0)\alpha x + F(\alpha, x)\alpha^2 + G(\alpha, x)\alpha x + H(x, y)x^2.$$

where F, G and H converge to 0 as (x, y) converges to 0. Set $\bar{\alpha}(p) = min\left\{\alpha(p), \left(\frac{2pq_b \Delta U_b}{C_{x\alpha^2}(0,0)}\right)^{\frac{1}{2}}\right\}$ and notice that $\bar{\alpha}(p) - \alpha(p) = O(p)$.

Integrating and discarding infinitesimal of higher order one obtains that

$$\tilde{x}(p) \approx \int_0^{\bar{\alpha}(p)} \frac{-C_{xx\alpha}(0,0)\alpha + \sqrt{\left[C_{xx\alpha}^2(0,0) - C_{x\alpha\alpha}(0,0)C_{xxx}(0,0)\right]\alpha^2 + 4C_{xxx}(0,0)pq_b \Delta U_b}}{C_{xxx}(0,0)} f(\alpha)\, d\alpha$$

from which we obtain $\tilde{x}(p) \approx Cp \log p$ for some constant $C < 0$.

The proof of the existence of a SBE with information acquisition in the case $q_b \Delta U_b = q_a \Delta U_a$ is unchanged. Let's consider informational efficiency:

Notice that $\lim_{n\to\infty} \frac{\tilde{x}_n}{p_{an}} = \infty$, furthermore $p_{an} \approx \frac{1}{\sqrt{\pi n}} e^{-4\left(\sqrt{n}\tilde{x}_n\right)^2}$ so $\lim_{n\to\infty} \sqrt{n}\tilde{x}_n = \infty$ We can conclude with Proposition 3.

Now assume $q_b \Delta U_b < q_a \Delta U_a$. In the proof of Theorem 2 we observed that a SBR with information acquisition and 2n+1 voters exists if $\left\{1 - 4\left[\tilde{'x}\left(\gamma_n^{II}\right) + \mu_n^{II}\left(1 - F\left(\gamma_n^{II}\right)\right)\right]^2\right\}^n \leq \frac{\Delta U_b q_b}{\Delta U_a q_a}$, where γ_n^{II} and μ_n satisfy $2\Delta U_b q_b 2^{-2n}\binom{2n}{n} = C_x\left(\gamma_n^{II}, 0\right)$ and $\mu_n^{II} = \frac{\tilde{x'}\left(\gamma_n^{II}\right)}{\left(1 - F\left(\gamma_n^{II}\right)\right)}$. From $\tilde{x}(p) \approx Cp \log p$ as $p \to 0$, follows $\tilde{x'}\left(\gamma_n^{II}\right) \approx \frac{C}{\sqrt{\pi n}} log\left(\frac{1}{\sqrt{\pi n}}\right)$, thus $\lim_{n\to\infty} \sqrt{n}\tilde{x}\left(\gamma_n^{II}\right) = \infty$ and $\lim_{n\to\infty} \sqrt{n}\mu_n^{II} = \infty$. This fact implies that $\lim_{n\to\infty}\left\{1 - 4\left[\tilde{x}\left(\gamma_n^{II}\right) + \mu_n^{II}\left(1 - F\left(\gamma_n^{II}\right)\right)\right]^2\right\}^n = 0$ so a SBEequilibrium with information acquisition exists for n large enough. Then a SBE with information acquisition exists for n large enough.

