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MULTI-PERIOD HEDGE RATIOS FOR A MULTI-ASSET PORTFOLIO WHEN ACCOUNTING FOR RETURNS COMOVEMENT

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Multi-period hedge ratios for a multi-asset portfolio when accounting for returns co-movement

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Abstract

This article presents a model to select the optimal hedge ratios of a portfolio comprised of an arbitrary number of commodities. In particular, returns dependency and heterogeneous investment horizons are accounted for by copulas and wavelets, respectively. We analyze a portfolio of London Metal Exchange metals for the period July 1993-December 2005, and conclude that neglecting cross correlations leads to biased estimates of the optimal hedge ratios and the degree of hedge effectiveness. Furthermore, when compared with a multivariate-GARCH specification, our methodology yields higher hedge effectiveness for the raw returns and their short-term components.

JEL: C22, G15; Keywords: hedge ratio, multivariate copulas, wavelets, multivariate GARCH.

1 Introduction

The determination of the optimal hedge ratio is an issue of both practical and theoretical interest as its impacts hedging effectiveness. Recent contributions to the literature on heterogeneous investors and the selection of an optimal hedge ratio has focused on a single commodity in isolation (e.g., Lien and Shrestha, 2007; In and Kim, 2006a, 2006 b). This article presents a generalization of such an approach, in which we consider the selection of optimal hedge ratios for a portfolio comprised of several net positions in commodities (i.e., cash position minus a proportion of a futures contract). In particular, we obtain an analytical expression for an optimal vector of hedge ratios and a measure of the degree hedge effectiveness (HE) for such a portfolio. Towards that end, we utilize wavelet and

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copula analysis in order to accommodate for the existence of heterogeneous investors and asset returns dependency, respectively.

Our empirical application considers a portfolio composed of cash and futures positions on London Metal Exchange (LME) metals for the sample period July 1993-December 2005. Our portfolio choice is based on the grounds of data availability on cash and futures prices at a high frequency. Indeed, wavelet analysis is a computationally intensive statistical tool, which requires long time series (e.g., Percival and Walden, 2000; Gençay, Whitcher, and Selçuk, 2002). On the other hand, given that the LME data has been earlier analyzed (e.g., McMillan, 2005), our conclusions can be contrasted with those of previous studies.

The simulations we carry out to characterize the distribution functions of the hedge ratios and the degree of HE are performed under two scenarios: with and without accounting for the cross correlations of returns on cash and futures contracts. As mentioned above, dependency is handled by copula analysis, a statistical tool that has recently gained ground in the finance field to extract the dependence structure from the joint probability distribution function of a set of random variables (e.g., Cherubini, Luciano, and Vecchiato, 2004).

Indeed, earlier attempts to determine optimal hedge ratios in a multi-asset setting have modeled assets returns dependency exclusively by means of either static or dynamic correlations. For instance, Lien (1988) focuses on hedging foreign exchange risk of a portfolio comprised of positions on Canadian dollars and German marks. Although Lien derives an analytical solution to the optimal vector of hedge ratios taking into account the covariance structure of spot and futures price changes, no dynamic consideration as to the evolution of such covariance structure through time is discussed. In that regard, the article by Gagnon, Lypny, and McCurdy (1998) represents a further contribution to the literature. Indeed, the authors consider a portfolio made up of spot and futures positions on the Deutsche Mark, the Swiss Franc, and the Japanese Yen, and model their time-varying co-variability by a trivariate BEKK model.

We move a step forward and model the joint probability density function of the spot and futures positions, not only their covariance structure.[2] Indeed, we carry out some simulation exercises which show that neglecting the correlation across commodities leads to biased estimates of the optimal hedge ratios and the degree of hedge effectiveness. In addition, we conclude that commodities markets are driven by the existence of heterogeneous investors, as has been documented in recent studies. Furthermore, when compared with a multivariate-GARCH specification, we find that our methodology yields higher hedge effectiveness for the raw returns (i.e., without accounting for the holding period of the investment) and for their lower wavelet-timescales (i.e., short-term components).

This article is organized as follows. Section 2 provides an overview of some recent studies on the determination of the optimal hedge ratio. Section 3, which covers methodological issues, is divided into three parts. Section 3.1 derives an optimal vector of hedge ratios for a portfolio made up of cash and futures positions of several commodities. Section 3.2 presents an overview of wavelet analysis, and it provides the definitions of wavelet variance and covariance. Section 3.3 in turn offers an introduction to copulas and an algorithm to simulate dependent asset returns. Section 4 presents an empirical application to cash and futures positions on metals. In particular, Section 4.1 presents descriptive statistics of the data whereas Section 4.2 reports simulations of the distribution

[2] If returns are not jointly normal, their covariance structure may not be an adequate measure of dependency.

functions of optimal hedge ratios and the degree of effectiveness of the hedging strategy, with and without accounting for returns cross correlations. Section 4.3 in turn compares, in terms of hedge effectiveness, the copula approach with the more standard tool of multivariate GARCH models. Finally, Section 5 presents our main conclusions.

2 Background

The existent literature on the determination of the optimal hedge ratio is quite broad, and early contributions in this area can be traced back to the late 1970's. For instance, Ederington (1979) resorted to variance minimization of the hedged portfolio to obtain a minimum-variance (MV) hedge ratio. Alternative derivations of the hedge ratio, which are consistent with mean-variance analysis, involve objective functions where both the expected return and the variance of the hedged portfolio are considered (e.g., Howard and D'Antonio, 1984; Ceccheti, Cumby, and Figlewski, 1988; Hsin, Kuo, and Lee, 1994). Such an optimal mean-variance hedge ratio will be equivalent to the MV hedge ratio if the futures price is a martingale process.

A key point to consider, nevertheless, is that the mean-variance hedge ratio approach will be consistent with expected utility maximization only if either the utility function is quadratic or returns are jointly normal. In order to work under more general assumptions, alternative optimization methods have been recently considered: mean-extended-Gini minimization and generalized-semivariance minimization, among others. Chen, Lee, and Shrestha (2006), in press, discuss these alternative procedures, and test whether the pure martingale and joint normality hypotheses hold for spot and futures returns. For a set of 25 commodities, Chen et al. find that joint normality tends to be rejected for all the commodities when the hedging horizon is relatively short, whereas it tends to hold only for a few of them when the hedging horizon is longer. As to the pure martingale hypothesis, they conclude that it holds for almost all of the commodities.

Simple and widely utilized approaches to compute the hedge ratio are the naïve one-to-one and ordinary least squares (OLS). The naïve hedge ratio consists of taking an equal and opposite position in futures relative to the position in cash, whereas the OLS-based hedge ratio is obtained from a linear regression model. Both routes assume that the hedge ratio will remain constant through time. However, for the past 20 years, the finance literature has reported in several studies the existence of time-varying (conditional) volatility and volatility clustering in assets returns. Hence, recent studies have resorted to GARCH-type (e.g., McMillan, 2005; Gagnon, Lypny, and McCurdy, 1998; Lee, 1999) and stochastic volatility models (e.g., Lien and Wilson, 2001) to characterize the behavior of hedge ratios over time

A novel approach, also aimed at capturing the time-varying nature of a hedge ratio, is wavelet analysis. This is a refinement of Fourier analysis which makes it possible to decompose a time series into its high- and low-frequency components (i.e., short- and long-term variation, respectively). Two recent studies by In and Kim (2006a, 2006b) have pioneered the use of this mathematical methodology to obtain a timescale decomposition of the hedge ratio and the hedge effectiveness. They conclude that there is a feedback between spot and futures markets regardless of the timescale. Also a recent application in this area can be found in Lien and Shrestha (2007).

3 Theoretical framework

3.1 Optimal hedge ratio

Let us consider an investor who wishes to hedge some of their cash position invested on ω_i unit of asset i by holding futures contracts:

$\omega_i r_i^c - \beta_i r_i^f$

where r_i^c and r_i^f are the returns on the cash and futures positions on asset i, respectively, and β_i represents the hedge ratio.

Furthermore, let us assume that the investor hold n such cash positions, which are hedged with futures contracts. Then his/her portfolio return r_p is given by:

$$r_p = \boldsymbol{\omega}'\mathbf{r^c} - \boldsymbol{\beta}'\mathbf{r^f} \quad (1)$$

where $\boldsymbol{\omega}'=(\omega_1\ \omega_2 \ldots \omega_n)$ is an n×1 vector unit spot holdings, $\boldsymbol{\beta}'=(\beta_1\ \beta_2 \ldots \beta_n)$ is an n×1 vector of hedge ratios, $\mathbf{r}^{c'} = (r_1^c\ r_2^c \ldots r_n^c)$ and $\mathbf{r}^{f'} = (r_1^f\ r_2^f \cdots r_n^f)$ are n×1 vectors of returns on the cash and futures positions, respectively.

