Uncertainty, Pay for Performance and Adverse Selection in a Competitive Labor Market

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Abstract

This paper develops a new rationale for the emergence of pay-for-performance contracts. The labor market is competitive, workers are risk averse and firms risk neutral. The paper shows that in stable environments more productive workers self-select into pay-for-performance jobs because risk is less costly to them than to their less productive counterparts which prefer fixed-salary contracts. When uncertainty is sufficiently large a pooling equilibrium emerges in which all workers have pay-for-performance contracts, thereby reducing more productive workers' costs of being pooled with less productive workers.

The model explains several empirical regularities unaccounted for by alternative models, such as markets where all observed contracts involve pay-for-performance, and also that such markets are more likely to emerge in highly uncertain environments.

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1 Introduction

The standard rationale for linking pay to performance is that pay-for-performance is used to align workers' incentives with those of the firm–the well-known incentive effect. The main consequence of this rationale is that risk and incentives are negatively related (Holmström and Milgrom, 1987). Pay-for-performance imposes risk on a risk-averse worker that results in a higher wage costs. The risk imposed increases with the uncertainty of the environment and therefore the standard test of the negative trade-off between risk and incentives shows that the power of incentives, as measured by the pay-for-performance sensitivity, is lower in more uncertain environments. While appealing, this rationale fails to account for two crucial empirical facts: the positive relationship between pay for performance and uncertainty (see, Prendergast, 1999) and the low pay-for-performance sensitivity among executives (see, Murphy, 1999).

In this paper an alternative theory for the use of pay-for-performance contracts is given, which is broadly consistent with the two facts mentioned above. In so doing, a simple model is developed in which identical risk neutral firms compete for risk averse workers of two different productivity levels. Each worker knows his productivity level but firms do not–firms only know the proportion of more productivite workers in the population, and a worker's output depends only on his productivity and a random shock. In addition, firms are allowed to offer a menu of contracts where each contract is restricted to a class of linear incentive contracts, and workers have exponential utility functions.

The reason for focusing on a linear model is three-fold. First, it allows for straightforward comparative static analysis, since the power of incentives induced by optimal contracts is specified by a single parameter. Second, it allows us to fully characterize the equilibrium contracts in terms of the main parameters of the model; for instance, the variance of the performance measure. And third, it allows comparisons of the shape of the optimal contracts with those from the standard linear agency model.

The paper shows that in more certain environments, firms offer a straight-salary contract and a pay-for-performance contract, more productive workers self-select into pay-for-performance jobs and less productive workers into straight-salary jobs. More productive workers self-select into pay-for-performance jobs because for them is it worth taking the risk associated with pay for performance. Thus, the main cost of self-selection is that more productive workers have to bear too much risk, but in exchange they get an expected compensation equal to their expected productivity. Whereas in more uncertain environments, all firms offer the same contract and both more and less productive workers choose that contract. Surprisingly, the contract offered is a pay-for-performance contract and not a straight-salary contract as the optimal allocation of risk would lead us to conclude. If a straight-salary contract were offered, competition would result in a salary such that firms would break even at the average productivity for the population, which implies that more productive workers compensation would be lower than their expected output while less productive workers compensation would be higher than their expected output. In other words, more productive workers would provide a kind of subsidy to less productive workers in an amount equal to the difference between the straight salary and less productive workers' expected output. If, instead, the contract involves a small pay-for-performance sensitivity, less productive workers' expected compensation would be smaller while more productive workers' expected compensation would be higher due to the difference in productivity. Thus, the cross-subsidy from more productive workers to less productive workers decreases, and since more productive workers dislike risk less than less productive workers do, the reduction in the subsidy outweighs the cost from the amount of risk imposed by the small pay-for-performance sensitivity.

Thus, in stable environments, pay-for-performance and straight-salary contracts coexist and workers self-select into different jobs, while when the environment is more uncertain, only pay-for-performance contracts are observed, which in turn reduce more productive workers' cost of being pooled with less productive workers. Therefore, competition stops firms from offering contracts that result in self-selection of workers into different jobs. This implies that competition is effective at reducing the costs of asymmetric information since it forces firms to use pay-for-performance contracts to minimize more productive workers' cost of being either pooled with less productive workers or separated from them. In fact, the equilibrium is constrained-Pareto efficient; that is, no central authority could do better than the market given the information available.

The evidence for a positive relationship between measures of uncertainty and incentives, which is reviewed in Prendergast (1999), comes from four different occupations. The most cited evidence comes from executives, where the evidence on the negative trade-off is mixed. Some authors like Aggarwal and Samwick (1999) find evidence in favor of it while others such as Garen (1994) find no relationship[1]. The evidence from agricultural sharecropping clearly points to a positive relationship (Allen and Lueck, 1995, 2000). Fixed rent contracts are more likely to be observed in crops with larger yield variance. Among franchisees strong output-based contracts are the norm, while in company owned stores variable pay is usually absent or minimal. The evidence clearly points to a positive relationship between uncertainty and pay for performance, which means that franchisees are much more common in uncertain industries (see, Lafontaine and Slade, 2001). Finally, the literature on salesforce compensation finds little evidence of either a negative or a positive relationship.

The model presented here also yields the following empirical predictions: (i) workers paid by output on average earn and produce more than salaried or hourly workers. The compensation and productivity differences ranged roughly from 5% to 37%[2]; (ii) pay-for-performance sensitivity and the variance of output are negatively related. The evidence for this comes mainly from executive data and is mixed. Yet the model suggests that this may be due to an omitted variables problem; (iii) pay-for-performance sensitivity is smaller in jobs where workers' productivity is more important. The evidence on executives and franchising points into this direction; and (iv) pay-for-performance sensitivity in jobs where all workers are paid by output could be quite small. The evidence showing that the pay-for-performance sensitivity for CEOs is rather low has been carefully reviewed by Murphy (1999). He concludes that evidence from several studies leaves us fairly certain that the estimated pay-for-performance sensitivity for CEO's is rather small, between 0.001 and 0.007, and the following quote from Jensen and Murphy (1990) reflects the same conclusion: "the lack of strong pay-for-performance incentives for CEOs indicated by our evidence is puzzling."..

[1] As mentioned by Lazear (2000) and Prendergast (2002) casual evidence also seems to suggest that incentive pay is used more frequently in more uncertain industries, such as the use of options in high-tech industries and bonuses in the financial sector.

[2] See, for instance, Brown, 1992; Lazear, 2001; Paarsh and Shearer, 1997; Petersen, 1991 and 1992; Seiler, 1984.

The literature on incentives is vast[3], so for the sake of brevity we will focus only on the most closely related papers. Lazear (1986) shows that the worst workers self-select into firms offering straight salaries and the best in firms offering piece rates[4]. The mechanism by which self-selection is achieved in Lazear's paper is quite different from the one in this paper. More productive workers self-select into pay-for-performance jobs because for them is it worth indirectly paying the (higher) monitoring costs associated with piece rates, while in our paper self-selection occurs only when the environement is sufficiently certain such that it is worth it for high-productivity workers to face the compensation risk imposed by pay for performance. Furthermore, in Lazear's paper no pooling equilibrium exists, uncertainty plays no role and the optimal pay-for-performance sensitivity is equal to one.

Prendergast (2002), while different from this paper, also explains why variable pay should be more prevalent in uncertain environments, in a model in which workers are risk neutral. Mainly in a multitasking setting, he argues that in low uncertainty environments, firms are content to assign workers to certain tasks and monitor their efforts. By contrast in more uncertain environments, they delegate the responsibility of choosing a task to workers and use variable pay to align their incentives or constraint their discretion. Thus, Prendergast's paper can also be seen as complementary to this one since his explanation is based on moral hazaed while ours is based on adverse selection.

The outline for the rest of the paper is as follows. Section 2 describes the model and the equilibrium concept. Section 3 provides the full information benchmark. Section 4 derives the equilibrium. The next section, Section 5, discusses the main empirical predictions of the basic model, compares them with those from the linear agency model and presents the evidence supporting them. Finally, concluding remarks are presented in the last section.

[3]For excellent surveys see Prendergast (1999) and Gibbons (1998).

[4]See, Matutes et al. (1994) for a similar result.

2 The Basic Model and The Equilibrium Concept

2.1 The Basic Model

Identical risk neutral firms compete for a fixed number of workers of unknown productivity, where the price of output is normalized to 1. Workers come in two types, high-productivity workers (H) and low-productivity workers (L), and each firm has the same production technology that depends on the worker's productivity parameter. In particular, the i-worker's output, which is assumed to be contractible, is given by $y_i = n + m\,(\theta_i + \varepsilon)$, where $(n, m) \in \Re^2_+$, θ_i is the i-worker's productivity parameter, ε is a random variable that is normally distributed with mean 0 and variance σ^2_ε, and a larger variance identifies a more uncertain environment. In what follows, it is assumed that $\theta_H > \theta_L$. Thus, a high-productivity worker's distribution of output risk-dominates a low-productivity worker's distribution of output. This assumption guarantees that the single-crossing property is satisfied.