In order to complete the proof we have to consider the information aggregation properties of the elections. Consider a sequence of SBE with information acquisition. Notice that $\lim_{p\to 0} \frac{\tilde{x}(p)}{p} = 0$ and so $\lim_{n\to\infty} \frac{\tilde{x}_n}{p_{\omega n}} = \infty$ for $\omega = a, b$. We have

$$p_{bn} \approx \frac{e^{n\log\left\{1-4[\tilde{x}_n-\mu_n(1-F(\alpha_n))]^2\right\}}}{\sqrt{\pi n}}.$$

Dividing both site of the equivalence by $\tilde{x}_n$ we obtain $\lim_{n\to\infty}\sqrt{n}\tilde{x}_n = \infty$. As $\mu_n \geq 0$ we obtain $\lim_{n\to\infty}\sqrt{n}\theta_{an} = \infty$. From $\tilde{x}(p_{bn}) \approx Cp_{bn}\log(p_{bn})$

$$\sqrt{n}\tilde{x}(p_{bn}) \approx -Ce^{n\log\left\{1-4[\tilde{x}_n-\mu_n(1-F(\alpha_n))]^2\right\}}\frac{1}{\sqrt{\pi}}n\left[\log\left\{1-4\left[\tilde{x}_n-\mu_n\left(1-F\left(\alpha_n\right)\right)\right]^2\right\}-\frac{1}{2n}\log\pi n\right]$$

From $\lim_{n\to\infty}\sqrt{n}\tilde{x}_n = \infty$ we have $\lim_{\to\infty}\sqrt{n}\theta_{bn} = \infty$. Thus the last claim follows from Proposition 3. We can conclude by

Now we consider the most general case. If at least one of the third derivatives of C is zero at $(0,0)$, there are C_1, C_2, C_3 positive such that for p small enough

(i) $C_x(\alpha, x) \leq C_1x^2 + 2C_2\alpha x + C_3\alpha^2$ uniformly for $\alpha \leq \alpha(p)$ and $x \leq x(0,p)$. [19].

(ii) The function $\frac{-C_1\alpha+\sqrt{\left(C_2^2-C_1C_3\right)\alpha^2+2pq_b\Delta U_b}}{C_1}$ is well defined is well defined and non negative for all $\alpha \leq \alpha(p)$.

Then $\frac{-C_1\alpha+\sqrt{\left(C_2^2-C_1C_3\right)\alpha^2+4pq_b\Delta U_b}}{C_1} \leq Dx(\alpha, p)$ for some $D > 0$.

Integrating like in the first part of the proof one obtains that $|p(logp)| = O(\tilde{x}(p))$ as $p \to 0$. Then the claim follows from the same arguments used above. ■

Proof of Theorem 4. In order to prove the claim it suffices to show that there is a profile of symmetric strategies that elect the best candidate with probability close to one, and such that expected costs converge to 0. Theorem 3 prove the result when Condition C2 holds. So, assume that condition C2 does not hold. For every $p > 0$, small enough, let $\beta(p)$ the solution of

$$C_x(\beta, 0) = (2n+1)(q_a\Delta U_a + q_b\Delta U_b)p.$$

[19] In the case where $C_{xxx}(0,0) = C_{xx\alpha}(0,0) = C_{x\alpha\alpha}(0,0) = 0$ the claim holds for every C_1, C_2, C_3 positive.

For $\alpha \leq \beta(p)$, let $y(\alpha, p)$, the solution of

$$C_x(\alpha, y) = (2n+1)(q_a \Delta U_a + q_b \Delta U_b) p$$

and set $y(\alpha, p) = 0$, if $\alpha > \beta(p)$. The function $y(\alpha, p)$ maximizes the social marginal benefits from acquiring the information for a voter of type α, when the probability of being pivotal at any state is p. Set $\tilde{y}(p) = \int_0^{\beta(p)} y(\alpha, p) f(\alpha) d\alpha$. Define