Under the above set of assumptions, the variance of the portfolio is given by

$$\mathrm{Var}(r_p) = \boldsymbol{\omega}'\boldsymbol{\Omega}^C\boldsymbol{\omega} + \boldsymbol{\beta}'\boldsymbol{\Omega}^{\mathbf{f}}\boldsymbol{\beta} - 2\boldsymbol{\omega}'\boldsymbol{\Omega}^{\mathbf{cf}}\boldsymbol{\beta} \quad (2)$$

where $\boldsymbol{\Omega}^c = \begin{pmatrix} \sigma_{11}^c & \sigma_{12}^c & \cdots & \sigma_{1n}^c \\ \sigma_{21}^c & \sigma_{22}^c & \cdots & \sigma_{2n}^c \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1}^c & \sigma_{n2}^c & \cdots & \sigma_{nn}^c \end{pmatrix}$ and $\boldsymbol{\Omega}^f = \begin{pmatrix} \sigma_{11}^f & \sigma_{12}^f & \cdots & \sigma_{1n}^f \\ \sigma_{21}^f & \sigma_{22}^f & \cdots & \sigma_{2n}^f \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1}^f & \sigma_{n2}^f & \cdots & \sigma_{nn}^f \end{pmatrix}$ are n×n variance-covariance matrices of the returns on the cash and futures positions, respectively, and $\boldsymbol{\Omega}^{cf} = \begin{pmatrix} \sigma_{11}^{cf} & \sigma_{12}^{cf} & \cdots & \sigma_{1n}^{cf} \\ \sigma_{21}^{cf} & \sigma_{22}^{cf} & \cdots & \sigma_{2n}^{cf} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1}^{cf} & \sigma_{n2}^{cf} & \cdots & \sigma_{nn}^{cf} \end{pmatrix}$ is an n×n matrix that contains the covariances between the returns on the cash and futures positions (i.e., $\sigma_{ij}^{cf} = \mathrm{cov}(r_i^c, r_j^f)$).

The optimal vector of hedge ratios solves the equation

$$\frac{d\mathrm{Var}(r_p)}{d\boldsymbol{\beta}} = (\boldsymbol{\beta}'\boldsymbol{\Omega}^f)' - (\boldsymbol{\omega}'\boldsymbol{\Omega}^{cf})' = \mathbf{0}$$

That is,

$$\boldsymbol{\beta}^{*} = (\boldsymbol{\Omega}^{f})^{-1}\boldsymbol{\Omega}^{cf'}\boldsymbol{\omega}$$

Without loss of generality, and as is customary in the literature, let us assume that ω_i=1, ∀i=1,.., n. That is,

$$\boldsymbol{\beta}^{*} = (\boldsymbol{\Omega}^{f})^{-1}\boldsymbol{\Omega}^{cf'}\boldsymbol{\iota} \tag{3}$$

where $\boldsymbol{\iota}$ is an n×1 vector of ones.

Notice that if n=1, we have the well-known result

$$\beta^{*} = \frac{\sigma_{11}^{cf}}{\sigma_{11}^{f}}$$

(See for instance, McMillan, 2005 and Chen, Lee, and Shrestha, 2006, in press).

On the other hand, if $\sigma_{ij}^{f} = \sigma_{ij}^{cf} = 0$, ∀i≠j, then

$$\boldsymbol{\beta}^{*} = \begin{pmatrix} \sigma_{11}^{cf}/\sigma_{11}^{f} \\ \sigma_{22}^{cf}/\sigma_{22}^{f} \\ \vdots \\ \sigma_{nn}^{cf}/\sigma_{nn}^{f} \end{pmatrix} \tag{3′}$$

That is, each beta can be estimated from a linear regression of the cash return on the futures returns by ordinary least-squares (OLS).

Equation (3) can be made consistent with the mean-variance framework if we consider, for instance, the mean-variance expected utility maximization problem utilized by Hsin, Kuo and Lee (1994) and In and Kim (2006b):

$$\text{Max}_{\beta}(E(r_p), \sigma_p; A)=E(r_p)-0.5A\,\sigma_p^2$$

where A is a parameter that measures the degree of the risk aversion (A>0).

In our notation, we seek to

$$\text{Max}_{\beta}\ E(\boldsymbol{\iota}'\mathbf{r}^{c} - \boldsymbol{\beta}'\mathbf{r}^{f}) - 0.5\,A\,(\boldsymbol{\iota}'\boldsymbol{\Omega}^{C}\boldsymbol{\iota} + \boldsymbol{\beta}'\boldsymbol{\Omega}^{f}\boldsymbol{\beta} - 2\boldsymbol{\iota}'\boldsymbol{\Omega}^{cf}\boldsymbol{\beta}) \tag{4}$$

From the first-order conditions, we obtain:

$$\tilde{\boldsymbol{\beta}} = -(\boldsymbol{\Omega}^{f})^{-1}\left(\frac{E(\mathbf{r}^{f})}{A} - \boldsymbol{\Omega}^{cf'}\,\boldsymbol{\iota}\right) \tag{5}$$

Therefore, if $A\rightarrow\infty$ (i.e., individuals are infinitely risk averse) or $E(\mathbf{r}^{f})=\mathbf{0}$ (i.e., future prices follow a simple martingale process), we are back to the result of equation (3).

Under our framework, we can generalize the definition of hedge effectiveness (HE) utilized by In and Kim, op cit.:

$$HE = 1 - \frac{Var(r_{p})}{Var(r_{p}^{c})}\bigg|_{\beta^{*}} = 1 - \frac{\boldsymbol{\iota}'\boldsymbol{\Omega}^{C}\boldsymbol{\iota} + \boldsymbol{\beta}'\boldsymbol{\Omega}^{f}\boldsymbol{\beta} - 2\boldsymbol{\iota}'\boldsymbol{\Omega}^{cf}\boldsymbol{\beta}}{\boldsymbol{\iota}'\boldsymbol{\Omega}^{c}\boldsymbol{\iota}}\bigg|_{\beta^{*}} = \frac{\boldsymbol{\iota}'(\boldsymbol{\Omega}^{cf}\boldsymbol{\Omega}^{f^{-1}}\boldsymbol{\Omega}^{cf})\boldsymbol{\iota}}{\boldsymbol{\iota}'\boldsymbol{\Omega}^{c}\boldsymbol{\iota}} \tag{6}$$

where $Var(r_p)$ is given by expression (2) and $Var(r_{p}^{c})$ is the variance of an equally-weighted portfolio invested on spot positions only.

Expression (6) boils down to In and Kim's HE for n=1, in which case $HE = \rho_{sf}^{2}$, i.e., the square of the correlation coefficient between the returns on the stock and futures prices.

When neglecting cross correlations between the spot and futures positions of the different assets, i.e., when OLS hedge ratios are utilized, equation (6) boils down to

$$HE = \frac{\boldsymbol{\iota}'(\text{diag}(\boldsymbol{\Omega}^{cf}) \times \text{diag}(\boldsymbol{\Omega}^{f^{-1}}) \times \text{diag}(\boldsymbol{\Omega}^{cf}))\boldsymbol{\iota}}{\boldsymbol{\iota}'\text{diag}(\boldsymbol{\Omega}^{c})\boldsymbol{\iota}} \tag{6′}$$

where "diag" indicates that only the elements of the main diagonal are considered and the elements off the main diagonal are all set to zero.

