# **Spatial Peak-load Pricing**\*

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# Abstract

This article extends the traditional electricity peak-load pricing model to include transmission costs. In the context of a two-node, two-technology electric power system, where suppliers face inelastic demand, we show that when the marginal plant is located at the energy-importing center, generators located away from that center should pay the marginal capacity transmission cost; otherwise, consumers should bear this cost through capacity payments. Since electric power transmission is a natural monopoly, marginal-cost pricing does not fully cover costs. We propose distributing the revenue deficit among users in proportion to the surplus they derive from the service priced at marginal cost.

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## 1. Introduction

The aim of this paper is to incorporate power transmission costs into the peakload pricing model for the electricity sector.<sup>1</sup> Peak-load pricing provides correct signals to sectoral stakeholders such that their short-term (operational) and long-term (investment and location) decisions both lead to an efficient market equilibrium. The standard peak-load pricing model envisages a spaceless electric power system with no transport costs, when in reality generators and consumers are spatially distributed. The challenge is therefore to determine the economic principles that should govern the pricing of electricity services, including power transmission, when the spatial dimension is taken into consideration.

Peak-load pricing along with centralized dispatching by merit order is a widespread feature of Latin American electricity regulations.<sup>2</sup> The main arguments adduced in favor of this approach are its efficiency properties, and the fact that it stipulates clear and simple rules that can be routinely applied, thereby facilitating price forecasting. In the transmission segment, Latin American countries have adopted a multilateral approach where all users finance a common grid; the owner of the grid receives a predefined payment that covers grid operation and maintenance costs, plus its long-run annualized replacement value. One source of revenues is the difference in energy and capacity prices at different nodes. However, such revenues cover less than 20% of total costs; hence the need to develop efficient rules to allocate the revenue shortfall in transmission among users of the system.

<sup>&</sup>lt;sup>1</sup> Boiteux (1960) and Steiner (1957) were earlier developers of peak-load pricing. Crew *et al* (1995) provide a thorough review of the subject.

<sup>&</sup>lt;sup>2</sup> In the US capacity payments based on each customer's peak power consumption have been widely used since the early twentieth century (Neufeld, 1987).

Latin American countries have approached the problem of distributing the revenue gap among users in several different ways, disagreeing, in particular, on who the users are and the criteria that should be used to distribute the revenue shortfall among them. For instance, while Argentina decided to charge transmission payments solely to generating companies (gencos), Colombia, Bolivia and Chile split these costs between gencos and consumers. Secondly, Latin American countries usually allocate the full capacity costs of transmission (net of the income due to price differentials) on the basis of some *ex ante* measure of network use during peak hours.

We extend peak-load pricing to an electric power system consisting of two nodes, with a single transmission line interconnecting them. To further simplify the analysis we (i) examine a deterministic world; (ii) assume inelastic demand; and (iii) restrict the number of generating technologies to two (both with linear costs). In what follows, the term "peaking technology" denotes the technology with highest operating unit cost (and lowest capacity cost per unit). Transmission is a natural monopoly because of the large economies of scale involved in its development; and to represent these, we assume that capacity cost has a fixed component and a constant marginal cost.

In this setting we show that the location of the dispatched plant that absorbs demand changes<sup>3</sup> (i.e. the marginal plant) is what determines whether the marginal capacity transmission cost should be borne by consumers or by generators. When the marginal plant is located in the energy-importing node, gencos located at the other node should pay for the marginal capacity cost of transmission; otherwise, consumers should bear this cost. The charge should be applied to the appropriate agents (gencos and/or

<sup>&</sup>lt;sup>3</sup> This plant has the highest (or equal highest) operating cost of all dispatched plants.

consumers) at the transmission peak, which does not necessarily coincide with the energy consumption peak.