$$S_n(p) = \binom{2n}{n} 2^{-2n} \left(1 - 4\tilde{y}^2(p)\right)^n.$$

Let $p_n \neq 0$ be a fixed point of $S_n(p_n)$, which exists from Brower's fixed point theorem. Set $\beta_n = \beta(p_n)$, and set $y_n(\alpha) = y(\alpha, p_n)$ for all $\alpha \in [0,1]$. Let γ_n satisfying $F(\gamma_n) - F(\beta_n) = 1 - F(\gamma_n)$. Define the strategy v_n as follows: $v_n(\alpha) = (A, B)$ if $\alpha \leq \beta_n = \beta(p_n)$, $v_n(\alpha) = (A, A)$ if $\alpha \leq \gamma_n$, $v_n(\alpha) = (B, B)$ if $\gamma_n < \alpha \leq 1$. Finally set $\tilde{y}_n = \tilde{y}(p_n)$. Thus, if every voter adopts strategy (y_n, v_n) their probability of being pivotal is $p_n = \binom{2n}{n} 2^{-2n} \left(1 - 4\tilde{y}_n^2\right)^n$ and $C_x(\alpha, y_n(\alpha)) = (2n+1)(q_a \Delta U_a + q_b \Delta U_b) p_n$ for every $\alpha \leq \beta_n$.

First observe that

$$C_x(\beta_n, 0) \approx \frac{2}{\sqrt{\pi}} e^{n\left[\log\left(1-4\tilde{y}_n^2\right) + \frac{1}{2n}\log n\right]}$$

so $\lim_{n\to\infty} \beta_n = 0$ which implies that individual expected costs converge to zero. Also, $\lim_{n\to\infty} \tilde{y}_n = 0$ because $\tilde{y}_n \leq x(0, y_n(0)) F(\beta_n)$.

Using the same techniques of the proofs of Theorems 1,2 and 3, it is easy to prove that $\tilde{y}_n \approx C\left((2n+1)p_n\right)^{1+\xi}$ for some ξ, $0 \leq \xi \leq 2$, and some $C > 0$. Thus $p_n \approx \frac{(\tilde{y}_n)^\delta}{C(2n+1)}$ for some δ, $\frac{1}{2} \leq \delta \leq 1$. We have $\frac{1}{\sqrt{\pi n}}\left(1 - 4\tilde{y}_n^2\right)^n \approx \frac{(\tilde{y}_n)^\delta}{C(2n+1)}$ or, equivalently $\frac{2Ce^{-4n\tilde{y}_n^2}}{\left(\sqrt{n}\tilde{y}_n\right)^\delta} \approx \sqrt{\pi} n^{-\delta-\frac{1}{2}}$. from which follows $\sqrt{n}\tilde{y}_n \to \infty$. Thus, from Proposition 3 it follows that the probability of taking the right decision approaches 1 for n large. ■

Proof of Proposition 5. Assume that $C_{xx}(0,0) \neq 0$ and $C_{x\alpha}(0,0) \neq 0$. (i) For every p_a, p_b small let $\beta(p_a, p_b)$ satisfying

$$C_x(\beta, 0) = p_a q_a \Delta U_a + p_b q_b \Delta U_b.$$

β is a continuous function of (p_a, p_b).

Set

$$P_n(p) = \binom{2n}{n} 2^{-2n} \left(1 - 4\tilde{x}^2 (\beta_n (p,p))\right)^n .$$

Let p_n be a fixed point of $P_n(p)$, which exists from Brower's Fixed Point Theorem. Observe that $p_n \neq 0$. Set $x_n(\alpha) = x(\alpha, p_n, p_n)$, set $v_n(\alpha, s_a) = A$ and $v_n(\alpha, s_b) = B$ for every $\alpha \in [0, \beta_n]$.

It is easy to check that (x_n, v_n) is a SBE with information acquisition and that p_n is the probability each voter is pivotal at any of the states in this equilibrium.