In order to have another benchmark with which to compare our minimum-variance hedge ratio, we compute the HE of the naïve hedge ratio. This equals 1 for all the futures positions in the portfolio. We consider two cases: one in which we account for the diversification benefits of having several positions in the portfolio, and another in which we do not. In the first case, the HE equals:

$$\mathrm{HE} = 1 - \frac{\mathrm{Var}(r_p)}{\mathrm{Var}(r_p^c)}\Big|_{\boldsymbol{\beta}=\boldsymbol{\iota}} = 1 - \frac{\boldsymbol{\iota}'\boldsymbol{\Omega}^{C}\boldsymbol{\iota} + \boldsymbol{\iota}'\boldsymbol{\Omega}^{f}\boldsymbol{\iota} - 2\boldsymbol{\iota}'\boldsymbol{\Omega}^{cf}\boldsymbol{\iota}}{\boldsymbol{\iota}'\boldsymbol{\Omega}^{c}\boldsymbol{\iota}} = 1 - \frac{\boldsymbol{\iota}'(\boldsymbol{\Omega}^{c} + \boldsymbol{\Omega}^{f} - 2\boldsymbol{\Omega}^{cf})\boldsymbol{\iota}}{\boldsymbol{\iota}'\boldsymbol{\Omega}^{c}\boldsymbol{\iota}} \tag{7}$$

whereas in the second case

$$\mathrm{HE} = 1 - \frac{\boldsymbol{\iota}'(\mathrm{diag}(\boldsymbol{\Omega}^{c} + \boldsymbol{\Omega}^{f} - 2\boldsymbol{\Omega}^{cf}))\boldsymbol{\iota}}{\boldsymbol{\iota}'\mathrm{diag}(\boldsymbol{\Omega}^{c})\boldsymbol{\iota}} \tag{7'}$$

3.2 Heterogeneous investors and wavelet analysis

A subject, which has received attention in recent studies and which also has important implications for portfolio management, is the existence of heterogeneous investors. In a recent article, Connor and Rossiter (2005) point out that, in the context of commodity markets, long-horizon traders will essentially focus on price fundamentals that drive overall trends, whereas short-term traders will primarily react to incoming information within a short-term horizon. Hence, market dynamics in the aggregate will be the result of the interaction of agents with heterogeneous time horizons. In order to model the behavior of financial series at different time spans, researchers have resorted to wavelet analysis, a mathematical tool developed in the early 1990's (e.g., Li and Stevenson 2001; Gençay, Whitcher, and Selçuk 2003, 2005; Karuppiah and Los, 2005; Connor and Rossiter, 2005; Fernandez, 2005; Fernandez, 2006a, 2006b; In and Kim, 2006a, 2006b; Fernandez and Lucey, 2007).

3.2.1. The discrete wavelet transformation

Wavelets are a refinement of Fourier analysis, which make it possible to decompose a time series into high- and low-frequency components (see, for instance, Percival and

Walden 2000). High-frequency components describe the short-term dynamics, whereas low-frequency components represent the long-term behavior of the series. Wavelets are classified into father and mother wavelets. Father wavelets capture the smooth and low-frequency parts of a signal, whereas mother wavelets describe its detailed and high-frequency parts.

Applications of wavelet analysis usually utilize a discrete wavelet transform (DWT). The DWT maps a vector of n observations to a vector of n smooth and detail wavelet coefficients,[3] which make it possible to capture the underlying smooth behavior of the data and the deviations from it. Given J levels, when the length of the data, n, is divisible by 2^J, there are n/2 wavelet coefficients at the finest scale 2^1, $n/2^2$ coefficients at the next finest scale 2^2, and etcetera.[4] The number of wavelet coefficients at a given scale is related to the width of the wavelet function. This implies that the lowest scales will mimic the short-term fluctuations of the original time series.

In particular, wavelet analysis enables us to decompose a time series into its fundamental components, where each of them contains information regarding the variability of the data at a particular scale. Such a decomposition is called a multi-resolution decomposition (MRD) of a time series y(t), which is the sum of the orthogonal components $S_J(t)$, $D_J(t)$, $D_{J-1}(t)$,.., $D_1(t)$ from scales 1 through J:

$$y(t) \approx S_J(t)+D_J(t)+D_{J-1}(t)+...+D_1(t), \tag{8}$$

where $S_J(t)$ and $D_J(t)$ are denominated the smooth and detail components, respectively. Wavelet scales are such that times are separated by multiples of 2^j, j=1,.., J. For instance,

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[3] $s_{J,k}$ and $d_{j,k}$, j=1,2,…, J, respectively, where J is the total number of levels. At level j=1, ..., J, the $n/2^j$-vector of the detail wavelet coefficients d_{jk} is associated with changes on a scale of length 2^{j-1}, whereas the $n/2^J$-vector of smooth wavelet coefficients s_{Jk} is associated with averages on a scale of length 2^J.

[4] These are denominated dyadic scales (see Percival and Walden, 2000, chapter 1).

for daily data, scale 1 is associated with 2-4 day dynamics, scale 2 with 4-8 day dynamics, scale 3 with 8-16 day dynamics, etcetera.

An application of wavelets, which is of particular interest to this study, is the decomposition of the variance of a time series into its time-scale components. Specifically, wavelet variance analysis enables us to identify which scales are the most important contributors to the overall variability of the data (Percival and Walden, 2000). In particular, let $x_1, x_2,..., x_n$ be a time series of interest, assumed to be a realization of a stationary process with variance σ_X^2. If $\upsilon_X^2(\tau_j)$ denotes the wavelet variance at scale $\tau_j \equiv 2^{j-1}$, then the following relationship holds:

$$\sigma_X^2 = \sum_{j=1}^{\infty} \upsilon_x^2(\tau_j) \tag{9}$$

where the square root of the wavelet variance is expressed in the same units as the original time series.

Let $n'_j = \lfloor n/2^j \rfloor$ be the number of DWT coefficients at level j, where n is the sample size, and let $L'_j \equiv \left\lceil (L-2)(1-\frac{1}{2^j}) \right\rceil$ be the number of DWT boundary coefficients[5] at level j (provided that $n'_j > L'_j$), where L is the width of the wavelet filter. An unbiased estimator of the wavelet variance based on the DWT is given by

$$\tilde{\upsilon}_X^2(\tau_j) \equiv \frac{1}{(n'_j - L'_j)2^j} \sum_{t=L'_j}^{n'_j-1} d_{j,t}^2 \ . \tag{10}$$

[5] $\lfloor x \rfloor$ and $\lceil x \rceil$ represent the greatest integer ≤x and the smallest integer ≥x, respectively. The boundary coefficients are those formed by putting together some values from the beginning and the end of the time series.

Given that the DWT de-correlates the data, the non-boundary wavelet coefficients at a given level ($\mathbf{d}_j$) are zero-mean Gaussian white-noise processes.

Similarly, the unbiased wavelet covariance between time series X and Y, at scale j, can be defined as

$$\tilde{\upsilon}^2_{XY}(\tau_j) \equiv \frac{1}{(n'_j - L'_j)2^j} \sum_{t=L'_j}^{n'_j-1} d^{(X)}_{j,t} d^{(Y)}_{j,t} \quad , \tag{11}$$

provided that $n'_j > L'_j$.

The sample properties of the DWT variance and covariance estimators are, however, inferior to those of non-decimated discrete wavelet transforms, also known as stationary wavelet transforms. The non-decimated DWT is a non-orthogonal variant of the DWT, which is time-invariant. That is, unlike the classical DWT, the output is not affected by the date at which we start recording a time series. In addition, the number of coefficients at each scale equals the number of observations in the original time series. A non-decimated form of the DWT is known as the maximal overlap DWT (MODWT).[6] The unbiased MODWT estimator of the wavelet variance is given by

$$\hat{\upsilon}^2_X(\tau_j) \equiv \frac{1}{M_j} \sum_{t=L_j-1}^{n-1} \tilde{d}^2_{j,t} \tag{12}$$

where $\tilde{d}^2_{j,t}$ is the MODWT-wavelet coefficient at level j and time t, $M_j \equiv n - L_j + 1$, $L_j \equiv (2^j - 1)(L - 1) + 1$ is the width of the MODWT filter for level j, and n is the number of observations in the original time series. While there are n MODWT-wavelet coefficients at

[6] The scaling $(\tilde{l}_k)$ and wavelet $(\tilde{h}_k)$ filter coefficients for the MODWT are rescaled versions of those of the DWT. Specifically, $\tilde{l}_k \equiv l_k / \sqrt{2}$ and $\tilde{h}_k \equiv h_k / \sqrt{2}$.

each level j, the first (L_j–1)-boundary coefficients are discarded. (Retaining such boundary coefficients leads to a biased estimate).