A natural monopoly requires marginal-cost pricing for efficiency, but this precludes full recovery of costs. The solution deployed to bridge the revenue shortfall is a two-part tariff, which consists of a lump-sum charge in conjunction with marginal-cost pricing.<sup>4</sup> The core of a bargaining solution consists of all those allocations of the revenue gap in which each user's lump sum is less than or equal to the surplus derived from the service when priced at marginal cost. Clearly, a necessary condition for a non-empty core is that the aggregate surpluses accruing to all transmission system users should exceed the revenue gap to be prorated between them. If this condition is not met, the facility is inefficient and will not be built.

We propose to distribute the revenue gap among users in proportion to the surplus they derive from the service. This is the only allocation criterion in the core that can be decentralized, i.e. the proportion of the revenue gap paid by each user depends only on his/her benefits and total benefits (Moulin, 1988).<sup>5</sup> Estimating the surplus obtained from the transmission system by each user, requires consideration of the three services it renders to users, namely to transport energy, improve the system's

<sup>&</sup>lt;sup>4</sup> An alternative strategy would be Ramsey pricing, which meets the break-even constraint through linear prices. This produces a second-best solution, since the efficiency condition (price = marginal cost) is lost. As the aim of this paper is to propose a set of pricing principles that will yield an efficient market equilibrium, we focus on two-part tariffs.

<sup>&</sup>lt;sup>5</sup> Some authors have proposed other criteria drawn from cooperative game theory to bridge the revenue gap, such as the Shapley value (see Lie and Tan, 2001, and Zolezzi and Rudnick (2002)). The Shapley value solution is not necessarily contained in the core, and it is also more taxing since requires computing the benefits of all possible coalitions that users can create.

reliability,<sup>6</sup> and reduce the market power of generators (Arellano and Serra, 2004).<sup>7</sup> In common with most studies, this paper only considers the transportation role.

With free entry to generation, gencos earn zero profits, and the revenue gap should be borne by consumers. If an additional charge were imposed on gencos, the result would be an inefficient generation portfolio since producers would over-invest in peaking technology in order to break even. However, if for some reason, such as resource availability, generation with the low-operating-cost technology is restricted, gencos using such technology will make profits. In this situation the revenue gap should be distributed between final consumers and those gencos, in proportion to the benefit obtained from the transmission line by each group.

The principles set out in this paper produce the correct transmission investment signal for the single-line system. Moreover, if transmission line owners know in advance that, besides the marginal capacity charge, they can also charge lump sum fees not exceeding the users' surpluses, they will not extend transmission lines that are not socially profitable. In addition, when there are alternative projects, the risk of being foreclosed by new entrants should give the correct incentives to ensure most efficient project is chosen.

Optimal transmission pricing and the financing of transmission investment costs have been analyzed in the literature for some time. Schweppe et al (1988) introduced the concept of optimal spot prices and later related this to the investment decision. They

<sup>&</sup>lt;sup>6</sup> Plant indivisibility gives transmission a role in improving system reliability.

<sup>&</sup>lt;sup>7</sup> Gencos could exercise market power, even when prices are regulated, through the composition of the generation portfolio (Arellano and Serra, 2004).

posit that the optimal investment level in a given transmission line or generation technology should be such that the costs of an additional incremental investment equals They posit that the optimal investment level in a given transmission line or generation technology should be such that the costs of an additional incremental investment equals the present value of the expected benefits derived from it. However, they do not develop a pricing system providing long-run signals for investors, as we do, albeit in a more restricted setting. Schweppe *et al* also fail to address the issue of how the transmission company –a natural monopoly– fully recovers its investment costs under linear pricing.

More recent work has focused in fully deregulated power industries. Within this context, the current consensus is that a coordinated spot market with locational marginal cost prices complemented by some form of financial transmission rights are a recipe for a successful market design (Hogan, 1992 and 2002).<sup>8</sup> In the presence of economies of scale, however, there is no guarantee that the pricing system will generate sufficient revenues to fully cover the costs of the transmission system. As in the case of a regulated industry analyzed in this paper, the problem of allocating the fixed cost of the transmission system has also been an important issue. For instance, while the FERC has recommended assigning the costs to the beneficiaries of the system, a rule that socializes the transmission cost among users (independently of the benefit each obtains from the transmission system) has been proposed for New England.<sup>9</sup>

The problem of determining the efficient level of investment in transmission capacity has also been discussed in the context of a deregulated power industry. Hogan

<sup>&</sup>lt;sup>8</sup> Oren et al (1996) show that contract network rights are redundant since they can be replicated by a composite financial instrument that combines short and long nodal forward contracts.