The probability a voter chooses the right alternative is $\frac{1}{2}+\tilde{x}_n$, where $\tilde{x}_n = \tilde{x}(\beta_n, p_n, p_n) = \frac{1}{F(\beta_n)} \int_0^{\beta_n} x(\alpha, p_n, p_n) f(\alpha) d(\alpha)$. Working as in Lemma 2 we obtain that $\beta_n \approx \frac{2p_n q_a \Delta U_a}{C_{x\alpha}(0,0)}$ and $\tilde{x}_n \approx \frac{3q_a \Delta U_a p_n}{C_{xx}(0,0)}$. Thus, $p_n \approx \frac{C_{xx}(0,0)}{3q_a \Delta U_a} \tilde{x}_n$. Furthermore $p_n \approx \frac{1}{\sqrt{\pi n}} e^{-4n\tilde{x}_n^2}$. We have

$$\sqrt{\pi n} \frac{C_{xx}(0,0)}{3q_a \Delta U_a} \tilde{x}_n \approx e^{-4n\tilde{x}_n^2}$$

form which follows the claim.

If

(i) Consider any β_n-epistocratic game.

From Lemma 1, in any SBE with information acquisition $p_{an} = p_{bn} = p_n$ and $\mu_n = 0$. Notice that in order to maximize the probability of electing the best candidaecandidate it is necessary to look for a β_n which is at most indifferent between acquiring information and not. Formally we have to look for β_n such that

$$C_x(\beta_n, 0) \leq 2p_n q_a \Delta U_a$$

Thus $\beta_n \leq \beta(p_n, p_n)$ where $\beta(p_a, p_b)$ has been defined in the proof of part (ii). Set $C_n = \frac{\beta_n}{p_n}$. Notice that $\bar{C} = \sup \lim_{n\to\infty} C_n \leq \frac{2q_a \Delta U_a}{C_{x\alpha}(0,0)}$.

Thus the information acquired by an elector of unknown type is $\tilde{x}_n = \frac{1}{F(\beta_n)} \int_0^{\beta_n} x(\alpha, p_n, p_n) f(\alpha) d(\alpha)$. The probability an elector votes for the best candidate at each state is $\frac{1}{2} + \tilde{x}_n$. Working as in Lemma 2 we obtain

$$\widetilde{x}_n \quad \approx \quad \left(\frac{C_n C_{x\alpha}(0,0) + 4q_a \Delta U_a}{2C_{xx}(0,0)}\right) p_n$$

So $p_n \approx \frac{2C_{xx}(0,0)}{C_n C_{x\alpha}(0,0)+4q_a\Delta U_a}\tilde{x}_n$. It follows that

$$\sqrt{\pi n}\frac{2C_{xx}(0,0)}{C_n C_{x\alpha}(0,0) + 4q_a \Delta U_a}\tilde{x}_n \approx e^{-4n\tilde{x}_n^2}$$

.

Without loss of generality assume that $C_n \to C$, Then $\sqrt{n}\tilde{x}_n \to l$ where

$$\sqrt{\pi}\frac{2C_{xx}(0,0)}{CC_{x\alpha}(0,0) + 4q_a \Delta U_a} l \approx e^{-4l^2}$$

The limit l is a decreasing function of l so it is maximized for $C = \bar{C}$. This yields a supremum of the possible values of $\bar{l}$ satisfying:

$$\sqrt{\pi}\frac{C_{xx}(0,0)}{3q_a \Delta U_a} l \approx e^{-4l^2}.$$

The case where $C_{x\alpha}(0,0) = 0$ and $C_{xx}(0,0) = 0$ is proved similarly through the approximation found in the proof of Lemma 2.

The strategy of proof for the case where either $C_{x\alpha}(0,0) = 0$ or $C_{x\alpha}(0,0)$, is identical and uses the approximations for $\tilde{x}(p)$ that were found in Theorems 1, 2 and 3. The details are available on request. ■

References

[1] Aidt, Toke, S., Voting and information, Electoral Studies 19, 349-362, 2000

[2] Althaus, Scott L., "Information Effects in Collective Preferences."American Political Science Review 92 (1998), 545-558.

[3] Althaus, Scott. L. "Collective Preferences in Democratic Politics: Opinion Surveys and the Will of the People." New York: Cambridge University Press, 2003.

[4] Austen-Smith, David and Banks, Jeffrey S., "Information aggregation, rationality, and the Condorcet jury theorem." American Political Science Review, 90 (1996), 34–45.