Employers know only that a worker's productivity takes one of the two possible values $\theta_i \in \{\theta_H, \theta_L\}$ and that the proportion of high-productivity workers is $\mu \equiv prob(H)$, while workers know their own productivity.

We assume that employers are risk neutral but that workers have the following exponential utility functions:

$$U(w\,(y)) = -\exp\left\{-rw\,(y)\right\}, \tag{1}$$

where $w\,(y)$ is the total compensation, which is allowed to depend on the realized output and $r > 0$ is the coefficient of absolute risk aversion[5].

Each firm is allowed to offer a menu of linear wage contracts in which each contract is of the following form: $w\,(y) = \alpha + \beta y$, where α specifies a fixed wage and β is the pay-for-performance slope,which is restricted to be non-negative. We identify β with the power of a worker's incentives. Thus, contract C is of the following form: $C \equiv (\alpha, \beta)$. When $\beta = 0$, C is a fixed-wage or straight-salary contract while when $\beta = 1$ and $\alpha = 0$, C is a pure piece-rate contract. Let us denote the contract $(0, 0)$ by C_0 and assume that workers' reservation utility

[5] The results are unchanged if it is assumed that the coefficient of absolute risk aversion is type dependent in the following way: $r_L \geq r_H$.

is given by $V_i(C_0) = -1$.

An i-worker's expected utility when he accepts contract C is:

$$V_i(C) = -E_i \exp\{-r[\alpha + \beta y]\}, \tag{2}$$

where E_i denotes the expectation when the worker is of type $i \in \{H, L\}$, and a firm's expected profit from employing an i-worker under contract C is given by,

$$\pi_i(C) = (1-\beta)(n + m\theta_i) - \alpha. \tag{3}$$

The timing of decisions adopted here was suggested by Hellwig (1987) and is as follows. At *Stage 1*, firms are symmetrically informed and simultaneously offer a menu of linear wage contracts that includes either a pay-for-performance or straight-salary contract or both for the upcoming period. At *Stage 2,* after offers have been made, each worker applies to a particular firm for the upcoming period. In the case that more than one firm offers the same contract, workers choose randomly between firms. At *Stage 3,* after each worker has chosen a contract and *firms have observed other firms' offers*, firms have the opportunity to either accept or reject a worker's application. Yet, once a worker has agreed to work for a particular firm and has been accepted, the terms of the agreement become binding for that period. At the final stage, output is produced and compensation takes place as specified in the contract.

2.2 The Equilibrium Concept

This section briefly explains the equilibrium concept that will be used and the importance of the timing adopted. Under the standard equilibrium concept (Perfect Bayesian Equilibrium) and the standard timing for screening games (stages 1 and 2), this type of model suffers from *non-existence of equilibrium* for some parameter values. The classic example of this is Rothschild's and Stiglitz's (1976) competitive insurance model, in which an equilibrium does not exist when the proportion of low risk individuals is sufficiently large[6].

[6] There are several other equilibrium concepts different from the one used in this paper that deal with the non-existence of equilibrium problem. The most common are Riley's Reactive Equilibrium and Wilson's

Hellwig (1987) added the third stage to the two-stage screening game in order to solve the competitive screening games' known *non-existence* of equilibrium problem[7]. Because the last two stages mimic a signaling game, however, Hellwig's timing effectively trades the problem of non-existence for the problem of multiple equilibria. In particular, as shown by Cho and Kreps (1987), signaling games have a plethora of Perfect Bayesian Equilibrium (hereafter, PBE) that are supported by unreasonable off-the-equilibrium path beliefs. In this paper, we adopt a signaling equilibrium refinement proposed by Mailath et al. (1993) to eliminate equilibria that are based on unreasonable beliefs. In particular, we will require that any PBE of the signaling sub-game (stages 2 and 3) must be *undefeated* among all possible PBEs that can arise from any first stage contract offers. This equilibrium refinement picks only those PBE that give the highest payoff to high-productivity workers. In other words, it picks only those equilibria that are constrained Pareto efficient.

We adopt the Undefeated Equilibrium refinement rather than the Intuitive Criterion, which is the most common refinement, and also over others like Divinity, because the equilibrium selected by the later remains unchanged for any positive proportion of high-productivity workers. That is, the equilibrium is not sensitive to the proportion of high-productivity workers unless this is exactly equal to 1, yet one can argue that it is unreasonable that the outcome of a game with a one worker in a million chance of a low-productivity worker differs significantly from a game in which there is no chance of such a worker. In addition, it seems reasonable to think that the distribution of types will not be certain, therefore, the model and the equilibrium of our model can be useful only if the predicted outcome is not overly sensitive to the description of the environment, in particular to the proportion of high-productivity workers.

3 The Full-Information Benchmark

Equations 2 and 3 tell us that under complete information, all contracts should be efficient straight-salary contracts. If β were different from 0, then a reduction in β with the ap-

Anticipatory Equilibrium. In addition, Dasgupta and Maskin (1984) derived conditions that guarantee the existence of a mixed strategy equilibrium.

[7] Grossman (1979) was the first to discuss this specification but in a non-sequential setting.

propriate increase in α would shift some of the risk of the contract from a worker to an employer, which is advantageous because employers are risk neutral, and workers are not. Therefore, in equilibrium, the contracts offered to both high- and low-productivity workers are straight-salary contracts, and competition among employers forces firms to pay each type his expected output. That is, $C_H^* = (n + m\theta_H, 0)$ and $C_L^* = (n + m\theta_L, 0)$.

In addition, a high-productivity worker gets a larger utility than a low-productivity worker; that is, $V_H(C_H^*) > V_L(C_L^*)$, and it is efficient for the two productivity types to participate; that is $V_i(C_i^*) > V_i(C_0)$ for $i \in \{H, L\}$.

4 Asymmetric Information

4.1 Preliminaries

Now consider the case in which employers do not know workers' productivity. The first thing to notice is that efficient contracts (C_H^*, C_L^*) are not incentive compatible. To see this first recall that the i-worker's efficient contract C_i^* is a straight-salary contract that pays $\alpha_i = n + m\theta_i$ irrespective of the output produced and that $V_H(C_H^*) > V_L(C_L^*)$. Suppose then that firms offer the menu (C_H^*, C_L^*). Then, a low-productivity worker will mimic a high-productivity worker since his utility from choosing C_H^* yields a utility equal to $V_L(C_H^*) = V_H(C_H^*) > V_L(C_L^*)$.

The next thing to notice is that indifference curves satisfy the single-crossing property in the (α, β) space. This is shown in the next lemma.

Lemma 1 *(i) A high- and a low-productivity worker's indifference curves in the (α, β) space cross only once; and (ii) a high-productivity worker's zero-profit locus is steeper than a low-productivity worker's zero-profit locus in the (α, β) space.*

Proof. Using the fact that if x is normally distributed with mean $\bar{x}$ and variance σ_x^2, then $E\{\exp(-rx)\} = \exp\left\{-r\bar{x} + \frac{r^2}{2}\sigma_x^2\right\}$, it is then easy to verify that:

$$V_i(C) = -\exp\left\{-r\left[\alpha + \beta(n + m\theta_i)\right] + \frac{r^2}{2}\beta^2 m^2 \sigma_\varepsilon^2\right\}.$$

The slope of the indifference curve in the (α, β) space for worker i is then $\left.\frac{d\alpha}{d\beta}\right|_{V_i=k} = -n - m\theta_i + r\beta m^2\sigma_\varepsilon^2$. Because $\theta_H > \theta_L$, $\left.\frac{d\alpha}{d\beta}\right|_{V_H=k} < \left.\frac{d\alpha}{d\beta}\right|_{V_L=k}$ for all $\beta \geq 0$ and, therefore, the indifference curves cross only once.

The slope of the zero-profit locus in the (α, β) space for worker i is $\left.\frac{d\alpha}{d\beta}\right|_{\pi_i=0} = -n - m\theta_i$. Because $\theta_H > \theta_L$, $\left.\frac{d\alpha}{d\beta}\right|_{\pi_H=k} < \left.\frac{d\alpha}{d\beta}\right|_{\pi_L=k}$ and, therefore, the zero-profit locuses cross only once.
■

The fact that high-productivity workers' indifference curves are steeper than low-productivity workers' indifference curves in the (α, β) space intuitively means the following: starting from a situation (α_0, β_0) an increase in β requires a decrease in α to keep the expected utility at the initial level, but the required decrease in α must be larger for high-productivity workers. In other words, low-productivity workers dislike the risk that pay for performance imposes more than high-productivity workers do since the latter have a higher expected productivity. This implies that firms could use the pay-for-performance sensitivity as a device to achieve self-selection of workers into different compensation methods.