Likewise, the unbiased MODWT estimator of the wavelet covariance can be obtained as

$$\hat{\upsilon}^2_{XY}(\tau_j) \equiv \frac{1}{M_j}\sum_{t=L'_j}^{n-1} \tilde{d}^{(X)}_{j,t}\tilde{d}^{(Y)}_{j,t} \,. \tag{13}$$

A wavelet-based correlation coefficient between X and Y at scale j can be defined as

$$\rho_{XY}(\tau_j) = \frac{\upsilon^2_{XY}(\tau_j)}{\sqrt{\upsilon^2_X(\tau_j)\upsilon^2_Y(\tau_j)}} \,.$$

3.2.2 Time-scale decomposition of the hedge ratio and hedge effectiveness

The minimum-variance hedge ratio at scale j can be determined as

$$\hat{\boldsymbol{\beta}}^*(\tau_j) = (\hat{\boldsymbol{\Omega}}^f(\tau_j))^{-1}\hat{\boldsymbol{\Omega}}^{cf\,\prime}(\tau_j)\boldsymbol{\iota} \tag{14}$$

where

$$\hat{\boldsymbol{\Omega}}^f(\tau_j) = \begin{pmatrix} \hat{\upsilon}^{2\ f}_{11}(\tau_j) & \hat{\upsilon}^{2\ f}_{12}(\tau_j) & \cdots & \hat{\upsilon}^{2\ f}_{1n}(\tau_j) \\ \hat{\upsilon}^{2\ f}_{21}(\tau_j) & \hat{\upsilon}^{2\ f}_{22}(\tau_j) & \cdots & \hat{\upsilon}^{2\ f}_{2n}(\tau_j) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\upsilon}^{2\ f}_{n1}(\tau_j) & \hat{\upsilon}^{2\ f}_{n2}(\tau_j) & \cdots & \hat{\upsilon}^{2\ f}_{nn}(\tau_j) \end{pmatrix}, \hat{\boldsymbol{\Omega}}^{cf}(\tau_j) = \begin{pmatrix} \hat{\upsilon}^{2\ cf}_{11}(\tau_j) & \hat{\upsilon}^{2\ cf}_{12}(\tau_j) & \cdots & \hat{\upsilon}^{2\ cf}_{1n}(\tau_j) \\ \hat{\upsilon}^{2\ cf}_{21}(\tau_j) & \hat{\upsilon}^{2\ cf}_{22}(\tau_j) & \cdots & \hat{\upsilon}^{2\ cf}_{2n}(\tau_j) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\upsilon}^{2\ cf}_{n1}(\tau_j) & \hat{\upsilon}^{2\ cf}_{n2}(\tau_j) & \cdots & \hat{\upsilon}^{2\ cf}_{nn}(\tau_j) \end{pmatrix}$$

by using the notation of the previous sub-section.

Likewise, the HE at scale j can obtained as

$$HE(\tau_j) = \frac{\boldsymbol{\iota}'(\boldsymbol{\Omega}^{cf}(\tau_j)\boldsymbol{\Omega}^{f^{-1}}(\tau_j)\boldsymbol{\Omega}^{cf}(\tau_j))\boldsymbol{\iota}}{\boldsymbol{\iota}'\boldsymbol{\Omega}^c(\tau_j)\boldsymbol{\iota}} \tag{15}$$

In this regard, we can think of the optimal hedge ratio at scale j in (14) as the result of a utility maximization process of an investor whose time horizon is given by τ_j:

$$\text{Max}_{\boldsymbol{\beta}(\tau_j)}\ E(\boldsymbol{\iota}'\mathbf{r}^c(\tau_j) - \boldsymbol{\beta}(\tau_j)'\mathbf{r}^f(\tau_j)) - 0.5A(\boldsymbol{\iota}'\boldsymbol{\Omega}^C(\tau_j)\boldsymbol{\iota} + \boldsymbol{\beta}(\tau_j)'\boldsymbol{\Omega}^f(\tau_j)\boldsymbol{\beta}(\tau_j) - 2\boldsymbol{\iota}'\boldsymbol{\Omega}^{cf}(\tau_j)\boldsymbol{\beta}(\tau_j))$$

for the special case where $A \to \infty$ or $E(\mathbf{r}^f(\tau_j))=\mathbf{0}$.

In this case, the asset return at scale j represents the scale-j component of a multi-resolution decomposition (MRD) of the original return series. As mentioned earlier, a MRD enables us to decompose a time series into its high- and low-frequency components. The former will be associated with the short-term fluctuations of a return series whereas the latter with its long-term fluctuations. In other words, the upper scales of the data will be associated with the trend components of the spot and futures prices. And, therefore, such scales will be relevant to investors with longer-term horizons. By contrast, the lower scales will be the focus of interest of investors with short-term horizons.

3.3 Copulas

Copulas have arisen as a new technique to measure the co-movement between financial markets. They are uniform distributions which enable us to extract the dependence structure from the joint probability distribution function of a set of random variables and, at the same time, to separate the dependence structure from the univariate marginal behavior. Examples of recent applications of copulas in finance are Cherubini and Luciano (2002, 2003a, b), Embrechts, Lindskog and McNeil (2003), Giesecke (2004), Junker, Szimayer, and Wagner (2006), Pachenko (2005), and Rosenberg and Schuermann (2006). A thorough discussion on the use of copulas in finance is provided in the textbook by Cherubini, Luciano, and Vecchiato (2004). In addition, the survey article by Frees and Valdez (1998) provides an excellent background on the use of copulas in a more general context.

A copula is defined as a multivariate distribution function (df) F of random variables $X_1,..., X_n$ with standard uniform marginal cumulative distribution functions $F_1,..., F_n$ (i.e., margins). That is, $X_i \sim F_i$, $i=1,..., n$.

In general, let us consider an $n \times 1$ random vector **X** with a joint df F and continuous margins F_i, which are not necessarily standard uniform.[7] Then

$$F(x_1,...,x_n) = Pr(X_1 \leq x_1, ..., X_n \leq x_n)$$
$$= Pr(F_1(X_1) \leq F_1(x_1), ..., F_n(X_n) \leq F_n(x_n))$$
$$= C(F_1(x_1),..., F_n(x_n)) \quad (16)$$

Equation (16) shows that the joint df F can be described by the margins F_1,.., F_n and the copula C. The latter captures the dependence structure among X_1,..., X_n. The existence of the function C is established by Sklar's theorem (see Nelsen, 1999, section 2.10).

Given that we consider a portfolio composed of an arbitrary number of assets, we need a multivariate copula to capture the dependence structure. To that end, we resort to a multivariate normal copula, a widespread choice in the finance field which is computationally tractable when the number of assets in the portfolio is relatively large. For its implementation, we utilize the Monte Carlo simulation algorithm presented by Wang (1999) in his discussion on Frees and Valdez (1998)'s article. This is as follows. Let $(X_1,..,X_n)$ be a set of correlated random variables with margins $F_{X_1},..,F_{X_n}$ and Kendall's tau $\tau_{ij}=\tau(X_i,X_j)$ or Spearman's rank correlation RankCorr(X_i,X_j).[8] If their dependence structure can be adequately described by a multivariate normal copula, a random sample of $(X_1,...,X_n)$ can be simulated as follows:

Step 1: Convert the given Kendall's tau or rank correlation coefficient to the pair-wise Pearson correlation coefficient, $\rho_{ij} = \sin\left(\frac{\pi}{2}\tau_{ij}\right) = 2\sin\left(\frac{\pi}{6}\text{RankCorr}(X_i,X_j)\right)$, and construct

[7] A well-known result in statistics establishes that if X_i is a random variable with a continuous distribution function F_i, the random variable $F_i(X_i)$ is standard-uniformly distributed, i.e., $F_i(X_i) \sim U(0,1)$.

[8] One disadvantage of the Pearson correlation coefficient, relative to Kendall´s τ and Spearman's rank correlation, is that it is not invariant under non-linear strictly increasing transformations of the data. Therefore, Kendall´s τ and Spearman's rank correlation are preferable as dependence measures.

the lower triangular matrix **B**, such that **Σ**=**BB′**,where **Σ** is the matrix of pair-wise Pearson correlation coefficients of $(X_1,.., X_n)$.

Step 2: Generate an n × 1 vector Y of standard normal variables

Step 3: Let **Z**=**BY** and set $u_i=\Phi(Z_i)$, i=1,..., n, where Φ(.) is the df of a standard normal.

Step 4: Set $X_i = F_{X_i}^{-1}(u_i)$, i=1,.., n.

In order to model the margins F_{X_i}, i=1,.., n, we resort to a semi-parametric procedure discussed by Carmona (2004). Specifically, a generalized Pareto distribution is fitted to the tails,[9] while the empirical distribution is used to model the center of the distribution. In other words, parametric and non-parametric approaches are used to model the tails and the center of the distribution, respectively.

It is worth stressing that, due to Sklar's theorem, the multivariate normal copula generates a multivariate standard normal distribution if and only if the margins are standard normal. As has been well-documented in the finance literature, individual asset returns rarely are normally distributed. Therefore, Carmona's approach enables us to capture the presence of fat tails and non-zero skewness in individual returns. The normal copula is used, as previously stated, for the sake of computational parsimony. However, the joint probability distribution of spot and futures returns will most likely differ from a multivariate normal distribution, as the margins are allowed to departure from normality.

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[9]This is given by $H_{\zeta,\beta(u)}(y) = \begin{cases} 1-\left(1+\frac{\zeta y}{\beta(u)}\right)^{-1/\zeta}, \ \zeta \neq 0; 1-\exp\left(-\frac{y}{\beta(u)}\right), \ \zeta = 0 \end{cases}$, where β(u)>0, and y≥0 when ζ≥0, and 0≤y≤−β(u)/ζ when ζ<0 (see, for example, Coles, 2001). If ζ>0, F is said to be in the Fréchet family and $H_{\zeta,\beta(u)}$ is a Pareto distribution. In most applications of risk management, the data comes from a heavy-tailed distribution, so that ζ>0.