[5] .Bartels, Larry M. 1996. "Uninformed Votes: Information Effects in Presidential Elections." American Journal of. Political Science 40:194-230.

[6] Berend, Daniel, Paroush Jacob, "When is Condorcet's jury theorem valid?" Social Choice and Welfare 15 (1998), 481-88.

[7] Blendon, R.J., Benson, J.M., Brodie, M., Morin, R., Altman, D.E., Gitterman, D., Brossard, M., James, M., "Bridging the gap between the public's and economists' view of the economy. Journal of Economic Perspectives 11 (1997), 105–118.

[8] Berg, Sven, "Condorcet's Jury Theorem, Dependency among Jurors." Social Choice and Welfare 10 (1993), 87-95

[9] Chow, Yuan Shih, Teicher Henry, "Probability Theory: Independence, Interchangeability, Martingales." Springer-Verlag, 1997

[10] Condorcet, Marquis de, "Essai sur l'application de l'analyse à la probabilité des décisions rendues à la probabilité des voix." Paris: De l'imprimerie royale, 1785.

[11] Delli Carpini, Michael X., Keeter, Scott, "What Americans Know About Politics and Why It Matters." New Haven: Yale University Press, 1996.

[12] Downs, Anthony, "An Economic Theory of Democracy." New York: Harper-Collins Publishers, 1957.

[13] Estlund, David, "Democratic Authority;" Princeton: Princeton University Press, 2007.

[14] Feddersen, Timothy J., Pesendorfer, Wolfgang, "The Swing Voter' s Curse." American Economic Review, 86 (1996), 408-424.

[15] Feddersen, Timothy J., Pesendorfer, Wolfgang , "Voting Behavior and Information Aggregation in Elections with Private Information." Econometrica, 65 (1997), 1029-1058.

[16] Feddersen, Timothy J., Pesendorfer, Wolfgang, "Abstention in Elections with Asymmetric Information and Diverse Preferences." American Political Science Review, 93 (1999), 381- 398.

[17] Gerardi, Dino, Yariv, Leeat, "Deliberative Voting." Journal of Economic Theory 134 (2007), 317-338.

[18] Grofman, Bernard and Feld, Scott L., "Rousseau's general will: a Condorcetian Perspective,."American Political Science Review, 82 (1988)

[19] Gilens Martin, "Political Ignorance and Collective Policy Preference." American Political Science Review 95 (2001), 379-396.

[20] Hamilton, Alexander, Madison James, Jay John. "The Federalist: A Commentary on the Constitution of the United States." Modern Library; New Ed edition (1788), 2000.

[21] Ladha, Krishna, "The Condorcet Jury Theorem, Free Speech, and Correlated Votes", American Journal of Political Science 36 (1992), 617-34.

[22] Ladha, Krishna, "Condorcet's Jury Theorem in the Light of de Finetti's Theorem: Majority Rule with Correlated Votes." Social Choice and Welfare 10 (1993), 69-86.

[23] Lau, Richard R., and David P. Redlawsk. "How Voters Decide: Information Processing During Election Campaigns." New York: Cambridge University Press (2006).

[24] Mandler, Michael, "The Fragility of Information Aggregation in Large Elections." Mimeo (2007).

[25] Martinelli, Cesar, "Would Rational Voters Acquire Costly Information?" Journal of Economic Theory 129 (2006), 225-251.

[26] Martinelli, Cesar, "Rational Ignorance and Voting Behavior." International Journal of Game Theory 35 (2007), 315-335.

[27] Mill, John Stuart, "Consideration on Representatiive Government". New York: H. Holt and Co (1890).

[28] Mukhopadhaya, Kaushik, "Jury Size and the Free Rider Problem.." Journal of Law, Economics and Organization 19 (2003), 24-44.

[29] Myerson, Roger B. “Extended Poisson Games and the Condorcet Jury Theorem.” Games and Economic Behavior 25 (1998), 111-131.