4.2 Separating Equilibrium

Suppose first that in equilibrium separation of the two types occurs. In a separating equilibrium competition for workers forces firms to offer a menu of contracts that contains a contract that maximizes high-productivity workers' expected utility (cream-skimming) and one that maximizes low-productivity workers' expected utility subject to the fact that neither type of worker has incentive to choose the contract designed for the other type. That is, in a separating equilibrium contracts must satisfy low- and high-productivity workers' incentive compatibility constraints. The former is given by:

$$\alpha_H + \beta_H (n + m\theta_H) - \frac{r}{2}\beta_H^2 m^2\sigma_\varepsilon^2 \geq \alpha_L + \beta_L (n + m\theta_H) - \frac{r}{2}\beta_L^2 m^2\sigma_\varepsilon^2, \tag{4}$$

and the latter by:

$$\alpha_L + \beta_L (n + m\theta_L) - \frac{r}{2}\beta_L^2 m^2\sigma_\varepsilon^2 \geq \alpha_H + \beta_H (n + m\theta_L) - \frac{r}{2}\beta_H^2 m^2\sigma_\varepsilon^2. \tag{5}$$

Adding the two incentive compatibility constraints and rearranging terms, a necessary condition for separation to be an equilibrium is:

$$0 \geq -\left(\beta_H - \beta_L\right) \triangle\theta, \tag{6}$$

where $\triangle\theta \equiv \theta_H - \theta_L$.

Notice that a necessary condition for self-selection to occur is that the contract tailored to high-productivity workers has a higher pay-for-performance sensitivity than the one tailored to low-productivity workers.

When the two incentive compatibility constraints are satisfied, workers self-select and therefore competition forces employers to pay each productivity type his expected output. This, plus the fact that a firm offering C_L^* will make non-negative profit irrespective of the attracted workers' productivity level, and that any other contract that either breaks even or makes a positive profit when chosen only by low-productivity workers yields a lower utility to low-productivity workers, implies that s contract tailored to low-productivity workers is the full information contract C_L^*. Because C_L^* is a straight salary contract–that is $\beta_L^* = 0$– equation 6 plus the single-crossing property implies that β_H must be non-negative. This plus the fact that the full-information contract C_H^* cannot sort workers out when offered together with C_L^*, implies that the contract tailored to high-productivity workers must be a pay-for-performance contract.

Plugging $\beta_L = 0$ in equation 5 implies that the pay-for-performance sensitivity that satisfies the low-productivity worker's incentive compatibility constraint is given by:

$$\beta_H^s = \frac{-\triangle\theta + \left(\triangle\theta^2 + 2mr\sigma_\varepsilon^2\triangle\theta\right)^{\frac{1}{2}}}{rm\sigma_\varepsilon^2}. \tag{7}$$

It is easy to verify that high-productivity workers' incentive compatibility constraint is satisfied at contract $\left(\left(1-\beta_H^s\right)\left(n+m\theta_H\right), \beta_H^s\right)$[8], and that this contract is the best contract for high-productivity workers among all those contracts that break even when chosen only by high-productivity workers. Let us denote this contract by C_H^s and the optimal contract for low-productivity workers by C_L^s, where s stands for separation. The next lemma characterizes these two contracts.

[8] If 5 is satisfied when $\beta_L = 0$, then $n + m\theta_L = n + m\theta_H - \beta_H m\triangle\theta - \frac{r}{2}\beta_H m^2\sigma_\varepsilon^2$. The RHS is lower than $n + m\theta_H - \frac{r}{2}\beta_H m^2\sigma_\varepsilon^2$, which is high-ability worker's payoff from (α_H, β_H), since $\theta_H > \theta_L$.

Lemma 2 *(i) C_H^s is a pay-for-performance contract with a pay-for-performance slope* $\beta_H^s = \frac{-\triangle\theta + \left(\triangle\theta^2 + 2mr\sigma_\varepsilon^2 \triangle\theta\right)^{\frac{1}{2}}}{rm\sigma_\varepsilon^2}$ *and fixed component* $\alpha_H^s = (1 - \beta_H^s)(n + m\theta_H)$*; and (ii)* $C_L^s = C_L^*$*.*

This result establishes that firms use the pay-for-performance sensitivity to skim the cream. For the risk imposed by a pay-for-performance contract is less attractive for low-productivity workers because they are less likely to produce a higher level of output. Thus, self-selection is achieved at the cost of imposing risk to high-productivity workers.

4.3 Pooling Equilibrium

Next suppose that the equilibrium is pooling–that is high- and low-productivity workers choose the same contract and contract choice does not reveal any information. In order for the equilibrium to be pooling, it must be true that firms do not have an incentive to offer a contract that skims the cream or attracts only high-productivity workers and makes positive profits. A necessary condition for an equilibrium to be pooling is that high-productivity workers' expected utility is larger under the pooling contract than under the separating equilibrium contracts, otherwise the pooling contract attracts no high-productivity workers and, therefore, loses money. Given this and that risk is less costly for high-productivity workers, there could be a cream-skimming contract that makes positive profits. The problem is that under the equilibrium strategies no worker would ever apply to the pooling contract offered by the non-deviating firms. Workers would not apply because they know that they are going to be rejected in the third stage since the non-deviating firms, after seeing a cream-skimming offer, will know that their offers would be attracting only low-productivity workers. Thus, firms offering the pooling contract must be getting a below-average sample of the population and since the pooling contracts break-even at the average population productivity, workers that apply to this contract must be rejected at stage 3. Workers anticipating this rejection would not apply to the pooling contract and would apply to the deviating firm offering the cream-skimming contract, which would make this contract unprofitable. Thus, the best pooling contract cannot be upset by a cream-skimming offer. In fact, any pooling contract that yields a large expected utility to high- and low-productivity workers than the one they get in a separating equilibrium cannot be upset by a cream-skimming offer as is the case in a two stage screening game. Furthermore, among all those pooling contracts, the only one selected by our equilibrium refinement is the one that offers the largest expected utility to high-productivity workers among those contracts that break-even at the population average productivity, $\widehat{\theta} \equiv \mu\theta_H + (1-\mu)\theta_L$. Let us denote this contract by C^p, where p stands for pooling. The reason for this is any other pooling contract that can be sustained as PBE does so by assuming unreasonable beliefs off-the-equilibrium path. In particular, a pooling contract different from C^p can be sustained as a PBE only if a deviating firm believes that this new pooling offer attracts a below average sample of workers.

The next lemma characterizes C^p.

Lemma 3 *C^p is a pay-for-performance contract with a pay-for-performance slope $\beta^p = \frac{(1-\mu)\triangle\theta}{rm\sigma_\varepsilon^2}$ and fixed component $\alpha^p = (1-\beta^p)\left(n + m\widehat{\theta}\right)$.*

It is interesting to note that C^p is a pay-for-performance contract and not a straight-salary contract as the optimal allocation of risk would lead us to conclude. To better understand why the contract is not a straight-salary contract it is useful to understand high-productivity workers' cost of being pooled with low-productivity ones under a straight-salary contract $(\alpha, 0)$. Competition implies that α must be such that firms break even at the population average productivity $\widehat{\theta}$. This implies that high-productivity workers' compensation would be lower than their expected output while low-productivity workers' compensation would be larger than their expected output. In other words, high-productivity workers provide a kind of subsidy to low productivity workers in an amount equal to the difference between $n + m\widehat{\theta}$ and low-productivity workers' expected output. If, instead, the pooling contract involves a small pay-for-performance slope, low-productivity workers' expected compensation would be smaller than $n+m\widehat{\theta}$ because they have a lower expected output than the average worker, while high-productivity workers' expected compensation would be larger because they have a larger expected output than the average worker. Thus, the cross-subsidy from high-productivity workers to low-productivity workers decreases and, since the pay-for-performance slope is small, high-productivity workers' gains from the reduction in the subsidy outweighs the cost from the small amount of risk imposed by the pay-for-performance contract.

Thus, in a pooling equilibrium, competition induces firms to offer all workers a pay-for-performance contract to reduce the more productive workers cost of being pooled with less productive workers.

4.4 The Equilibrium

In a pooling equilibrium high-productivity workers' expected compensation is lower than their expected output and they face little risk, while in a separating equilibrium their expected compensation is equal to their expected output, but they face much more risk. Sorting is more expensive for high-productivity workers when the variance of output is large since the pay-for-performance sensitivity needed to achieve self-selection is larger than the one needed to minimize high-productivity workers' cost of being pooled with low-productivity workers. Hence, when uncertainty is sufficiently large, competition forces firms to pool workers under a pay-for-performance contract while in less uncertain environments the more productive workers' cost of working under a high-powered incentive contract is lower and, therefore, competition forces firms to offer a menu of contracts that induces high-productivity workers to self-select into pay-for-performance jobs and low-productivity workers to self-select into straight-salary jobs.

Define $\sigma_{\varepsilon}^{2}(\mu)$ as the minimum variance that leaves high-productivity workers indifferent between contract C^{p} and contract C_{H}^{s}, and leaves low-productivity workers better-off than when they choose C_{L}^{s}. That is $V_{H}(C_{H}^{s}) = V_{H}(C^{p})$ and $V_{L}(C_{L}^{s}) < V_{L}(C^{p})$. Then the following is proposed and formally shown in the appendix.