<sup>&</sup>lt;sup>9</sup> Hogan (2003b) points out that this rule will alter the mix and location of investments in generating capacity; and, at deeper level, it will represent a move away from the normal operation of a real market.

(2003a) argues against centralizing all the investment decisions in transmission capacity, and proposes a "market failure test" to draw a line between centrally determined transmission investments and merchant transmission. Only in the former case a cost allocation rule is needed

The rest of this paper is organized as follows. Section 2 presents the standard peak-load pricing model; section 3 analyzes a two-node system in which demand is concentrated in one node; the fourth section extends the model to allow for demand in both centers; and the final section draws conclusions.

## 2. Standard peak-load pricing

We begin by summarizing the traditional spaceless peak-load pricing system with inelastic demand,<sup>10</sup> assuming that there are two generation technologies denoted by i (i = 1,2) both with linear costs. In what follows,  $c_i$  is the operating cost per unit and  $f_i$ the capacity cost per unit for generation technology i. Without loss of generality, we assume that  $f_1 > f_2$ , and  $c_1 < c_2$ . Thus technology 2 is the peaking technology. Function q(t) denotes the load curve of the system, i.e., q(t) designates consumption at the t-th highest hourly consumption. Lastly, we assume that (i) plant factors are equal to 1 for both technologies, (ii) generation at each plant can be varied instantaneously and costlessly, and (iii) no failures occur.

Generating plants are dispatched in merit order, i.e. they are brought on line successively from the lowest operating cost to the highest, until demand is satisfied.<sup>11</sup> With this set of assumptions, the problem of minimizing the total cost of the electric system is formalized as follows:

<sup>&</sup>lt;sup>10</sup> Balasko (2001) performs a theoretical general equilibrium analysis of pricing with elastic demand.

$$\operatorname{Min}_{k_1,k_2} \left\{ f_1 k_1 + f_2 k_2 + c_2 \int_0^{t(k_1)} (q(t) - k_1) dt + c_1 k_1 t(k_1) + c_1 \int_{t(k_1)}^T q(t) dt \right\}$$

s.t.:  $k_1 + k_2 \ge q^M$ 

where  $q^M$  designates the system's peak demand,  $k_i$  the installed capacity of technology type *i*, and *T* the number of hours in the year. This statement of the problem assumes optimal use of installed capacity. In fact, between hours  $t(k_1)$  and *T*, demand is met by type-1 plant generation, since installed capacity makes this feasible and it is cheaper than generating with the type-2 plant. Between hours 0 and  $t(k_1)$ , type-2 plants come on stream to satisfy the demand not covered by type-1 plants (see figure 1). From the consumer's point of view, the relevant variable is  $t(k_1)$ , since this shows the number of hours during which they must pay a higher energy price.

Denoting by  $\lambda$  the Lagrange multiplier for the capacity constraint, the Kuhn-Tucker conditions for the above problem are:

$$f_1 - t(k_1)\Delta c - \lambda \ge 0 \qquad \qquad k_1(f_1 - t(k_1)\Delta c - \lambda) = 0$$

$$f_2 - \lambda \ge 0 \qquad \qquad k_2(f_2 - \lambda) = 0$$

where  $\Delta f = f_1 - f_2$  and  $\Delta c = c_2 - c_1$ , with  $\Delta c$  and  $\Delta f$  positive under the given assumptions. As the objective function is convex, the solution that minimizes the overall cost of the system is:

$$t^* = Min\left(\frac{\Delta f}{\Delta c}, T\right),$$
  $k_1^* = q(t^*),$   $k_2^* = q^M - k_1^*.$ 

<sup>&</sup>lt;sup>11</sup> The reality is somewhat more complex, however, since indivisibilities in plant operation can alter their natural order of entry. This has given rise to what the literature has called the "Unit Commitment Problem" (see Fischer and Serra, 2002).