This is important given the evidence found by Chen, Lee and Shrestha (2006, in press) on the non-normality of spot and futures returns, referred to in the Background.

4 Empirical results

4.1 The data

The data series utilized in this study are daily returns on cash and futures on London Metal Exchange (LME) metals (July 1993-December 2005). The sampled metals include aluminum, copper, lead, nickel, zinc, and tin. Cash and 3-month futures are those quoted by the LME. Descriptive statistics are presented in Table 1. As we see, all return series exhibit fat tails and negative skewness, suggesting a departure from normality.[10]

The impact of the wavelet-scale on returns cross-correlations is reported in Table 2. As we see, the correlation matrix tends to be stable over scales. However, we observe that the co-movement between spot and futures markets increases at the upper scales. For instance, the correlation of the returns on the nickel cash and futures contracts increases from 0.97 at scale 2 to 0.99 at scale 4. In the raw data, the correlation coefficient between the two series is 0.96. Such features of the data highlight the importance of focusing on different timescales or investment horizons.

Table 3 presents wavelet-based hedge ratios for the six metals categories, based on equation (3), which accounts for cross correlations (minimum-variance hedge ratio), and equation (3′), which does not (OLS hedge ratio). In general, we see that neglecting cross correlations translates into an underestimation of the hedge ratios, particularly in the raw data and at the lowest scales. As the scale increases, the hedge ratios approximate 1 (in

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[10] All the programming involved in this and the following subsections was implemented in S-Plus 7.0 and its modules S+Wavelets 2.0 and S+Finmetrics 2.0.

some cases, they even slightly exceed 1), regardless whether we take cross correlations into account. This result is in agreement with In and Kim (2006a, 2006b)'s findings.

The degree of hedge effectiveness (HE), as defined in equations (6) through (7′), is reported in Table 4. If we focus on the first two rows of the table, we observe that HE tends to 1 as we move to the upper scales 4 and 5, even when cross correlations are ignored (i.e., $HE = \rho_{sf}^2$). Again, this is congruent with In and Kim's conclusions. However, HE is greater and closer to 1 when the portfolio diversification gains of having several net positions are taken into consideration (e.g., scales 3-5). For instance, at scale 5, the computed HE is 0.99 when accounting for cross correlations, whereas it reaches only 0.93 when they are disregarded. Rows three and four of Table 4 report the HE for the naïve-hedge ratio with and without accounting for cross correlations. As we see, the minimum-variance hedge ratio outperforms the naïve estimate, after accounting for portfolio diversification gains, at all scales. Similarly, the OLS hedge ratio is preferable to its naïve counterpart for the single-commodity portfolio case. In this regard, our results are congruent with those reported by In and Kim (2006a) in their in-sample comparisons of hedge effectiveness of naive, OLS and wavelet (minimum variance) hedge ratios (Table 2 in their article).

In order to have a more complete picture of the implication of the existence of heterogeneous investors to hedge effectiveness, we evaluate the utility function $E(\boldsymbol{\iota}'\mathbf{r}^{c} - \boldsymbol{\beta}'\mathbf{r}^{f}) - 0.5A(\boldsymbol{\iota}'\boldsymbol{\Omega}^{C}\boldsymbol{\iota} + \boldsymbol{\beta}'\boldsymbol{\Omega}^{f}\boldsymbol{\beta} - 2\boldsymbol{\iota}'\boldsymbol{\Omega}^{cf}\boldsymbol{\beta})$ at different estimates of the hedge ratios—i.e., minimum variance, OLS, and naïve, and timescales of the data. The expected values of returns are replaced by the median of the wavelet coefficients at each corresponding timescale[11], whereas the wavelet-based variance-covariance matrices are

[11] We use the median, rather than the mean, because is more robust in the presence of outliers.

computed as described in Section 3.2.2. The results are reported in Table 5. The magnitudes of the risk-aversion parameter, A, are the same as those utilized by In and Kim (2006a), in addition to the value of 500, which would represent an extremely high-risk averse individual.

The simulations exhibit some distinctive patterns. First of all, utility tends to be monotonically increasing with the timescale, with the exception of low risk-averse individuals, whose maximum level of utility is attained at the lower scales (i.e., scale 2). This finding implies that their hedging horizon tends to be short. Second, very high-risk averse individuals will select longer-term hedging (i.e., scale 5). And, finally, the discrepancy between the investor's welfare yielded by the three different hedging methods becomes more apparent as the degree of risk aversion increases. For instance, the utility value obtained from the MV and OLS hedge ratios do not differ much for A=0.001, 0.5, and 1, but they noticeably do for A=100 and 500.Our first two findings coincide with In and Kim's op cit in their analysis of one-single commodity portfolio, but they only report the minimum-variance case.

4.2 Simulations

In this section, we analyze the behavior of the hedge ratios and the degree of HE beyond the sample. In doing so, we resort to the mathematical tools of wavelets and copulas. We consider two scenarios: one in which cross correlations of the net positions in the portfolio are neglected, and another in which they are taken into account. Returns dependency is modeled by a multivariate normal copula and Carmona's semi-parametric method to fit the marginal distributions, as discussed in Section 3.3. As in the previous subsection, wavelets are utilized to quantify the impact of the time horizon on the portfolio investment.

Table 6 reports the simulation results based on 500 iterations[12] for the hedge ratios. With the exception of aluminum, the mean hedge ratio tends to be higher when cross correlations are taken into consideration. In addition, we find that the empirical distributions exhibit more dispersion—as measured by the interquartile range, which equals the third minus the first quartile—when accounting for cross correlations. In general, there is no regularity regarding the magnitude of the quartiles of the different empirical distributions under either scenario. For instance, in the raw data, the first quartile of aluminum is smaller when cross correlations are taken into consideration (0.59) than when they are not (0.64), whereas for lead the opposite holds (1.1 versus 1.07, respectively). However, a distinctive property we find is that the maximum hedge ratio is greater when accounting for cross correlations, whichever the commodity is. And, that the maximum hedge ratio increases as we move to the upper scales.

Descriptive statistics of the simulation results for hedge effectiveness (HE) are reported in Table 7.[13] Panel (a) characterizes the HE of the minimum-variance and naïve hedge ratios when accounting for returns dependency, whereas Panel (b) reports the corresponding results for the OLS and naïve hedge ratios when dependency is ignored. The evidence found was to be expected. Indeed, if we look at the left-hand side of Table 7 (i.e., minimum-variance (MV) versus OLS hedge ratios), we see that the empirical distribution of the HE when cross correlations are accounted for is always to the right to that of the HE when cross correlations are disregarded. That is, holding several net commodity positions

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[12] The computer time required to produce the table was about 2 hours on a Pentium 4.0 with 768 MB of RAM.

[13] For the sake of computational time, the number of simulations was reduced to 100. Indeed, the time required to produce the output of the table approached 4 hours.

contributes unambiguously to risk reduction. Similar conclusions are drawn for the naïve case.

In order to verify whether there are statistical differences between the HE of the MV hedge ratio and a more parsimonious approach, such as the naïve one, we conduct a test of difference in mean of the HE with and without accounting for returns dependency (Panels (a) and (b) of Table 8, respectively). As we see, the MV-hedge ratio approach provides with a higher HE than the naïve one, in either case, at all scales of the data. However, the magnitude of the statistic tends to decrease as we move to the upper scales. This suggests that at longer-term horizons the naïve approach tends to performs relatively better.

4.3 An alternative methodology to measure hedge effectiveness: MGARCH models

The aim of this section is to compare our methodology to others that have been used in the literature to assess hedge performance, such as multivariate GARCH (MGARCH) models. Specifically, we focus on two commodities: petroleum and aluminum. These two commodities have been previously studied by Lien and Wilson (2001) and MacMillan (2005), respectively.

For the case of petroleum spot and futures returns, we jointly analyze their behavior and that of spot and futures energy indices by means of a diagonal VEC (DVEC) model. These commodity indices are constructed by Goldman Sachs, and they cover the sample period 1983-2005 at a weekly frequency. The aluminum spot and futures data series are those previously utilized in our study.