Proposition 1 *There exists a threshold for the variance of output denoted by $\sigma_{\varepsilon}^{2}(\mu)$ such that: (i) if $\sigma_{\varepsilon}^{2} \leqq \sigma_{\varepsilon}^{2}(\mu)$, then in equilibrium firms offer a menu with two contracts: the straight-salary contract C_{L}^{s} and the pay-for-performance contract C_{H}^{s}. Low-productivity workers self-select into straight-salary jobs while high-productivity workers self-select into pay-for-performance jobs; while (ii) if $\sigma_{\varepsilon}^{2} > \sigma_{\varepsilon}^{2}(\mu)$, then in equilibrium all firms offer the pay-for-performance contract C^{p} and both types of workers participate.*

This proposition states that when the environment is more certain, firms choose to separate workers offering a pay-for-performance and a straight-salary contract. While when the environment is more uncertain attempting to sort is too costly for high-productivity workers relative to pooling and, therefore, competition forces firms to pool workers under the same contract. Thus, when pay-for-performance and straight-salary contracts coexist the former are used as self-selection device, while when only pay-for-performance contracts are observed, those are used as mechanism to reduce the more productive workers' cost of being pooled with less productive workers.

This result also shows that competition is effective at reducing the costs of asymmetric information since it forces firms to use pay-for-performance contracts to minimize high-productivity workers' cost of being either pooled with low-productivity workers or separated from low-productivity workers. In fact, the equilibrium is constrained-Pareto efficient.

Before ending this section, it is worthwhile to notice that it is easy to extend the analysis to more than two productivity types. If we restrict ourselves to either pure separating or pure pooling equilibrium only, with more than two types the analysis yields the following. In the separating equilibrium only the lowest-productivity type receives a straight salary, while all other productivity types receive pay-for-performance. Because of the single-crossing property, the pay-for-performance sensitivity β_i^s increases with workers' productivity; that is, for any two workers of productivity θ_i and $\theta_{i'}$ with $\theta_i > \theta_{i'}$, the β^s from the contract tailored to an i-worker is larger than the β^s from the contract tailored to an i'-worker. This readily follows from the sum of a i-worker and an i'-worker's incentive compatibility constraints, $0 \geq -\left(\beta_i^s - \beta_{i'}^s\right)\left(\theta_i - \theta_{i'}\right)$ which results in that $\beta_i^s \geq \beta_{i'}^s$ must be satisfied. In the pooling equilibrium all firms offer the same pay-for-performance contract and workers are indifferent between firms. The pay-for-performance contract offered in equilibrium is the one that the highest-productivity worker prefers, i.e., the best among all those contracts that break even on the average population productivity, and is accepted by all workers. As in the two type case, a unique pooling equilibrium exists when each worker's productivity type prefers the pooling equilibrium to the separating one, with strict preferences for at least one worker type. Therefore, results similar to the ones in proposition 1 hold even with more than two productivity types.

5 Empirical Predictions and Evidence

In this section the empirical predictions of the model, the evidence supporting them and their comparison with those of the linear agency model are presented.

5.1 Empirical Predictions

We begin by focussing on the predictions from the agency model as developed by Holmstrom and Milgrom (1987), where linear contracts are generated by assuming a dynamic model with exponential utility and normally distributed errors. For our purposes this model, as well as the one in this paper, has as the particular feature that the power of the incentives induced by optimal contracts is specified by a single parameter, and this depends on a unique measure of risk that is the variance of output. It is straightforward to show that the optimal contract in this model has a pay-for-performance sensitivity given by $\beta_i^M = \frac{1}{1+c_i r \sigma_\varepsilon^2}$ when output is assumed to be $y = n + m\,(e + \varepsilon)$, the cost of effort is $\frac{c_i e^2}{2}$ where c_i is known to everyone, and the market is competitive. This yields the simple prediction that the power of incentives decreases with the variance of output and the coefficient of absolute risk aversion and is independent om the marginal productivity of effort, m.

The next table shows the main predictions of this paper and those from agency theory concerning the pay-for-performance sensitivity with respect with the main parameters of interest.

Parameters	β_L^s	β_H^s	β^p	β^M
σ_ε^2	0	$-$	$-$	$-$
r	0	$-$	$-$	$-$
m	0	$-$	$-$	0
μ	0	0	$-$	$n.a.$

Surprisingly, pay-for-performance sensitivity behaves with respect to the variance of output and the coefficient of absolute risk aversion in the same way in both models. That is, there is a negative trade-off between risk (degree of risk aversion) and incentives as measured by the pay-for-performance sensitivity. The intuition is simple. When equilibrium is separating, more risk makes the contract tailored to high-productivity workers less attractive to low-productivity workers and, therefore, a lower pay-for-performance sensitivity is needed to induce workers to self-select. Whereas when the equilibrium is pooling, less compensation risk is needed since the reduction in the cross-subsidy obtained by using pay for performance cannot compensate for the extra compensation risk. The intuition for the effect of an increase in the absolute risk aversion coefficient is similar.

Thus, this prediction coupled with the fact that the equilibrium is more likely to be pooling in more uncertain environments implies that pay for performance is more likely to be observed in uncertain environments, but the pay-for-performance sensitivity is smaller in those environments. In addition, the pay-for-performance sensitivity of any given worker decreases with the riskiness of the environment.

An increase in μ decreases the pay-for-performance sensitivity when the equilibrium is pooling because the larger the proportion of high-productivity workers the smaller is the cross-subsidy and, therefore, the benefit of imposing compensation risk is smaller.

The intuition for why pay-for-performance decreases with m is less straightforward. An increase in m implies that the difference in productivity between workers is more important and that for a positive pay-for-performance the compensation risk is larger. Thus, when the equilibrium is separating, less compensation risk is needed to induce self-selection because low-productivity workers' benefit from pay for performance relative to high-productivity workers is lower, but the compensation risk is the same. Whereas when the equilibrium is pooling, an increase in m increases the compensation risk and, therefore, less risk is imposed.

The model also predicts that $\sigma_{\varepsilon}^{2}(\mu)$ is decreasing in μ. A larger proportion of high-productivity workers implies that high-productivity worker's expected compensation is larger and, therefore, being pooled with low-productivity workers is less costly. Thus, the model predicts that in occupations in which the proportion of high-productivity workers is larger, the likelihood of observing only pay-for-performance workers is larger.

Finally considering total compensation, both models predicts that pay-for-performance workers on average earn and produce more than straight-salary workers.

5.2 Empirical Evidence

The most important prediction of the model is that pay for performance is more likely to be observed in uncertain environments, but that pay-for-performance sensitivity is smaller in those environments. The only evidence that we are aware off that test this prediction comes from the franchising literature. In particular from Lafontaine (1992) who considers how uncertainty affects both: (i) the decision to franchisee and (ii) the royalty rate offered to franchisees. She consider 548 franchisors in 14 different sectors of which 117 franchise all their retail outlets. She reports in table 5 that the decision to franchise is positively related and significant to the uncertainty measured by the likelihood of bankruptcy, while the royalty rate is negatively related but is not significant (test t=1.44). This is quite consistent with the model proposed here, except the fact that the relationship between royalty rate and risk is not significant. This may be due to an omitted variables problem.

Lafontaine (1992) runs the following regression:

$$\beta_j = \gamma_0 + \gamma_1 \sigma_j^2 + \varepsilon_j, \tag{8}$$

where β_j is the royalty rate for the j-store , σ_j^2 is measure of risk for the j-store, and ε_j is the error term[9]. The prediction of the principal agent model is that $\gamma_1 < 0$.

The simplest of all models that can be used to test our theory assuming that the econometrician knows whether a worker is paid by output is the following:

$$\beta_j = \left(\gamma_0^H + \gamma_1^H \sigma_j^2\right) D_j^S D_j^E\left(\sigma_j^2\right) + \left[\gamma_0 + \gamma_1 \sigma_j^2\right]\left(1 - D_j^E\left(\sigma_j^2\right)\right) + \epsilon_j, \tag{9}$$

where $D_j^E\left(\sigma_j^2\right)$ is a dummy variable that takes the value 1 when the j-store is a franchise from a company where company owned and franchise stores co-exists and takes the value 0 otherwise, and D_j^S is equal 1 when the j-store is a franchise from a company in which there is no company owned stores and is equal to 0 otherwise. Thus, there is an omitted variables problem.

Let us define $x_1 = \triangle D_j^E\left(\sigma_j^2\right) D_j^S$, $x_2 = \triangle \sigma_j^2 D_j^E\left(\sigma_j^2\right) D_j^S$, $x_3 = \triangle D_j^E\left(\sigma_j^2\right)$ and $x_4 = \triangle \sigma_j^2 D_j^E\left(\sigma_j^2\right)$, where $\triangle$ means difference with respect to the mean. If for the sake of simplicity

[9] Other control variables are used, but for the sake of brevity they are ignored in the discussion.