When  $t^* = T$ , only type-2 plants are set up. Notice that  $t^*$  depends only on the relative fixed and operating costs of the available technologies, and is independent of shape of the load duration curve (Wenders, 1976).

Peak-load pricing consists of an energy charge equal to the unit operating cost of the marginal plant, and a capacity charge applied only to consumers' demand at system peak, equal to the marginal cost of increasing capacity. Usually the latter corresponds to the per unit capacity cost of the technology with the lowest per unit capacity cost. Peak-load pricing leads a decentralized system to the optimal solution. When the price of energy is  $c_1$ , only plants with type-1 technology are willing to produce, whereas both types of plant are willing when the price is  $c_2$ . The composition of the generation portfolio is also optimal and neither type of plants obtains rents. Indeed, type-2 plants never make economic profits; and, assuming free entry to the generation industry, type-1 plants will enter up to the point when they also make zero profits; which happens when they operate at full capacity during  $t^*$  hours.

#### **3.** Peak-load pricing with transmission

In the previous section we assumed that power plants and consumers were all located in the same place. This section now analyzes a two-node electric power system with a single transmission line. To represent scale economies in transmission, we assume that the capacity cost has a fixed component F and a constant variable component denoted by  $f_t$  (the long run marginal transmission cost). The total capacity cost of a transmission line with capacity  $k_t$  is therefore  $F+f_tk_t$ . For simplicity we assume that transmission operating costs are negligible (i.e. there are no losses).

We also assume that demand is concentrated at one node (referred to as the "demand center"). As the location of the marginal plant will prove to be important, we consider two different cases: case 1, in which the marginal plant is located at the demand center; and case 2 in which it is located at the other end of the line.<sup>12</sup> In both cases, the demand center is the energy-importing region.

## Case 1: Marginal plant located at demand center

When type-2 plants are located at the demand center with type-1 plants at the other end of the transmission line, the problem of minimizing the cost of the integrated system can be stated formally as follows:

$$\operatorname{Min}_{k_1,k_2} \left\{ f_1 k_1 + f_2 k_2 + c_2 \int_0^{t(k_1)} (q(t) - k_1) dt + c_1 k_1 t(k_1) + c_1 \int_{t(k_1)}^T q(t) dt + f_t k_1 + F \right\}$$

s.t.:  $k_1 + k_2 \ge q^M$ 

Note that, without loss of generality, transmission capacity has been assumed to match demand ( $k_t = k_I$ ). The Kuhn-Tucker conditions are:

$$f_1 + f_t - \Delta c t(k_1) - \lambda \ge 0 \qquad \qquad k_1(f_1 + f_t - \Delta c t(k_1) - \lambda) = 0$$
  
$$f_2 - \lambda \ge 0 \qquad \qquad k_2(f_2 - \lambda) = 0$$

The optimal solution is therefore characterized by the condition:

$$\hat{t} = Min\left(\frac{\Delta f + f_t}{\Delta c}, T\right), \qquad \hat{k}_1 = q(\hat{t}), \qquad \hat{k}_2 = q^M - \hat{k}_1$$

In this case the inclusion of transmission has no effect on peak-load pricing; so, the pricing scheme for energy and capacity that leads to an optimal decentralized solution is akin to that derived for the no-transmission case. In other words, the price of

<sup>&</sup>lt;sup>12</sup> These different locations may reflect the geographical location of fuels or hydrological resources, or environmental restrictions that preclude the construction of plants with a certain type of technology

energy is  $c_2$  when type-2 plants are operating, and  $c_1$  otherwise. The capacity charge applied to peak consumers—is given by  $f_2$ . Generators located outside the consumption center thus pay the marginal capacity cost of transmission  $f_t$ .