The econometric specification we consider is of the general form:

$$\mathbf{r}_t = \mathbf{c} + \sum_{l=0}^{L} \boldsymbol{\beta}_l \mathbf{r}_{t-l} + \boldsymbol{\varepsilon}_t \qquad t=2, \ldots, T \qquad (17)$$

where $\mathbf{r}_t$ is a k × 1 vector of returns, **c** is a k × 1 vector of constant terms, $\mathbf{r}_{t-1}$ is an m × 1 vector of first lags, β is a k × m matrix containing the coefficients on $\mathbf{r}_{t-1}$, and ε_t is a k × 1 white-noise vector with zero mean. The matrix variance-covariance of ε_t in a DVEC(p, q) is given by

$$\mathbf{\Sigma}_t = \mathbf{A}_0 + \sum_{i=1}^{p} \mathbf{A}_i \otimes (\boldsymbol{\varepsilon}_{t-i}\boldsymbol{\varepsilon}_{t-i}') + \sum_{j=1}^{q} \mathbf{B}_j \otimes \mathbf{\Sigma}_{t-j} \qquad (18)$$

where $\mathbf{A}_0$, $\mathbf{A}_i$ (i=1, 2, …, p) and $\mathbf{B}_j$ (j=1, 2,.., q) are k × k matrices, $\mathbf{\Sigma}_t$ and $\boldsymbol{\varepsilon}_{t-j}\varepsilon_{t-j}'$ are symmetric matrices, and ⊗ denotes the Hadamard product (e.g., Zivot and Wang, 2003, chapter 13).

A particular case of (18) is when p=q=1, i.e., DVEC(1,1), which turns out to fit the data well. Table 9 shows some diagnostic tests for the petroleum and energy system (Panel (a)) and the aluminum equations (Panel (b)). According to the Shapiro and Wilk statistic, normality is rejected in all equations except for that of the energy futures index. As stated earlier, normality is usually rejected in individual returns series. On the other hand, no evidence of missing ARCH effects is found at the 1-percent significance level.

The simulations carried out in this section are as follows. We generate artificial data from the corresponding DVEC(1,1) models by assuming that these are a reasonable approximation of the true data generating process, as Table 9 suggests. We next compute the hedge effectiveness (HE) ratio accounting for the dependence of the returns series, according to equation (6). We repeat this exercise 500 times. Similarly, we utilize the copula technique of Section 3.3 to generate returns artificial data and compute the HE ratio 500 times. The outcome of our simulations is reported in Table 10. In particular, we conduct a difference-in-mean test to verify whether one method statistically outperforms the other, on average. The evidence shows that when looking at the raw data, i.e., without distinguishing between investment horizons, and by assuming that the data is generated by a normal copula with generalized Pareto margins, an investor obtains a higher HE ratio than if he/she assumes a DVEC(1,1) model as the true data-generating process. This is particularly the case for aluminum: the copula-HE ratio equals 0.41 whereas that yielded by the DVEC(1,1) model amounts to only 0.35.

If we decompose the data according to investment horizons, we find that copulas outperforms the DVEC(1,1) model at scales 1 through 3 for aluminum, and at scales 1 and 2 for petroleum and energy. For the latter, at scale 3 both methods perform identically in statistical terms at the 10-percent significance level. At scales 4-5 we find mixed evidence. Indeed, at the 1-percent significance level, copulas and the DVEC(1,1) model are equivalent in terms of hedge effectiveness at scale 4 for petroleum and energy, whereas for aluminum the DVEC(1,1) specification seems more suitable. At scale 5, copulas are unambiguously a second-best choice in either case.

In sum, it appears that copulas are a more powerful tool to model returns dependency than a multivariate-GARCH model in the raw data and at its lower scales, based on simulations of hedge effectiveness. At the higher scales of the data, however, the evidence is mixed and it may be the case that a multivariate-GARCH model yields higher hedge effectiveness than copulas. In other words, the performance of copulas relative to a multivariate-GARCH specification may depend upon the holding period of the investment.

5 Conclusions

In this article, we develop an analytical framework to select the optimal hedge ratios for a portfolio comprised of an arbitrary number of net positions in commodities. In doing so, we explicitly model returns dependency by resorting to copulas, which enables us to account for all pair-wise correlation coefficients across cash and futures positions. In addition, we consider the existence of heterogeneous investment horizons by resorting to wavelets. Our empirical application deals with a portfolio composed of cash and futures positions on London Metal Exchange (LME) metals (aluminum, copper, lead, nickel, zinc, and tin) for the sample period July 1993-December 2005. Our simulations show that the hedge effectiveness provided by a portfolio of net positions of several commodities is greater than that of a single net position. This represents evidence in favor of diversification gains.

In order to enrich our analysis, we carry out some simulations to compare copulas with a multivariate GARCH model, in terms of their hedge effectiveness. To that end, we analyze the petroleum and energy spot and futures indices elaborated by Goldman Sachs and the LME aluminum data. We find that copulas appear to be a more powerful tool to model returns dependency than a multivariate-GARCH model in the raw data and at its lower scales. At higher scales of the data, however, the evidence is less clear-cut and it is possible that a multivariate-GARCH model yields higher hedge effectiveness than copulas.

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Tables

Table 1 Returns on sampled metals: July 1993-December 2005

	Aluminum 3months	Aluminum Cash	Copper 3month	Copper Cash	Lead 3month	Lead Cash
Minimum	–0.082	–0.067	–0.110	–0.104	–0.100	–0.105
1st Qu	–0.005	–0.005	–0.007	–0.008	–0.007	–0.009
Median	0.000	0.000	0.000	0.000	0.000	0.000
Mean	0.000	0.000	0.000	0.000	0.000	0.000
3rd Qu.	0.006	0.006	0.007	0.008	0.008	0.009
Maximum	0.061	0.063	0.079	0.070	0.060	0.093
Excess kurtosis	4.663	3.068	4.928	4.264	3.404	3.325
Skewness	–0.338	0.184	–0.287	–0.255	–0.368	–0.112
Observations	3,261	3,261	3,261	3,261	3,261	3,261
	Nickel 3months	Nickel Cash	Zinc 3month	Zinc Cash	Tin 3month	Tin Cash
Minimum	–0.144	–0.141	–0.124	–0.127	–0.113	–0.111
1st Qu.	–0.010	–0.010	–0.006	–0.006	–0.005	–0.006
Median	0.000	0.000	0.000	0.000	0.000	0.000
Mean	0.000	0.000	0.000	0.000	0.000	0.000
3rd Qu.	0.010	0.010	0.006	0.007	0.006	0.006
Maximum	0.105	0.107	0.072	0.073	0.072	0.070
Excess kurtosis	4.552	4.059	6.903	7.264	8.205	6.449
Skewness	–0.221	–0.171	–0.558	–0.614	–0.689	–0.476
Observations	3,261	3,261	3,261	3,261	3,261	3,261

Note: The data source is Datastream, and the returns are daily.

Table 2 Wavelet-based correlation coefficients

Raw data

	AL3MTH	ALCASH	CU3MTH	CUCASH	PB3MTH	PBCASH	NI3MTH	NICASH	ZN3MTH	ZNCASH	SN3MTH	SNCASH
AL3MTH	1.00	0.69	0.62	0.60	0.47	0.44	0.47	0.46	0.57	0.56	0.41	0.41
ALCASH	0.69	1.00	0.46	0.44	0.35	0.33	0.32	0.31	0.41	0.40	0.31	0.31
CU3MTH	0.62	0.46	1.00	0.94	0.50	0.46	0.48	0.47	0.55	0.53	0.39	0.37
CUCASH	0.60	0.44	0.94	1.00	0.49	0.47	0.46	0.46	0.53	0.52	0.39	0.37
PB3MTH	0.47	0.35	0.50	0.49	1.00	0.93	0.42	0.42	0.56	0.55	0.39	0.38
PBCASH	0.44	0.33	0.46	0.47	0.93	1.00	0.40	0.40	0.53	0.53	0.36	0.35
NI3MTH	0.47	0.32	0.48	0.46	0.42	0.40	1.00	0.96	0.46	0.45	0.39	0.37
NICASH	0.46	0.31	0.47	0.46	0.42	0.40	0.96	1.00	0.45	0.44	0.38	0.36
ZN3MTH	0.57	0.41	0.55	0.53	0.56	0.53	0.46	0.45	1.00	0.97	0.41	0.40
ZNCASH	0.56	0.40	0.53	0.52	0.55	0.53	0.45	0.44	0.97	1.00	0.39	0.39
SN3MTH	0.41	0.31	0.39	0.39	0.39	0.36	0.39	0.38	0.41	0.39	1.00	0.96
SNCASH	0.41	0.31	0.37	0.37	0.38	0.35	0.37	0.36	0.40	0.39	0.96	1.00