X_j is ignored, then one can easily verify that:

$$E\left(\hat{\gamma}_1\right)-\gamma_1=\frac{\sum_j \triangle\sigma_j^2\left[\gamma_0^H\triangle x_1+\gamma_1^H\triangle x_2-\gamma_0\triangle x_3-\gamma_1\triangle x_4\right]}{\sum_j\left(\triangle\sigma_j^2\right)^2}.$$

The model predicts that the covariance between $D_j^E\left(\sigma_j^2\right)$ and σ_j^2 is negative since as the variance increases the equilibrium is more likely to be pooling. Under the assumption that $D_j^S=1$ with probability μ, then the covariance between $D_j^S D_j^E\left(\sigma_j^2\right)$ and σ_j^2 is equal to $\mu C\left(D_j^E\left(\sigma_j^2\right),\sigma_j^2\right)<0$, where $C\left(\cdot\right)$ denotes the covariance. Finally, the $C\left(\sigma_j^2 D_j^E\left(\sigma_j^2\right)D_j^S,\sigma_j^2\right)=\mu C\left(\sigma_j^2 D_j^E\left(\sigma_j^2\right),\sigma_j^2\right)\lesseqqgtr 0$. Given this $E\left(\hat{\gamma}_1\right)-\gamma_1$ becomes

$$\frac{\sum_j \triangle\sigma_j^2\left[\left(\gamma_0^H\mu-\left(1-\mu\right)\gamma_0\right)C\left(D_j^E\left(\sigma_j^2\right),\sigma_j^2\right)+\left(\gamma_1^H\mu-\gamma_1\right)C\left(\sigma_j^2 D_j^E\left(\sigma_j^2\right),\sigma_j^2\right)\right]}{\sum_j\left(\triangle\sigma_j^2\right)^2}.$$

Assuming for the sake of simplicity that $C\left(\sigma_j^2 D_j^E\left(\sigma_j^2\right),\sigma_j^2\right)=0$, then the bias is positive as long as $\mu<\frac{\gamma_0}{\gamma_0+\gamma_0^H}$. In addition, for small μ, $C\left(\sigma_j^2 D_j^E\left(\sigma_j^2\right),\sigma_j^2\right)$ is more likely to be negative since $\sigma_\varepsilon^2\left(\mu\right)$ decreases in μ. This implies that for small μ the second term in the bias is likely to be positive. Thus, the omitted variables problem is likely to bias the coefficient on the measure of risk towards zero.

The evidence concerning the negative relationship between the pay-for-performance sensitivity and the variance of output coming from executive data is mixed, yet if a negative sign is found it cannot be concluded that the data is explained by agency theory since our model also predicts a negative relationship. For instance, Aggarwal and Samwick (1999) find a negative relationship and concludes that the data is explained by agency theory. However, most studies using volatility of returns as a measure of risk find no relationship[10]. Our model suggests that this finding could be a consequence of omitted variables.

Since in the executive compensation literature there is no information on pay-for-performance sensitivity, one way of testing for the existence of a negative relationship between the variance of the performance measure and the pay-for-performance sensitivity is to run a regression of the following form:

$$w_{ijt}=\gamma_0+\gamma_1 y_{jt}+\gamma_2 F\left(\sigma_{jt}^2\right)y_{jt}+\lambda_i+\varepsilon_{ijt}, \tag{10}$$

[10] There are two papers that use different measures of risk that find a positive relationship.

where w_{ijt} is the i-worker's total compensation in firm j, y_{jt} is the i-worker's performance measure (usually firm value in the CEO's compensation literature), λ_i is a fixed effect and ε_i is the error term[11]. The term $F\left(\sigma_{jt}^2\right)$ is some function of the variance of the i-worker's performance measure. For instance, in Aggarwal and Samwick (1999) the function F is the cumulative distribution function of the variance of returns for firms in the sample. By using this F function they can transform the estimated values of γ_1 and γ_2 into pay-for-performance sensitivities at any percentile of the distribution of variances. The estimated pay-for-performance sensitivity is $\gamma_1 + \gamma_2 F\left(\sigma_{jt}^2\right)$, and the predictions of the principal agent model are that $\gamma_1 > 0$ and $\gamma_2 < 0$.

Notice that this model imposes the same slope for all workers. This implies that the coefficients γ_1 and γ_2 are averages of workers with γ_2 equal to 0, workers with γ_2 equal γ_2^H, and workers with γ_2 equal γ_2. Since for low variances both methods of pay coexist while for large variances only workers paid by the output are observed, then γ_2 is biased towards zero when there are sufficient workers with γ_2 equal to 0.

Another prediction of the model is that in those occupations in which only pay-for-performance workers are observed the pay-for-performance sensitivity could be quite small. This result can also be predicted by the linear agency model when the variance of output is sufficiently large. Yet only pay-for-performance workers are observed within an occupation when the variance is sufficiently large. In addition, the pay-for-performance sensitivity is smaller the larger the proportion of high-productivity workers. Thus, in those occupations in which only pay-for-performance workers are observed and the proportion of high-productivity workers is large, the pay-for-performance sensitivity tends to be small. For instance, if the schooling level is correlated with the proportion of high-productivity workers, then within an occupation like CEOs, the pay-for-performance sensitivity is likely to be small. There is evidence that provides some support for this prediction. For instance, the pay-for-performance sensitivity for CEOs, occupation in which all workers are paid pay-for-performance, is rather low. Jensen and Murphy (1990) find that CEO's wealth changes \$3.25 for every \$1000 change in shareholder value, which implies a pay-for-performance sensitivity of *0.003* and conclude that the "the lack of strong pay-for-performance incentives for CEOs indicated by

[11] For illustrative purposes we ignore the other control variables used.

our evidence is puzzling"[12]. Murphy (1999), in a review of the CEO literature, concludes that evidence from several studies and samples leaves us fairly secure that the estimated pay-for-performance sensitivity for CEO's is rather small, between 0.001 and 0.007.

If higher jobs in a hierarchy are those in which m is larger–that is productivity or ability is more important- then the model predicts that when the equilibrium is either pooling or separating, the pay-for-performance sensitivity is smaller the higher ranking the job. The parameter m can also be thought of as proxy for firm size. For instance, m may depend positively on the amount of capital that a given firm has. Under this interpretation, a larger firm is one where the marginal contribution of a worker is larger the larger is his productivity parameter. In this case, the power of incentives should be smaller in larger firms. There is evidence on this coming from the franchise literature and executive compensation. Lafontaine and Slade (2001) report evidence from several studies in favor of this prediction and the studies on executives that control for firm size also find a negative relationship; e.g., Core and Guay (2003).

Finally, the prediction that workers under a straight salary have a lower average productivity and a lower average compensation than workers under pay-for-performance is fully borne-out in the data. For instance, the evidence shows that on average pay-for-performance workers earn more and have a higher average productivity than straight-salary workers do. The compensation and productivity differences ranged roughly from 5% to 37%.[13] Foster and Rosenzweig (1996), using detailed data from an agricultural labor market in which workers can work in either a piece-rate or a straight-salary occupation find that workers are sorted out according to their comparative advantages; that is, the more skillful workers work in the piece-rate sector while the less skillful ones work in the straight-salary sector.

6 Conclusions

Pay-for-performance contracts represent a significant portion of compensation contracts. The standard rationale for the existence of pay-for-performance is the well-known incentive

[12] Similar evidence is found in Kaplan (1994), Gibbons and Murphy (1990) and Murphy (1985, 1986).

[13] See, for instance, Brown, 1992; Lazear, 2001; Paarsh and Shearer, 1997; Petersen, 1991 and 1992; Seiler, 1984.

effect. Although incentives for effort are important, incentive theory does not mesh well with a number of empirical facts. In this paper, an alternative theory is put forth based only on asymmetric information about workers' productivity that explains why pay should vary with output and which is broadly consistent with the evidence. In particular, this theory explains among other things why variable pay is more prevalent in more uncertain environments.

We do not claim that the rationale set forth here can explain all the facts that incentive theory cannot explain, but the extreme focus on this theory leads to a lack of understanding of the role played by pay-for-performance contracts and has biased the empirical work that tries to identify the effects of contracts on productivity and compensation. Thus, it seems more attention should be paid to the effect that asymmetric information may have on pay-for-performance contracts.

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Appendix

A The Equilibrium Concept: Notation and Definitions

The main goal of the appendices is to state and prove formally all the propositions in the text and show that we have uniqueness of equilibrium when pure strategies are allowed.[14]

The three stage screening game is described as follows. There are two set of players, firms and workers. We denote the set of types by $I \in \{H, L\}$, with the common knowledge prior probability of $i = H$ given by $\mu \in [0, 1]$. Each firm offers a contract $C_k \in \Psi = \{(\alpha_k, \beta_k) \in \Re_+^2$ for $k = 1, ..., N\}$. Workers seeing the set of contracts offered $C = \cup_{k=1}^{k=N} C_k$ decides to which contract to apply. Firms seeing that, respond with either an acceptance of rejection of each contract application. An i-worker's pure strategy is denoted by $\sigma_i = (\sigma_{i1,...,}\sigma_{ik})$, where σ_{ik} : $I \times C \to \{\text{not apply, apply}\}$ and a firm's pure strategy is denoted by $(\rho, \gamma) = (\rho, \gamma_{1,}..., \gamma_N)$, where $\rho : I \times \Psi \to \{\text{reject, accept}\}$ and $\gamma_k : \Psi \to \{\text{not offer, offer}\}$. We assume that workers can apply to one contract at the time.

Let $\widehat{\mu}(H \mid C_k)$ be firms' belief about the probability that a worker applying to contract C_k is a high-ability worker.