The intuition for the above result is as follows. Maximum use of the transmission system coincides with the system's consumption peak. During the peak period, consumers pay energy and capacity charges of  $c_2$  and  $f_2$ , respectively. They do not pay for the marginal transmission capacity cost because they are indifferent as to who supplies the energy they consume. The effective marginal capacity cost of type-1 plants should include the marginal cost of transmission capacity, i.e. it should be equal to  $(f_1 + f_1)$ .

The increase in the per unit capacity cost differential between the two types of plant leads to a reduction in the installed capacity of type-1 plants. So, with transmission, type-2 plants are in operation for longer than when both plants are located in the consumption center and no transmission is involved ( $\hat{t} > t^*$ ). This allows type-1 plants to obtain higher revenues from energy sales per unit of installed capacity. These additional revenues, in turn, allow them to pay for the marginal capacity transmission  $\cosh^{13}$  (see figure 2).

As in the no-transmission case, the pricing system produces the right signals for investment and operation decisions, leading a decentralized system to the optimal solution. Note that in the social welfare maximizing solution, while generating

near consumption zones.

<sup>&</sup>lt;sup>13</sup> Strictly speaking, there is also the case in which the optimal solution without transmission involves the operation of peaking-technology plants only; in this case, however, no transmission is required.

companies have zero profits, consumers do benefit from the transmission system since it allows for a lower energy price when only type-1 plants are being dispatched. The social welfare gain produced by the transmission line is given by:

$$\Delta W^{c} = \Delta c \int_{\hat{t}}^{T} q(t) dt$$

Transmission priced at marginal cost results in a revenue gap equal to the fixed capacity cost of transmission F. Who should pay for this? If type-1 generating plants were forced to pay the fixed cost, their installed capacity would fall back to the point where they break even. As a result of the consequent underinvestment in type-1 technology, the higher operating cost plants would operate for longer.<sup>14</sup> Thus, while consumers would pay a higher average price for energy (and the same capacity charge), the genco's profits would remain zero, and social welfare would therefore be lower.

Accordingly, if the transmission line is socially profitable (i.e. if  $\Delta W^c > F$ ), then the fixed transmission capacity cost should be borne by consumers. Under our proposals, the fixed cost each consumer pays should be proportional to the surplus he/she derives from the transmission line, which in this case equals his/her consumption between hours  $\hat{t}$  and T, times the energy price differential  $\Delta c$ . Hence each consumer's lump-sum should be proportional to his or her consumption in that period. Note that each consumer's surplus is totally unrelated to consumption during the peak hour.

Thus far, we have implicitly assumed that there are no supply constraints. To illustrate the impact of capacity constraints on the results, we now assume that the

<sup>14</sup> Their operating time is 
$$\tilde{t} = Min\left(\frac{\Delta f + f_t}{\Delta c} + \frac{F}{\Delta cq(\tilde{t})}, T\right)$$
 with  $\hat{t} < \tilde{t}$ .

maximum capacity of technology-1 plants is limited to  $k^w < \hat{k}_1$ . Restrictions may be due to the availability of fuel or hydro resources, or to legal regulations (zones saturated with a particular pollutant, for example). Accordingly, assuming that a transmission line is actually built, the economic profit made by technology-1 plants' is given by:

$$\pi = (t(k^w) - \hat{t})k^w \Delta c > 0,$$

and consumers' welfare gains, compared to the benefits obtained in a no-transmission scenario, are:

$$\Delta W^{c} = \Delta c \int_{t(k^{w})}^{T} q(t) dt$$

Construction of the line is justified provided the aggregate surpluses that accrue to generators and consumers exceed the fixed capacity cost of the line, i.e.  $\pi + \Delta W^c \ge F$ . With *F* distributed on the basis of the respective benefits, the fraction of the line's fixed cost that should be paid by generating companies located outside the consumption zone is:

$$\frac{\pi}{\Delta W^c + \pi} = \frac{\left(t^w - \hat{t}\right)k^w}{(t^w - \hat{t})k^w + \int_{t(k^w)}^{t} q(t)dt}$$

The fraction of the fixed capacity cost of transmission that would be borne by technology-1 generating companies depends essentially on how far below the optimal level their installed capacity is.