Scale 2

	AL3MTH	ALCASH	CU3MTH	CUCASH	PB3MTH	PBCASH	NI3MTH	NICASH	ZN3MTH	ZNCASH	SN3MTH	SNCASH
AL3MTH	1.00	0.72	0.63	0.60	0.50	0.47	0.47	0.47	0.57	0.55	0.41	0.41
ALCASH	0.72	1.00	0.47	0.45	0.38	0.37	0.36	0.36	0.43	0.42	0.30	0.31
CU3MTH	0.63	0.47	1.00	0.94	0.51	0.48	0.48	0.48	0.55	0.53	0.38	0.36
CUCASH	0.60	0.45	0.94	1.00	0.49	0.47	0.46	0.47	0.52	0.51	0.38	0.36
PB3MTH	0.50	0.38	0.51	0.49	1.00	0.94	0.41	0.42	0.59	0.58	0.42	0.41
PBCASH	0.47	0.37	0.48	0.47	0.94	1.00	0.41	0.42	0.57	0.56	0.38	0.37
NI3MTH	0.47	0.36	0.48	0.46	0.41	0.41	1.00	0.97	0.49	0.48	0.41	0.39
NICASH	0.47	0.36	0.48	0.47	0.42	0.42	0.97	1.00	0.48	0.48	0.40	0.39
ZN3MTH	0.57	0.43	0.55	0.52	0.59	0.57	0.49	0.48	1.00	0.97	0.40	0.39
ZNCASH	0.55	0.42	0.53	0.51	0.58	0.56	0.48	0.48	0.97	1.00	0.39	0.39
SN3MTH	0.41	0.30	0.38	0.38	0.42	0.38	0.41	0.40	0.40	0.39	1.00	0.97
SNCASH	0.41	0.31	0.36	0.36	0.41	0.37	0.39	0.39	0.39	0.39	0.97	1.00

Scale 4

	AL3MTH	ALCASH	CU3MTH	CUCASH	PB3MTH	PBCASH	NI3MTH	NICASH	ZN3MTH	ZNCASH	SN3MTH	SNCASH
AL3MTH	1.00	0.72	0.64	0.61	0.43	0.36	0.50	0.50	0.55	0.51	0.32	0.31
ALCASH	0.72	1.00	0.46	0.44	0.25	0.20	0.35	0.34	0.37	0.36	0.23	0.22
CU3MTH	0.64	0.46	1.00	0.96	0.37	0.31	0.47	0.46	0.57	0.54	0.36	0.34
CUCASH	0.61	0.44	0.96	1.00	0.36	0.31	0.44	0.44	0.55	0.52	0.36	0.34
PB3MTH	0.43	0.25	0.37	0.36	1.00	0.94	0.47	0.47	0.55	0.53	0.35	0.33
PBCASH	0.36	0.20	0.31	0.31	0.94	1.00	0.42	0.42	0.47	0.47	0.31	0.30
NI3MTH	0.50	0.35	0.47	0.44	0.47	0.42	1.00	0.99	0.45	0.42	0.40	0.39
NICASH	0.50	0.34	0.46	0.44	0.47	0.42	0.99	1.00	0.44	0.42	0.39	0.38
ZN3MTH	0.55	0.37	0.57	0.55	0.55	0.47	0.45	0.44	1.00	0.95	0.41	0.37
ZNCASH	0.51	0.36	0.54	0.52	0.53	0.47	0.42	0.42	0.95	1.00	0.37	0.33
SN3MTH	0.32	0.23	0.36	0.36	0.35	0.31	0.40	0.39	0.41	0.37	1.00	0.98
SNCASH	0.31	0.22	0.34	0.34	0.33	0.30	0.39	0.38	0.37	0.33	0.98	1.00

Notes: AL3MTH= aluminum 3- month futures, ALCASH: aluminum cash, CU3MTH= copper 3- month futures, CUCASH: copper cash, PB3MTH= lead 3-month futures, PBCASH: lead cash, NI3MTH= nickel 3-month futures, NICASH: nickel cash, ZN3MTH= zinc 3- month futures, ZNCASH: zinc cash, SN3MTH= tin 3- month futures, SNCASH: tin cash, Wavelet scales are such that scale 2: 4-8 days and scale 4: 16-32 days. .

Table 3 In-sample wavelet-based hedge ratios

	Raw	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
Minimum variance						
Aluminum	0.725	0.743	0.725	0.629	0.688	0.741
Copper	1.059	1.066	1.065	1.020	1.036	1.080
Lead	1.141	1.156	1.148	1.122	1.146	1.028
Nickel	0.977	0.944	1.027	1.033	1.016	0.900
Zinc	1.086	1.126	1.055	1.003	0.949	1.182
Tin	1.023	1.014	1.007	1.021	1.064	0.994
OLS						
	Raw	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
Aluminum	0.682	0.627	0.728	0.775	0.733	0.851
Copper	0.841	0.841	0.829	0.827	0.869	0.874
Lead	0.799	0.783	0.813	0.816	0.792	0.824
Nickel	0.935	0.925	0.931	0.945	0.961	0.979
Zinc	0.881	0.878	0.895	0.881	0.859	0.890
Tin	0.907	0.891	0.919	0.941	0.933	0.941

Note: The minimum-variance hedge ratio is computed according to equation (3), whereas the OLS hedge ratio according to equation (3′).

Table 4 In-sample degree of hedging effectiveness

	Raw	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
Minimum-variance hedge ratio	0.965	0.960	0.969	0.971	0.975	0.986
OLS hedge ratio	0.871	0.851	0.884	0.892	0.904	0.930
Naïve accounting for cross correlations	0.952	0.947	0.956	0.953	0.960	0.972
Naïve neglecting cross correlations	0.861	0.841	0.874	0.880	0.893	0.920

Note*:* The degree of hedging effectiveness is calculated according to equations (6) through (7′).

Table 5 In-sample utility comparisons of hedging effectiveness

(a) Minimum variance	A = 0.001	A = 0.5	A = 1	A = 4	A = 10	A = 100	A = 500
Scale 1	–0.0246	–0.0260	–0.0275	–0.0362	–0.0536	–0.2904	–1.4522
Scale 2	–0.0075	–0.0080	–0.0085	–0.0115	–0.0174	–0.0994	–0.4968
Scale 3	–0.0120	–0.0122	–0.0123	–0.0132	–0.0149	–0.0290	–0.1449
Scale 4	–0.0104	–0.0105	–0.0106	–0.0109	–0.0115	–0.0110	–0.0549
Scale 5	–0.0100	–0.0100	–0.0100	–0.0101	–0.0102	–0.0019	–0.0095
(b) OLS	A = 0.001	A = 0.5	A = 1	A = 4	A = 10	A = 100	A = 500
Scale 1	–0.0250	–0.0264	–0.0279	–0.0366	–0.0541	–0.3157	–1.4787
Scale 2	–0.0075	–0.0080	–0.0085	–0.0115	–0.0175	–0.1073	–0.5065
Scale 3	–0.0121	–0.0122	–0.0124	–0.0134	–0.0154	–0.0451	–0.1771
Scale 4	–0.0099	–0.0100	–0.0100	–0.0104	–0.0110	–0.0209	–0.0648
Scale 5	–0.0110	–0.0110	–0.0110	–0.0110	–0.0110	–0.0113	–0.0126
(c) Naïve	A = 0.001	A = 0.5	A = 1	A = 4	A = 10	A = 100	A = 500
Scale 1	–0.0290	–0.0317	–0.0344	–0.0507	–0.0832	–0.5709	–2.7382
Scale 2	–0.0085	–0.0096	–0.0107	–0.0174	–0.0308	–0.2319	–1.1257
Scale 3	–0.0116	–0.0121	–0.0125	–0.0153	–0.0208	–0.1036	–0.4716
Scale 4	–0.0113	–0.0115	–0.0117	–0.0128	–0.0149	–0.0470	–0.1896
Scale 5	–0.0105	–0.0106	–0.0106	–0.0111	–0.0120	–0.0257	–0.0866

Note: The utility function is given by $E(\boldsymbol{\iota}'\mathbf{r}^c - \boldsymbol{\beta}'\mathbf{r}^f) - 0.5A(\boldsymbol{\iota}'\boldsymbol{\Omega}^C\boldsymbol{\iota} + \boldsymbol{\beta}'\boldsymbol{\Omega}^f\boldsymbol{\beta} - 2\boldsymbol{\iota}'\boldsymbol{\Omega}^{cf}\boldsymbol{\beta})$. The expected values of returns are replaced by the median of the wavelet coefficients at each corresponding timescale, whereas the variance-covariance matrices are computed at each timescale.