Definition 1 $\Upsilon \equiv (\widehat{\mu}, \sigma_i, \rho, C)$ *is a pure Perfect Bayesian Equilibrium if* (σ_i, ρ, C) *are best responses to the other players' strategies and* $\widehat{\mu}(\cdot)$ *is Bayesian consistent with the prior belief* μ*, firms' and workers' equilibrium strategies and observed actions along the equilibrium path, otherwise* $\widehat{\mu}(\cdot)$ *is arbitrarily chosen.*

As usual the game is solved backwards, starting from stage 3 and rolling back the optimal strategies up to stage 1.

We denote the signalling sub-game starting in the second stage by G and the set of pure strategy PBEs for the signalling sub-game by $PSE\,(G)$.

Definition 2 $\Lambda \equiv (\widehat{\mu}, \sigma_i, \rho) \in PSE\,(G)$ *defeats* $\Lambda' \equiv (\widehat{\mu}', \sigma_i', \rho_k') \in PSE\,(G)$ *if* $\exists C_k \in C$ *such that:*

[14] The same result holds when mix strategies are allowed. For the sake of simplicity we focus on pure strategies. For a more formal justification of why focus only in pure strategies see Mailath (1992).

C1: $\forall i \in I : \sigma_i^{'}(C_k) \neq 0$ and $K \equiv \{i \in I : \sigma_i(C_k) = 1\} \neq \phi$,

C2: $\forall i \in K : V_i(\Lambda) \geq V_i(\Lambda^{'})$ and $\exists i \in K : V_i(\Lambda) > V_i(\Lambda^{'})$; and

C3: $\exists i \in K : \widehat{\mu}'(\theta \mid C_k) \neq \frac{\mu(i)\beta(i)}{\sum_{i^{'} \in I} \mu(i^{'})\beta(i^{'})} \equiv \mu(i, \beta(i))$ for any $\beta : I \to [0, 1]$ satisfying

(i) $i^{'} \in K$ and $V_{i'}(\Lambda) > V_{i'}(\Lambda')$, $\beta(i^{'}) = 1$ and

(ii) $i^{'} \notin K$, $\beta(i^{'}) = 0$.

We say that a PBE Λ is undefeated if there is no other PBE Λ' that defeats Λ.

Definition 3 *The three stage screening game has an equilibrium if the set of contracts offer give rises to an undefeated PBE of the signalling sub-game;* i.e.*, stages 2 and 3, with respect to all possible PBEs that may arise from any feasible set of contracts that firms may offer in stage 1.*

Lemma 4 *(Pure adverse selection) In any PBE, denoted by Υ, low-ability workers' equilibrium payoff is at least as large as the payoff that they would obtain under perfect information; that is, $V_L(\Upsilon) \geq V_L(C_L^*)$.*[15]

Proof. We will prove this by contradiction. Suppose not, then there exists a PBE, Υ', such that $V_L(\Upsilon^{'}) < V_L(C_L^*)$. Then by continuity of preferences and risk-aversion, there is a full insurance contract, C', that provides low-ability workers with at least the same expected payoff than $\Upsilon^{'}$ does, $V_L(\Upsilon^{'}) = V_L(C^{'})$, and yields positive expected profits when is chosen by either low- or high-ability workers or both ($\pi_i(C') > 0, \forall i \in I$). This implies that C' yields positive expected profits for any beliefs that firms could hold, and therefore, all firms offering C' accept all the applicants to C'. Then a firm entering the market can offer C', make positive profits for any $\widehat{\mu} \in [0, 1]$, and attract all low-ability workers and may be some high-ability workers. Therefore, no matter which beliefs the incoming firm holds it has a profitable deviation contradicting that Υ' is PBE. ■.

Lemma 5 *For any PBE it is always possible to find another PBE in which along the equilibrium path at most two of the contracts offered at stage 2 are chosen,* i.e.*, there are at most two $C_k \in C$ with either $\sigma_H(C_k) > 0$ or $\sigma_L(C_k) > 0$.*

[15] With some abuse of notation $V_i(\Upsilon)$ is the expected payoff that an i-worker gets in the PBE Υ.

Since we are more concerned with equilibrium outcomes than equilibrium strategies, the proof consists in finding for any PBE new strategies that yields the same equilibrium outcomes but at most two contracts are chosen in equilibrium.

Proof. Take any PBE and denote it by Υ. Suppose that at least 3 contracts offered in stage 1 are chosen, *i.e.*, either $\sigma_H(C_k) > 0$ or $\sigma_L(C_k) > 0$ or both for at least three contracts.

Let C_H be the contract, among all $C_i \in C$ such that $\sigma_H(C_k) > 0$, that gives rises to the highest stage 3 beliefs $\widehat{\mu}(H \mid C_i)$ and let define C_L in the same way, but C_L maximizes $\widehat{\mu}(L \mid C_k)$, i.e., C_H (C_L) is the contract most often chosen by high (low) ability workers.

Let $\underline{\mu} = \widehat{\mu}(L \mid C_L)$ and $\overline{\mu} = \widehat{\mu}(H \mid C_H)$ and define the following strategies;

$\sigma'_H(C_H) = \frac{\overline{\mu}(\mu-\underline{\mu})}{\widehat{\mu}(\overline{\mu}-\underline{\mu})}$, $\sigma'_H(C_L) = \frac{\underline{\mu}(\overline{\mu}-\mu)}{\widehat{\mu}(\overline{\mu}-\underline{\mu})}$, $\sigma'_L(C_H) = \frac{(1-\overline{\mu})(\mu-\underline{\mu})}{(1-\widehat{\mu})(\overline{\mu}-\underline{\mu})}$ and $\sigma'_L(C_L) = \frac{(1-\underline{\mu})(\overline{\mu}-\mu)}{(1-\widehat{\mu})(\overline{\mu}-\underline{\mu})}$.

Consider the following strategies:

Stage 1: Firms offer the same sets of contracts.

Stage 2: High ability workers' play C_H with probability $\sigma'_H(C_H)$ and C_L with probability $1 - \sigma'_H(C_H)$, and $\sigma_H(C_k) = 0$ otherwise.

Low ability workers' play C_H with probability $\sigma'_L(C_H)$ and C_L with probability $1 - \sigma'_L(C_H)$, and $\sigma_L(C_i) = 0$ otherwise.

Stage 3: $\rho'_i(C_k) = \rho_i(C_k)$, $\forall C_k \in C$.

It is easy to show that this new strategies give rises to same beliefs in stage 3, therefore if $\rho_i(C_k)$ was an equilibrium under Υ it must be an equilibrium under the new strategies. In stage 2, a worker of type θ is willing to randomize between two or more contracts if and only if $V_i(C''_k)\rho_i(C''_k) = V(C'_k)\rho_i(C'_k)$. Since, stage 3 strategies have not changed, firms are offering the same sets of contracts at stage 1, $\pi(\widehat{\mu}, C_i) = 0$, $\forall C_k \in C$ such that $\sigma'_i(C_i) > 0$, $\forall i \in I$ and all of them give the same expected utility, then it must be case that choosing at most two contracts yields the same expected utility to an i-worker, $\forall i \in I$ and the same expected profits to each firm ■

Given this lemma from now on we will concentrate in the case in which each firm offer at most two contracts.[16]

[16] There is some lost of generality here, since there are some equilibria that use more than two contracts, though these equilibria have the same outcomes than the corresponding equilibria with two contracts. Given that our main concern are the equilibrium outcome and that the refinement that we use in the SSG select as

Lemma 6 *In any fully separating PBE, denoted by Υ, low-ability workers' equilibrium payoff, is $V_L(C_L^*)$, the payoff that they would obtain in the perfect information case.*

Proof. Let the contract chosen only by low-ability workers be C_L^s. By lemma 4 the equilibrium payoff of this contract is so that $V_L(C_L^s) \geq V_L(C_L^*)$. In addition, because preferences satisfy the single-crossing property on the (α, β)-space and the (α, e)-space and and a high-ability worker's zero-profit locus is steeper than a low-ability worker's zero-profit locus (see lemma 1), any contract $C_L \neq C_L^s$ that satisfies $V_L(C_L) \geq V_L(C_L^s)$ yields negative expected profits when chosen by low ability workers only, $\pi_L(C_L) < 0$, and therefore any applicant to contrat C_L is rejected. So, applicants to C_L will be accepted if and only if C_L yields non-negative profits when chosen by only low-ability workers. This, implies that $V_L(C_L) < V_L(C_L^s)$, contradicting lemma 4. Therefore, the only possible contract that is chosen only by low-ability workers and applications are accepted in equilibrium is C_L^*. This proves that $V\left(C_L^s\right) = V_L(C_L^*)$. ■

Lemma 7 *In any fully separating UPBE, denoted by Λ, high-ability workers' equilibrium payoff is at least as large as $V_H(C_H^s)$.*

Proof. Suppose not, then there exists fully separating, Λ', such that $V_H(\Lambda') < V_H(C_H^s)$.