#### **Case 2: Marginal plant located outside the demand center**

We now consider the case where technology-1 plants are located in the consumption center, while technology-2 plants are sited away from the consumption zone and require a transmission line to reach consumers. We also assume that  $f_2 + f_t < f_1$ , because otherwise the efficient solution would only contain technology-1 plants. Under these conditions, the problem of minimizing the cost of the system is:

$$\begin{aligned}
& \underset{k_{1},k_{2}}{\text{Min}} \left\{ f_{1}k_{1} + f_{2}k_{2} + c_{2} \int_{0}^{t(k_{1})} (q(t) - k_{1})dt + c_{1}k_{1}t(k_{1}) + c_{1} \int_{t(k_{1})}^{T} q(t)dt + F + f_{t}k_{2} \right\} \\
& \text{s.t.:} \quad k_{1} + k_{2} \ge q^{M}
\end{aligned}$$

Without loss of generality, transmission capacity has been assumed to match demand  $(k_t = k_2)$ . The Kuhn-Tucker conditions are:

$$f_1 - t(k_1)\Delta c - \lambda \ge 0 \qquad \qquad k_1(f_1 - t(k_1)\Delta c - \lambda) = 0$$
  
$$f_2 + f_t - \lambda \ge 0 \qquad \qquad k_2(f_2 + f_t - \lambda) = 0$$

So the optimal solution is given by:

$$\breve{t} = Min\left(\frac{\Delta f - f_t}{\Delta c}, T\right), \qquad \breve{k}_1 = q(\breve{t}), \qquad \breve{k}_2 = q^M - \breve{k}_1$$

To attain the optimal solution, the pricing system must specify a payment for energy equal to  $c_2$  when technology-2 plants are in operation (between t=0 yt), and  $c_1$ otherwise, and a capacity payment of  $f_2 + f_t$ . This pricing scheme corresponds to peakload pricing. In fact a capacity payment of this type corresponds to the marginal cost of expanding the capacity of the electric power system, since increasing capacity at minimum cost requires investments both in technology-2 plants and in transmission. Notice that consumers bear the marginal capacity cost of transmission through the capacity payment. The larger capacity charge reduces the optimal capacity of type-2 plants compared to when they are located at the demand center ( $\tilde{t} < \hat{t}$ ).

The fixed transmission cost should be borne by users who benefit from the line; the generators, who make zero profits in this case, cannot pay for it. On the other hand, consumers benefit from the transmission line since it lowers the capacity payment (though this benefit is partially offset by a higher price of energy during between t=0and  $\tilde{t}$ ). The transmission line is thus socially profitable if the consumers' benefit exceeds the fixed capacity cost of transmission, i.e.  $\Delta W^c \ge F$ , where:

$$\Delta W^{c} = (\Delta f - f_{t})q^{M} - \Delta c \int_{0}^{t(k_{1})} q(t)dt$$

The benefit perceived by each consumer is equal to his/her demand at peak-time multiplied by the capacity charge differential  $(\Delta f - f_t)$ , minus his/her consumption between t=0 and  $\check{t}$  times the energy charge differential. Note that some consumers could be made worse off by construction of the transmission line and should therefore be compensated.

## 4. Two demand centers

We now extend the model of the previous sections to allow for energy consumption at both nodes, denoted by superscripts *A* and *B*. Peak demand occurs at a different instant at each node. In the non-interconnected solution, i.e. no transmission, each center's installed capacity matches its respective peak demand. In what follows we describe the optimal interconnected solution. The problem can be written as:

where  $k_i^{j}$  is the installed generating capacity of technology *i* at node *j* (*j* = *A*, *B*) and  $q^{j}$  is peak demand at node *j*. Variables q(t),  $k_i$ , and  $q^{M}$  have the same meaning as before but applied to the integrated system ( $k_i = \sum_j k_i^{j}$ ). Denoting by  $\lambda$  the Lagrange multiplier of