Table 6 Simulations results for hedge ratios

	Accounting for cross correlations						Neglecting cross correlations					
	Raw data						Raw data					
Statistic	Aluminum	Copper	Lead	Nickel	Zinc	Tin	Aluminum	Copper	Lead	Nickel	Zinc	Tin
Minimum	0.29	0.90	0.71	0.63	0.75	0.71	0.48	0.97	0.77	0.65	0.95	0.76
1st Q	0.59	1.04	1.10	0.98	1.08	0.97	0.64	1.06	1.07	0.98	1.04	0.97
Median	0.64	1.08	1.13	1.01	1.13	1.01	0.66	1.07	1.09	0.99	1.06	0.99
Mean	0.64	1.09	1.13	1.00	1.13	1.01	0.66	1.08	1.09	0.99	1.06	0.99
3rd Q	0.69	1.13	1.17	1.03	1.18	1.04	0.69	1.09	1.11	1.01	1.08	1.01
Maximum	0.89	2.09	1.30	1.11	1.38	1.19	0.79	1.83	1.16	1.05	1.25	1.10
Kurtosis–3	1.03	27.40	4.49	5.55	1.13	1.05	1.32	65.64	18.09	21.45	6.16	4.28
Skewness	–0.43	3.68	–0.85	–1.52	–0.14	–0.22	–0.32	6.91	–2.41	–3.52	1.58	–0.86
	Scale 2						Scale 2					
Statistic	Aluminum	Copper	Lead	Nickel	Zinc	Tin	Aluminum	Copper	Lead	Nickel	Zinc	Tin
Minimum	0.24	0.85	0.69	0.62	0.71	0.68	0.50	0.97	0.76	0.66	0.95	0.74
1st Q	0.57	1.02	1.09	0.97	1.06	0.94	0.63	1.05	1.07	0.98	1.04	0.96
Median	0.64	1.10	1.14	1.00	1.13	1.00	0.66	1.07	1.09	0.99	1.06	0.99
Mean	0.64	1.10	1.14	1.00	1.13	1.00	0.66	1.08	1.09	0.99	1.06	0.99
3rd Q	0.71	1.15	1.19	1.04	1.20	1.06	0.70	1.09	1.11	1.01	1.08	1.01
Maximum	0.90	2.10	1.34	1.15	1.42	1.30	0.78	1.84	1.17	1.05	1.27	1.11
Kurtosis–3	0.26	15.39	1.67	3.31	0.32	0.34	–0.04	61.14	12.41	18.69	5.73	3.26
Skewness	–0.31	2.34	–0.33	–0.99	–0.20	–0.04	–0.14	6.50	–1.81	–3.20	1.52	–0.67
	Scale 4						Scale 4					
Statistic	Aluminum	Copper	Lead	Nickel	Zinc	Tin	Aluminum	Copper	Lead	Nickel	Zinc	Tin
Minimum	0.06	0.59	0.66	0.60	0.63	0.57	0.35	0.95	0.81	0.66	0.94	0.72
1st Q	0.49	0.99	1.05	0.92	0.98	0.91	0.61	1.04	1.05	0.96	1.04	0.95
Median	0.64	1.10	1.13	1.00	1.11	1.00	0.66	1.07	1.09	0.99	1.06	0.99
Mean	0.64	1.10	1.14	0.99	1.11	1.00	0.66	1.08	1.08	0.99	1.07	0.99
3rd Q	0.79	1.22	1.24	1.07	1.24	1.09	0.72	1.10	1.12	1.02	1.09	1.02
Maximum	1.32	2.11	1.62	1.26	1.75	1.43	0.95	1.84	1.25	1.08	1.30	1.13
Kurtosis–3	–0.08	1.42	0.07	–0.05	–0.13	0.12	0.40	36.36	1.44	7.65	2.05	1.49
Skewness	0.17	0.32	0.03	–0.29	0.15	0.00	0.01	4.39	–0.42	–1.75	0.81	–0.39

Table 7 Simulations of hedge effectiveness

(a) Accounting for cross correlations												
	Minimum-variance						Naïve					
Statistic	Raw	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Raw	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
Minimum	0.955	0.953	0.950	0.944	0.931	0.896	0.902	0.903	0.902	0.924	0.918	0.889
1st Q	0.963	0.963	0.963	0.961	0.958	0.953	0.947	0.946	0.945	0.944	0.944	0.939
Median	0.967	0.967	0.966	0.968	0.969	0.968	0.949	0.949	0.950	0.948	0.951	0.949
Mean	0.969	0.969	0.969	0.968	0.968	0.969	0.948	0.948	0.948	0.948	0.949	0.948
3rd Q	0.972	0.972	0.974	0.973	0.978	0.985	0.951	0.952	0.953	0.953	0.956	0.959
Maximum	1.042	1.039	1.045	1.051	1.056	1.068	0.957	0.958	0.959	0.965	0.971	0.977
Stdev	0.011	0.011	0.012	0.013	0.018	0.026	0.006	0.006	0.007	0.008	0.011	0.016
Kurtosis–3	24.470	19.808	18.361	17.934	5.189	1.541	34.063	23.978	15.967	0.795	0.431	1.358
Skewness	4.176	3.666	3.293	2.771	0.951	0.316	–4.667	–3.632	–2.923	–0.515	–0.834	–0.838
(b) Neglecting cross correlation												
	OLS						Naïve					
Statistic	Raw	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Raw	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
Minimum	0.835	0.834	0.830	0.834	0.829	0.783	0.761	0.757	0.764	0.764	0.762	0.732
1st Q	0.861	0.860	0.860	0.859	0.854	0.850	0.847	0.848	0.844	0.843	0.837	0.833
Median	0.866	0.865	0.867	0.866	0.868	0.865	0.852	0.851	0.853	0.851	0.857	0.847
Mean	0.865	0.865	0.865	0.866	0.866	0.866	0.850	0.850	0.850	0.851	0.851	0.847
3rd Q	0.869	0.870	0.871	0.874	0.878	0.883	0.857	0.856	0.859	0.862	0.865	0.863
Maximum	0.885	0.885	0.886	0.902	0.898	0.917	0.868	0.872	0.873	0.885	0.895	0.901
Stdev	0.007	0.008	0.009	0.012	0.016	0.024	0.015	0.016	0.016	0.016	0.020	0.028
Kurtosis–3	2.970	2.017	1.477	0.475	–0.699	0.416	19.934	19.516	12.976	7.357	2.622	2.633
Skewness	–0.531	–0.282	–0.747	–0.036	–0.290	–0.319	–3.948	–3.835	–3.062	–1.523	–1.036	–0.843

Table 8 Difference-in mean test between minimum-variance and naïve hedge effectiveness

(a) Accounting for cross correlations						
	Raw	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
statistic	36.871	35.535	33.105	29.437	21.091	15.455
p-value	0.000	0.000	0.000	0.000	0.000	0.000
(b) Without accounting for cross correlations						
	Raw	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
statistic	20.177	19.802	18.554	16.570	13.405	11.296
p-value	0.000	0.000	0.000	0.000	0.000	0.000

Note: In Panels (a) and (b), the test is conducted by taking the difference of the hedge effectiveness of the minimum-variance hedge approach and that of the naïve one.

Table 9 Specification tests of multivariate-GARCH models

(a) Petroleum and energy (weekly returns)				
Commodity index	Shapiro-Wilk normality test	Pr	ARCH-effects Lagrange multiplier test	Pr
Energy futures index	0.989	0.771	6.199	0.906
Energy spot index	0.764	0.000	4.133	0.981
Petroleum futures index	0.919	0.000	23.139	0.027
Petroleum spot index	0.902	0.000	3.824	0.986
(b) Aluminum (daily returns)				
Commodity index	Shapiro-Wilk normality test	Pr	ARCH-effects Lagrange multiplier test	Pr
Aluminum futures index	0.969	0.000	9.469	0.662
Aluminum spot index	0.971	0.000	26.058	0.011

Note: (1) AR(1)-DVEC(1,1) models are fitted to the data. (2) 'Pr' stands for probability value.

Table 10 Simulation results of hedge effectiveness: Multivariate-GARCH model versus copulas

	(a) Petroleum and energy (weekly returns)				(b) Aluminum (daily returns)			
	copula mean HE	MGARCH mean HE	mean test	Pr	copula mean HE	MGARCH mean HE	mean test	Pr
Raw data	0.947	0.946	7.577	0.000	0.410	0.349	11.035	0.000
Scale 1	0.948	0.946	8.206	0.000	0.410	0.355	9.846	0.000
Scale 2	0.948	0.946	8.519	0.000	0.412	0.370	7.838	0.000
Scale 3	0.947	0.946	1.262	0.104	0.411	0.400	2.346	0.009
Scale 4	0.945	0.946	–2.175	0.015	0.411	0.429	–3.679	0.000
Scale 5	0.945	0.948	–3.375	0.000	0.413	0.460	–7.690	0.000

Notes: (a) Simulations of multivariate-GARCH data (MGARCH) are based on the models reported in Table 9. (b) For the MGARCH and copula cases, the mean hedge effectiveness (HE) is computed on the basis of 500 hundred iterations, according to equation (6).

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