[30] Nannestad, P., Paldam, M., 2000. “What do voters know about the economy? A study of Danish data, 1990–93.” Electoral Studies 29, 363–391.

[31] Nordin, Mattias “The effect of information on voting behavior”, Working paper, Uppsala University (2009).

[32] Oliveros, Santiago, “Who abstains in equilibrium?” Mimeo, (2009).

[33] Oliveros, Santiago, “Endogenous information in committees and aggregation of information in large elections.” Mimeo (2008).

[34] Persico, Nicola, “Committee design with endogenous information.” Review of Economic Studies 71 (2004), 165-191.

[35] Polybius, Histories, Loeb Classical Library, 1992.

[36] Riker, William. “Liberalism Against Populism.” New York: Freeman (1982).

[37] Rousseau, Jean-Jacques, “Social Contract” ed. Roger D. Masters, trans. Judith R. Masters, New York: St. Martin’s Press (1978).

[38] Schumpeter, Joseph A., “Capitalism, Socialism, and Democracy.”, third edition, New York: Harper-Collins Publishers (1950).

[39] Paroush, Jacob, “Stay away from fair coins: A Condorcet jury theorem.” Social Choice and Welfare 15 (1998): 15- 20.

[40] Wittman, Donald, “Why Democracies Produce Efficient Results.” Journal of Political Economy 97 (1989), 1395-1424.

[41] Wittman, Donald, “The Myth of Democratic Failure: Why Political Institutions are Efficient,” University of Chicago Press, 1995.

[42] Yariv, Leeat. "When majority rule yields majority ruin." Mimeo (2004).

[43] Young, H. P. "Condorcet's theory of voting." American Political Science Review 82 (1988), 1231—1244.

Centro de Economía Aplicada
Departamento de Ingeniería Industrial
Universidad de Chile

2010

267. Costly information acquisition. Better to toss a coin?
Matteo Triossi

266. Firm-Provided Training and Labor Market Institutions
Felipe Balmaceda

2009

265. Soft budgets and Renegotiations in Public-Private Partnerships
Eduardo Engel, Ronald Fischer y Alexander Galetovic

264. Information Asymmetries and an Endogenous Productivity Reversion Mechanism
Nicolás Figueroa y Oksana Leukhina

263. The Effectiveness of Private Voucher Education: Evidence from Structural School Switches
Bernardo Lara, Alejandra Mizala y Andrea Repetto

262. Renegociación de concesiones en Chile
Eduardo Engel, Ronald Fischer, Alexander Galetovic y Manuel Hermosilla

261. Inflation and welfare in long-run equilibrium with firm dynamics
Alexandre Janiak y Paulo Santos Monteiro

260. Conflict Resolution in the Electricity Sector - The Experts Panel of Chile
R. Fischer, R. Palma-Behnke y J. Guevara-Cedeño

259. Economic Performance, creditor protection and labor inflexibility
Felipe Balmaceda y Ronald Fischer

258. Effective Schools for Low Income Children: a Study of Chile's Sociedad de Instrucción Primaria
Francisco Henríquez, Alejandra Mizala y Andrea Repetto

257. Public-Private Partnerships: when and how
Eduardo Engel, Ronald Fischer y Alexander Galetovic

2008

256. Pricing with markups in industries with increasing marginal costs
José R. Correa, Nicolás Figueroa y Nicolás E. Stier-Moses

255. Implementation with renegotiation when preferences and feasible sets are state dependent
Luis Corchón y Matteo Triossi

254. Evaluación de estrategias de desarrollo para alcanzar los objetivos del Milenio en América Latina. El caso de Chile
Raúl O'Ryan, Carlos J. de Miguel y Camilo Lagos