the first constraint and  $\mu_j$ , the multipliers of the second set of constraints, the Kuhn-Tucker conditions are as follows:

$$f_{1} - t(k_{1})\Delta c - \lambda - \mu_{j} \ge 0 \qquad k_{1}(f_{1} - t(k_{1})\Delta c - \lambda - \mu_{j}) = 0, \qquad j = A, B$$

$$f_{2} - \lambda - \mu^{j} \ge 0 \qquad k_{2}(f_{2} - \lambda - \mu^{j}) = 0 \qquad j = A, B$$

$$f_{i} \ge \mu_{A} + \mu_{B} \qquad k_{i}(f_{i} - \mu_{A} - \mu_{B}) = 0$$

$$k_{1} + k_{2} \ge q^{M} \qquad \lambda(k_{1} + k_{2} - q^{M}) = 0$$

$$k_{1}^{j} + k_{2}^{j} + k_{i} \ge q^{j} \qquad \mu_{i}(k_{1}^{j} + k_{2}^{j} + k_{i} - q^{j}) = 0 \qquad j = A, B$$

Assuming that the solution is interior and that all constraints are active,<sup>15</sup> the optimal solution is characterized by the following conditions:

$$t^{*} = Min\left(\frac{\Delta f}{\Delta c}, T\right)$$

$$k_{1}^{*} = q(t^{*}), \qquad k_{2}^{*} = q^{M} - q(t^{*})$$

$$k_{t}^{*} = \frac{q^{A} + q^{B} - q^{M}}{2} \qquad \qquad k_{1}^{j^{*}} + k_{2}^{j^{*}} = q^{j} - k_{t}^{*} \quad j = A, B$$

$$\lambda = f_2 - \frac{f_t}{2} \qquad \qquad \mu_j = \frac{f_t}{2} \qquad j = A, B$$

When  $t^* = T$ , only technology-2 plants are set up. Note that the interconnection of the two electric power systems makes it possible to reduce the generating capacity requirements. Necessary and sufficient conditions for achieving an interior solution, where the three capacity constraints are active, are therefore:  $\Delta f < T\Delta c$  and  $f_2 > f_1/2$ .

According to these equations, a rise in a node's peak demand (without increasing the demand of the integrated system) is met by transferring generation from the other node and increasing transmission capacity—in both cases by an amount equal to half the peak demand boost. On the other hand, a surge in consumption during the integrated system's peak demand requires an expansion of local generating capacity while allowing for a reduction in transmission capacity of half the amount of the consumption increase.

The optimal solution is consistent with a pricing system where consumption at the system's peak pays a capacity charge of  $f_2 - f_t/2$ , and peak consumption at each node pays a capacity charge of  $f_t/2$ . Broadly speaking, this pricing scheme is an extension of standard peak-load pricing, since consumers pay a capacity charge for peak consumption (the cost of increasing capacity). There are two differences with respect to the standard model, however. Firstly, there are three peak hours to consider in this case: the system's global peak and the two local peaks. Secondly, demand increases are met

<sup>&</sup>lt;sup>15</sup> This is feasible because we assumed that peak demand occurs at different times in the two markets.

by a combination of additional generating and transmission capacity, depending on the timing of the increase; so the relevant cost of increasing capacity differs.

Consumers who demand energy during the system peak period pay the marginal cost of increasing generation, discounted by the reduction in transmission made possible by the expansion of generating capacity. Customers that demand transmission during the local peak, pay half the marginal cost of transmission, since the required increase in transmission capacity is equal to half the demand increase. Hence the marginal capacity cost of transmission is borne by customers in each node consuming at the local peak, which coincides the time when the transmission line is used to the maximum. Note also that in this case the marginal plant is located at the energy-exporting center. In fact plants located at that center absorb any demand changes.