Let $C' = \{C_1', C_L^*, C_1\}$, where $V_H(C_1) \geq V_H(C_H^{As})$. By lemmas 4, 5 and 6, it must be the case that $V_H(C_1') > V_H(C_L^*)$ and $V_L(C_L^*) \geq V_L(C_1')$. Hence, $\sigma_H'(C_1') = 1$, $\sigma_L'(C_1') \in [0, 1)$, $\sigma_L'(C_L^*) \in [0, 1]$, $\widehat{\mu}'(L \mid C_L^*) = 1$ and $\widehat{\mu}'(H \mid C_1') \in (\mu, 1]$. This is an equilibrium if and only if $V_H(C_1) < V_H(C_H^s)$ and $V_L(C_L^*) \geq V_L(C_1)$, which requires that $\pi(\widehat{\mu}'(H \mid C_1), C_1) = 0$. Therefore, $\widehat{\mu}'(H \mid C_1) \in [0, \widehat{\mu}'^*(H \mid C_1)]$, where $\widehat{\mu}'^*(H \mid C_1)$ solves $\pi(\widehat{\mu}'(H \mid C_1), C_1) = 0$. Notice that $\widehat{\mu}'^*(H \mid C_1) < \mu$, otherwise some firms will offer C_1 and accept all the applicants to C_1 and at least break even. These firms break even since all high- and low-ability workers will apply to C_1 because $V_H(C_1) \geq V_H(C_H^s) > V_H(C_1')$ and any contract such that $V_H(C_1) \geq V_H(C_H^s)$ satisfies the following $V_L(C_1) \geq V_L(C_1')$.

Let $C = \{C_1, C_L^*, C_1'\}$ be the set of offers in the PBE Λ. By lemmas 4, 5 and 6, it must be the case that $V_H(C_1) > V_H(C_L^*)$, $V_H(C_1) \geq V_H(C_H^{As})$ and $V_L(C_L^*) \geq V_L(C_1)$. Hence,

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unique equilibria the best separating equilibrium and the pareto optimal pooling equilibrium, this restriction has no affect on the solution of the whole game.

$\sigma_H(C_1) = 1$, $\sigma_L(C_1) \in [0,1]$, $\sigma_L(C_L^*) \in (0,1]$, $\widehat{\mu}(L \mid C_L^*) = 1$ and $\widehat{\mu}(H \mid C_1) \in (\mu, 1]$.

Since in Λ', $\sigma_\theta(C_1) = 0$, $\forall i \in I$, and in Λ, $\sigma_H(C_1) = 1$, $K = \{H\}$ (if $\sigma_L(C_1) > 0$, then $K = \Theta$), which is non-empty, satisfying C1 of our refinement concept. Condition C2 is satisfied because $V_L(\Lambda) = V_L(\Lambda')$ and $V_H(\Lambda) > V_H(\Lambda')$. In the case in which $K = \{H\}$ condition C3 imposes that $\beta(H) = 1$, $\beta(L) = 0$, therefore $\mu(H, \beta(H)) = 1$ which is different from $\widehat{\mu}'(H \mid C_1)$. Therefore, Λ defeats any separating PBE, Λ', in which $V_H(\Lambda') < V_H(C_H^s)$. If $K = \Theta$ condition C3 imposes that $\beta(H) = 1$, $\beta(L) \in [0,1]$, therefore $\mu(H, \beta(H)) \in [\mu, 1]$ and $\mu(L, \beta(L)) \in [0, \mu]$, which are different from $\widehat{\mu}'(H \mid C_1)$ and $\widehat{\mu}'(L \mid C_1)$ because of $\widehat{\mu}'^*(H \mid C_1) < \mu$.

This proves that any fully separating UPBE that offers to high-ability workers an expected payoff lower than $V(C_H^{As})$ is defeated by a PBE that offers at least $V_H(C_H^{As})$ to high-ability workers and $V_L(C_L^*)$ to low-ability workers. ■

Lemma 8 *In any fully separating UPBE, denoted by Λ, high-ability workers' equilibrium payoff is equal to the expected payoff from contract C_H^{As}, $V_H(C_H^s)$.*

Proof. Suppose there exist a fully separating PBE, denoted by $\Lambda^{'}$, so that $V_H(\Lambda^{'}) > V_H(C_H^s)$. By lemmas 7 and 6, in any fully separating equilibrium, $\forall C_k \in C$ such that $\sigma_L(C_k) > 0$, $V_L(C_k) = V_L(C_L^*)$ and $\forall C_k \in C$ such that $\sigma_H(C_k) > 0$, $V_H(C_k) \geq V_H(C_H^s)$ and $\pi_H(C_k) \geq 0$. If a contract C_k such that $\sigma_H(C_k) > 0$, $V_H(C_k) > V_H(C_H^s)$ and $\pi_H(C_k) \geq 0$ exists, then C_H^{As} cannot be the contract that maximizes high-ability workers when only high-ability workers apply to this contract. This plus the fact that in any UPBE $V_H(\Lambda) \geq V_H(C_H^{As})$ implies that there is no fully separating UPBE where $V_H(\Lambda^{'}) \neq V_H(C_H^s)$. ■

Lemma 9 *The contract C^{Ap} is a PBE of the signalling sub-game.*

Proof. To prove that there is a PBE of the SSG that sustain C^p as a PBE, notice first that by definition C^{Ap} maximizes high-ability workers' expected payoff and breaks even only at the population average probability of success, therefore $\pi_L(C^p) < 0$. To prove that C^p can be supported as PBE consider the following strategies:

Stage 1: $C_i = \{C^p\}$ for $i \leqq k < N$ and $C_i = \{C'\}$ for $i > k$.

Stage 2: $\sigma_H(C^p) = 1$ and $\sigma_L(C^{Ap}) = 1$; that is, high- and low-ability workers apply to contract C^{Ap}.

Stage 3: $\rho_i(C^p) = 1$ and $\rho_i(C') = 1$, $\forall C' \in C$ such that $\pi_L(C') \geq 0$; that is, all the applicants to contract C^{Ap} are accepted and applicants to contract C' are also accepted.

On-the-equilibrium-path beliefs: $\widehat{\mu}(H \mid C^p) = \mu$.

Off-the-equilibrium-path beliefs: $\widehat{\mu}(H \mid C') = 0$, $\forall C' \neq C^p$ and $\pi_L(C') \geq 0$.

It is easy to check that these strategies satisfy the PBE requirements. ■

Lemma 10 *The contracts C_L^s and C_H^s are a PBE of the signalling sub-game.*

Proof. In order to sustain the solution to program II as a PBE consider the following strategies and beliefs.

Stage 1: $C_i = \{C_L^s\}$ for $i \leqq k < N$ and $C_i = \{C_H^s\}$ for $i > k$.

Stage 2: $\sigma_H(C_H^s) = 1$ and $\sigma_L(C_L^s) = 1$; that is, low-ability workers apply to C_L^s while high-ability workers apply to C_H^{As}.

Stage 3: $\rho_i(C_H^s) = 1$ and $\rho_i(C_L^s) = 1$; that is, all the applicants to contract C_H^s and C_L^s are accepted.

On-the-equilibrium-path beliefs: $\widehat{\mu}(H \mid C_H^s) = 1$ and $\widehat{\mu}(L \mid C_L^s) = 1$.

Off-the-equilibrium-path beliefs: $\widehat{\mu}(H \mid C') = 0$, $\forall C' \neq C_H^s$.

It is easy to check that these strategies satisfy the PBE requirements. ■

Proof of Proposition 1 There exists a threshold for the variance of output such that:

> (i) if $\sigma_\varepsilon^2 \leqq \sigma_\varepsilon^2(\mu)$, then the unique equilibrium contracts are the straight-salary contract C_L^s and the pay-for-performance contract C_H^s. Low-productivity workers choose the straight-salary contract C_L^s, while high-productivity workers choose the pay-for-performance contract C_H^s; while (ii) if $\sigma_\varepsilon^2 > \sigma_\varepsilon^2(\mu)$, the unique equilibrium contract is the pay-for-performance contract C^p and the two types of workers participate.

Proof.

Part 1: $\sigma_\varepsilon^2 \leq \sigma_\varepsilon^2(\mu)$.

Suppose there exits a PBE Λ that defeats Λ^s[17]; that is, there exists a contract $C_k \in C$ such that $\sigma_i^s(C_k) = 0$, $\forall i \in I$ and $\sigma_i(C_k) > 0$ for some $i \in I$ so that $C2$ and $C3$ are satisfied.

[17] Λ^sdenotes the PBE of SSG that sustain the contracts C_H^{As} and C_L^{As} as a PBE.