253. Welfare in models of trade with heterogeneous firms
Alexandre Janiak

252. Firm-Provided Training and Labor Market Policies
Felipe Balmaceda

251. Emerging Markets Variance Shocks: Local or International in Origin?
Viviana Fernández y Brian M. Lucey

250. Economic performance, creditor protection and labor inflexibility
Ronald Fischer

249. Loyalty inducing programs and competition with homogeneous goods
N. Figueroa, R. Fischer y S. Infante

248. Local social capital and geographical mobility. A theory
Quentin David, Alexandre Janiak y Etienne Wasmer

247. On the planner's loss due to lack of information in bayesian mechanism design
José R. Correa y Nicolás Figueroa

246. Política comercial estratégica en el mercado aéreo chileno
Ronald Fischer

245. A large firm model of the labor market with entry, exit and search frictions
Alexandre Janiak

244. Optimal resource extraction contracts under threat of expropriation
Eduardo Engel y Ronald Fischer

2007

243. The behavior of stock returns in the Asia-Pacific mining industry following the Iraq war
Viviana Fernandez

242. Multi-period hedge ratios for a multi-asset portfolio when accounting for returns comovement
Viviana Fernández

241. Competition with asymmetric switching costs
S. Infante, N. Figueroa y R. Fischer

240. A Note on the Comparative Statics of Optimal Procurement Auctions
Gonzalo Cisternas y Nicolás Figueroa

239. Parental choice and school markets: The impact of information approximating school effectiveness
Alejandra Mizala y Miguel Urquiola

238. Marginal Cost Pricing in Hydro-Thermal Power Industries: Is a Capacity Charge Always Needed?
M. Soledad Arellano and Pablo Serra

237. What to put on the table
Nicolas Figueroa y Vasiliki Skreta

236. Estimating Discount Functions with Consumption Choices over the Lifecycle
David Laibson, Andrea Repetto y Jeremy Tobacman

235. La economía política de la reforma educacional en Chile
Alejandra Mizala

234. The Basic Public Finance of Public-Private Partnerships
Eduardo Engel, Ronald Fischer y Alexander Galetovic

233. Sustitución entre Telefonía Fija y Móvil en Chile
M. Soledad Arellano y José Miguel Benavente

232. Note on Optimal Auctions
Nicolás Figueroa y Vasiliki Skreta.

231. The Role of Outside Options in Auction Design
Nicolás Figueroa y Vasiliki Skreta.

230. Sequential Procurement Auctions and Their Effect on Investment Decisions
Gonzalo Cisternas y Nicolás Figueroa

2006

229. Forecasting crude oil and natural gas spot prices by classification methods
Viviana Fernández

228. Copula-based measures of dependence structure in assets returns
Viviana Fernández

227. Un Análisis Econométrico del Consumo Mundial de Celulosa
José Ignacio Sémbler, Patricio Meller y Joaquín Vial

226. The Old and the New Reform of Chile's Power Industry. (Por aparecer en el International Journal of Global Energy Issues (forthcoming 2007)).
M. Soledad Arellano

225. Socioeconomic status or noise? Tradeoffs in the generation of school quality information. (Por aparecer en el Journal of Development Economics).
Alejandra Mizala, Pilar Romaguera y Miguel Urquiola.

224. Mergers and CEO power
Felipe Balmaceda

123. Task-Specific Training and Job Design.
Felipe Balmaceda

122. Performance of an economy with credit constraints, bankruptcy and labor inflexibility
Felipe Balmaceda y Ronald Fischer

121. Renegotiation without Holdup: Anticipating spending and infrastructure concessions
Eduardo Engel, Ronald Fischer y Alexander Galetovic

220. Using School Scholarships to Estimate the Effect of Government Subsidized Private Education on Academic Achievement in Chile
Priyanka Anand, Alejandra Mizala y Andrea Repetto

219. Portfolio management implications of volatility shifts: Evidence from simulated data
Viviana Fernandez y Brian M Lucey

218. Micro Efficiency and Aggregate Growth in Chile
Raphael Bergoeing y Andrea Repetto

* Para ver listado de números anteriores ir a http://www.cea-uchile.cl/.