The interconnection of the two centers allows for a reduction in generation capacity equal to  $(q^A + q^B - q^M)$ , but also requires a transmission line of capacity  $(q^A + q^B - q^M)/2$ . Constructing the line therefore increases social welfare provided the benefit associated with a decrease in installed generation capacity is less than the cost of the line. Hence the following condition must be met for welfare to increase as a result of construction of the line:

$$\left(f_2 - \frac{f_t}{2}\right)(q^A + q^B - q^M) > F$$
.

If construction of the line is to be welfare-enhancing, consumers must finance the fixed transmission cost, since generators earn zero profits and capacity charges only finance generating capacity costs and the variable part of transmission capacity costs. Let  $x_{j}^{h}$  denote the power demand of consumer *h* located at node *j* during the system's aggregate demand peak, and  $y_{j}^{h}$  his/her demand during peak demand at the node he/she lives. Then the benefit that this consumer derives from the transmission line is given by:

$$\left(f_2-\frac{f_t}{2}\right)(y_j^i-x_j^i)$$

The benefit each consumer obtains from the transmission line is thus proportional to the difference between his/her consumption during the local demand peak and during the aggregate demand peak. Note that users with high consumption during the aggregate system's peak period may be left worse-off by the transmission line, whereas those consumers with high consumption when their local systems are at a peak will benefit from it. In this example each consumer's benefit grows with his/her consumption during one of the transmission peaks, but the benefit is not proportional to this consumption.

# 5. Conclusions

In this paper we have extended the peak-load pricing model to embrace the spatial dimension of electric power systems. To be consistent with peak-load pricing, transmission prices must be set equal to marginal cost, and the marginal transmission capacity cost should be charged to consumption during the transmission peak, which does not necessarily coincide with the demand peak.

The transmission line is used to transport energy from producers to consumers, so it is not obvious who should pay the marginal transmission capacity cost. However, a general rule can be put forward for this simple one-line, two-generation technology electric power system. When the marginal plant is located outside the energy-importing demand center, the consumers should pay for it through capacity charges. Otherwise, it should be paid for by the gencos that use the line to transmit energy to the energyimporting demand center.

Since electric power transmission is a natural monopoly, if the variable charge of transmission is set at marginal cost, additional fixed charges are needed in order to bridge the revenue shortfall. In this paper we suggest that each user should pay a lump-sum charge proportional to his/her surplus when price equals marginal cost. In a competitive scenario, with free entry into the generation segment and no capacity constraints, generators' make zero profits and the fixed cost of transmission should be entirely borne by consumers. In a situation where the gencos obtain positive profits, the fixed charge should be shared between consumers and generators in proportion to their surpluses. This occurs, for example, when the capacity of low-operating cost plants is restricted.

The principles set out in this paper set up the correct pricing signals that would lead decentralized investment decisions to produce a socially efficient generating portfolio. Nonetheless, an independent agent is needed to compute the surplus of each market participant, in order to allocate the fixed transmission cost. Inefficient lines will not be built because users will not pay fixed charges that exceed their surpluses.

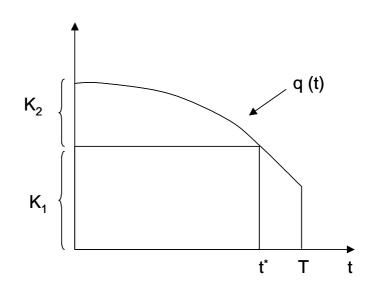
The results reported in this paper depend on a number of crucial assumptions. For one thing, the grid in our model is oversimplified; the difficulty of identifying the beneficiaries and the extent of their surpluses increases exponentially as the grid becomes more complex. The same transmission line might benefit consumers or generators, depending on time of day, season, hydrology, or other conditions. The problem is greater still when the other two functions performed by the transmission system (substitution of generating capacity and promotion of competition in the generating segment) are taken into account. The second simplification we made is that demand is inelastic. Perhaps the most restrictive assumption, however, is the deterministic world; in a future paper we intend to relax this assumption.

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# FIGURE 1





# FIGURE 2



(including a Transmission Line)