By lemmas 6 and 7 in any fully separating PBE, $V_H(\Lambda) = V_H(C_H^s)$ and $V_L(\Lambda) = V_L(C_L^{As})$. This implies that there is no fully separating PBE different from Λ^s that satisfies conditions $C2$ and $C3$. Therefore, there is no separating PBE that defeats Λ^s. By definition of $\sigma_\varepsilon^2(\mu)$, when $\sigma_\varepsilon^2 \leqq \sigma_\varepsilon^2(\mu)$, the highest payoff that high-ability workers can get in a pooling PBE is such that $V_H(C^p) < V_H(C_H^s)$. Since C^p will be a pooling equilibrium it is the case that $C_k \in C$ such that $\sigma_i^s(C^p) = 0$, $\forall i \in I$ and $\sigma_i(C^{Ap}) > 0$, therefore, $K = \Theta$ and condition $C2$ is immediately violated. Therefore, when $\sigma_\varepsilon^2 \leq \sigma_\varepsilon^2(\mu)$ there is no PBE that defeats Λ^s. Uniqueness follows from the fact if a semi-separating equilibrium exists only low-ability workers play mix strategies in which case, the only contract that breaks even when chosen only by high-ability workers is C_H^s. Furthermore, any contract chosen by low-ability workers must promise a payoff of $V_L(C_L^s)$ which is equal to $V_L(C_H^s)$.

Part 2: $\sigma_\varepsilon^2 > \sigma_\varepsilon^2(\mu)$

When $\sigma_\varepsilon^2 > \sigma_\varepsilon^2(\mu)$, by definition of $\sigma_\varepsilon^2(\mu)$ the highest payoff that high-ability workers can get in a pooling PBE is such that $V_H(C^p) > V_H(C_H^s)$ and $V_L(C^p) > V_L(C_L^s)$. Therefore, it is trivial to show that Λ^s is defeated by Λ^p[18]. The question is whether there is another pooling equilibrium besides Λ^p that is undefeated.

Suppose there exists a PBE denoted by $\Lambda^{p'}$ that defeats Λ^p. Recall that by definition, C^p is, among all possible contracts used in any PBE, the contract that yields the highest payoff to high-ability workers.

Let $C' = \{C^p, C^{p'}\}$ and $\sigma_i'(C^p) = 0$, $\forall i \in I$ and $C = \{C^p, C^{p'}\}$ and $\sigma_i(C^{p'}) = 0$, $\forall i \in I$. In this case $K = I \neq \phi$, then $C2$ fails since any pooling PBE $\Lambda^{p'}$ different from Λ^p is such that $V_H(\Lambda^{p'}) < V_H(\Lambda^p)$.

Finally we need to prove that $\sigma_\varepsilon^2(\mu)$ exists. Notice first that by plugging the optimal value of β^p in $\triangle V_L(\sigma_\varepsilon^2) \equiv V_L(C_L^s) - V_L(C^p)$ one gets that $\triangle V_L(\sigma_\varepsilon^2) = -\mu\triangle\theta\left(\frac{rm\sigma_\varepsilon^2-(1-\mu)\triangle\theta}{rm\sigma_\varepsilon^2}\right) + \frac{(1-\mu)^2\triangle\theta^2}{2rm\sigma_\varepsilon^2}$. Then, it is easy to verify that $\triangle V_L(\sigma_\varepsilon^2) \geq 0$ for all $\sigma_\varepsilon^2 \leq \frac{\left(1-\mu^2\right)\triangle\theta}{2\mu rm}$. Thus, a low-productivity worker's expected utility is larger under the pooling contract if and only if $\sigma_\varepsilon^2 > \frac{\left(1-\mu^2\right)\triangle\theta}{2\mu rm}$.

Next we need to find conditions under which $V_H(C_H^s) \geq V_H(C^p)$. Plugging the optimal value of β^p in $V_H(C^p)$ and β_H^s in $V_H(C_H^s)$, and after a few steps of simple algebra it is easy

[18] We denote by Λ^p the PBE that sustains C^p as a PBE.

to verify that $\triangle V_H\left(\sigma_\varepsilon^2\right) \equiv V_H(C_H^s) - V_H(C^p)$ is equal to

$$-2\triangle\theta mr\sigma_\varepsilon^2\mu - \triangle\theta^2\left(2+(1-\mu)^2\right) + 2\triangle\theta\left(\triangle\theta^2 + 2\triangle\theta mr\sigma_\varepsilon^2\right)^{\frac{1}{2}}.$$

Differentiating $\triangle V_H\left(\sigma_\varepsilon^2\right)$ with respect to σ_ε^2, one verifies that $\triangle V_H\left(\sigma_\varepsilon^2\right)$ is increasing in σ_ε^2 for all $\sigma_\varepsilon^2 \leq \frac{\left(1-\mu^2\right)\triangle\theta}{2rm\mu^2}$. In addition the second derivative of $\triangle V_H\left(\sigma_\varepsilon^2\right)$ is

$$-2\,(mr)^2\,\triangle\theta^3\left(\triangle\theta^2 + 2\triangle\theta mr\sigma_\varepsilon^2\right)^{-\frac{3}{2}} < 0$$

. Thus, $\triangle V_H\left(\sigma_\varepsilon^2\right)$ is a strictly concave function of σ_ε^2 for all $\sigma_\varepsilon^2 \geq 0$. Furthermore, $\triangle V_H\left(\sigma_\varepsilon^2 = 0\right) = -\triangle\theta^2\left(1-\mu\right)^2 < 0$ and $\lim_{\sigma_\varepsilon^2\to\infty}\triangle V_H\left(\sigma_\varepsilon^2\right) < 0$. In addition, $\triangle V_H\left(\sigma_\varepsilon^2 = \frac{\left(1-\mu^2\right)\triangle\theta}{2rm\mu^2}\right) = \triangle\theta^2\left[\frac{1+3\mu^2-3\mu-\mu^3}{\mu}\right] > 0$ for all $\mu \in [0,1)$. Thus, $\triangle V_H\left(\sigma_\varepsilon^2\right)$ has two real roots greater than 0, one smaller than $\frac{\left(1-\mu^2\right)\triangle\theta}{2rm\mu^2}$ and one larger than that value. Let us denote the smaller of the roots by $\sigma_{\varepsilon 0}^2$ and the larger of the two by $\sigma_{\varepsilon 1}^2$. Thus, $\triangle V_H\left(\sigma_\varepsilon^2\right) < 0$ for all $\sigma_\varepsilon^2 < \sigma_{\varepsilon 1}^2$, $\triangle V_H\left(\sigma_\varepsilon^2\right) \geq 0$ for $\sigma_{\varepsilon 1}^2 \geq \sigma_\varepsilon^2 > \sigma_{\varepsilon 0}^2$, and $\triangle V_H\left(\sigma_\varepsilon^2\right) < 0$ for all $\sigma_\varepsilon^2 > \sigma_{\varepsilon 1}^2$. This implies that a high-productivity workers gets a larger expected payoff under the pooling contract when the variance is either small or large. Finally, notice that

$$\triangle V_H\left(\sigma_\varepsilon^2 = \frac{\left(1-\mu^2\right)\triangle\theta}{2\mu rm}\right) = \triangle\theta^2\left[-4+2\mu+2\left(\frac{\mu+1-\mu^2}{\mu}\right)^{\frac{1}{2}}\right] > 0$$

for all $\mu \in [0,1)$. Because of this, $\triangle V_L\left(\sigma_{\varepsilon 0}^2\right) > 0$ and therefore, the pooling contract is preferred by both, low- and high-productivity workers as long as $\sigma_\varepsilon^2 < \sigma_\varepsilon^2\left(\mu\right) \equiv \sigma_{\varepsilon 1}^2$.

Notice that

$$\frac{\partial\sigma_\varepsilon^2\left(\mu\right)}{\partial\mu} = -\left(\frac{\partial\triangle V_H\left(\sigma_\varepsilon^2\left(\mu\right)\right)}{\partial\mu}\Big/\frac{\partial\triangle V_H\left(\sigma_\varepsilon^2\left(\mu\right)\right)}{\partial\sigma_\varepsilon^2}\right),$$

where $\frac{\partial\triangle V_H\left(\sigma_\varepsilon^2(\mu)\right)}{\partial\mu} = 2\triangle\theta\left(\left(1-\mu\right)\triangle\theta - mr\sigma_\varepsilon^2\right) < 0$ and $\frac{\partial\triangle V_H\left(\sigma_\varepsilon^2(\mu)\right)}{\partial\mu} < 0$ since $\sigma_\varepsilon^2\left(\mu\right) > \frac{\left(1-\mu^2\right)\triangle\theta}{2rm\mu^2}$. Thus, $\frac{\partial\sigma_\varepsilon^2(\mu)}{\partial\mu} < 0$.

Notice also that $\lim_{\mu\to 1}V_H(C^p) = V_H(C_H^*) > V_H(C_H^s)$, $\lim_{\mu\to 1}V_L(C^p) = V_L(C_H^*) > V_L(C_L^s)$, $\lim_{\mu\to 0}V_L(C^p) = V_L(C_L^*) = V_L(C_L^s)$ and $\lim_{\mu\to 0}V_H(C^p) = V_H(C_L^*) < V_H(C_H^s)$, where the last inequality follows from the incentive compatibility constraint. Thus, by continuity of $V_H(C^p)$ and $V_L(C^p)$ with respect to μ, there exists $\widetilde{\mu}$ such that for all $\mu > \widetilde{\mu}$, $V_L(C^p) > V_L(C_L^s)$ and $V_H(C^p) > V_H(C_H^s)$. $\blacksquare$.