

# Estimating Discount Functions with Consumption Choices over the Lifecycle

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August 11, 2005\*

## Abstract

Intertemporal preferences are difficult to measure. In this paper we attempt to estimate time preferences using a structural buffer stock consumption model and the Method of Simulated Moments. The model includes stochastic labor income, liquidity constraints, child and adult dependents, liquid and illiquid assets, revolving credit, retirement, and discount functions that allow short-run and long-run discount rates to differ. Field data on retirement wealth accumulation, credit card borrowing, and consumption-income comovement identify the model. Our benchmark estimates imply a 40% short-term annualized discount rate and a 4.3% long-term annualized discount rate. All specifications reject the restriction to a constant discount rate. Our quantitative results are sensitive to assumptions about the return on illiquid assets and the coefficient of relative risk aversion.

JEL classification: D91 (Intertemporal Consumer Choice; Lifecycle Models and Saving), E21 (Consumption; Saving)

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# 1 Introduction

Though intertemporal preferences play a critical role in most important economic decisions, economists have not identified a reliable method for measuring them (Frederick, Loewenstein and O'Donoghue 2002). The vast majority of research on time preferences has used laboratory studies in which the experimenter controls the choices that subjects face. Laboratory experiments often ask subjects to weigh immediate rewards against delayed rewards. A typical study asks subjects if they would prefer  $\$X$  now or  $\$Y$  at a specified future date.

Despite the advantages of controlled laboratory experimentation, such studies may confound time preferences with other considerations, like the trustworthiness of the experimenter or the outside investment options of the subject. It is not clear whether laboratory experiments measure the discount function, market interest rates, curvature of the utility function, some combination of these factors, or something else entirely.

Research using structural modelling and *field* data has its own strengths and weaknesses. Field data reflect choices from real-world markets and hence have greater external validity than abstract and unfamiliar laboratory decisions. Research with field data can also take advantage of existing large datasets on household behavior. However, field data are difficult to interpret since the researcher does not know exactly what tradeoffs households actually face in the marketplace. Structural modelling helps to pin down some of these tradeoffs but such modelling relies on a large set of explicit and implicit assumptions.

Given all of these considerations, laboratory and field research complement each other. Hence it is surprising that efforts to estimate discount rates have primarily used laboratory evidence.<sup>1</sup> This imbalance is particularly true of the recent research on *generalized* time preferences (i.e., discount functions that are not restricted to the class of exponential functions). Hundreds of studies beginning with Chung and Herrnstein (1967) and reviewed in Ainslie (1992) and Frederick et al. (2002) have estimated generalized discount functions with laboratory evidence while only a handful have attempted to do this with field data.<sup>2</sup>

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<sup>1</sup>Notable exceptions include Carroll and Samwick (1997), Gourinchas and Parker (2002), Hausman (1979), Lawrence (1991), Samwick (1998), Viscusi and Moore (1989), and Warner and Pleeter (2001).

<sup>2</sup>Prominent field studies include Attanasio and Weber (1995), Attanasio, Banks, Meghir and Weber (1999), Paserman (2004), Fang and Silverman (2004), and Shui and Ausubel (2004).

The current paper contributes to the literature estimating generalized discount functions using field data. We use lifecycle consumption data to estimate time preferences and to formally test the restriction to exponential discounting. Specifically, we use numerical methods to recursively solve and simulate a structural “buffer stock” model of lifecycle consumption and investment choices (Deaton 1991, Carroll 1992, Carroll 1997). Our model includes a rich array of financial instruments, constraints, demographic factors, and stochastic events – e.g., liquid and illiquid assets, revolving credit, liquidity constraints, household dependents, retirement, Social Security, and stochastic labor income – and thus controls for a number of relevant factors that affect intertemporal decisions.

We estimate the model’s time preference parameters using a two-stage Method of Simulated Moments (MSM) procedure (McFadden 1989, Pakes and Pollard 1989, Duffie and Singleton 1993), which was first used to study lifecycle consumption behavior by Gourinchas and Parker (2002).<sup>3</sup> The MSM procedure extends the Generalized Method of Moments (GMM) to account for numerical simulation error. In the first stage of the MSM procedure we estimate inputs to the life-cycle model, including the parameters of the stochastic labor income process, interest rates, credit card borrowing limits, and parameters that describe variation in household size over the lifecycle. In the second stage of the MSM procedure we use the simulation model to estimate time preference parameters. These preference parameters are identified by empirical patterns of wealth accumulation, credit card borrowing, and consumption-income comovement. Uncertainty in estimates of the first stage parameters propagates to the standard errors for the time preference parameters estimated in the second stage. Formal incorporation of the first stage is critical since it raises our second-stage standard errors by nearly an order of magnitude.

Our analysis has three goals. First, this paper uses field data to estimate time preference parameters for both the (restricted) exponential discount function and an (unrestricted) generalization that nests the exponential case. The unrestricted *quasi-hyperbolic* discount function allows the discount rate to differ in the short-run and the long-run. Second, we formally test the restricted and unrestricted models, using both t-tests and overidentification tests. Finally, we ask whether

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<sup>3</sup>Gourinchas and Parker (2002) and French (2005) use MSM to estimate different aspects of consumption models. Gourinchas and Parker identify the exponential discount function and the coefficient of relative risk aversion from lifecycle consumption profiles. French assesses how the opportunity to save and self-insure affects the impact of legislated Social Security and Medicare eligibility ages on the retirement decision. Most other applications of MSM have been in the industrial organization literature.

these models accurately predict important empirical regularities in the lifecycle literature.

When we estimate a restricted (exponential) discount function, the MSM procedure estimates a single annual discount rate of 15%. By contrast, when we estimate an unrestricted (quasi-hyperbolic) discount function, the MSM procedure estimates a short-run annualized discount rate of 40% and a long-run annualized discount rate of only 4%. All of these estimates are statistically significant at the 1% level. Our estimates also imply a formal rejection of the restricted case: i.e., we reject the hypothesis that the short-run discount rate is equal to the long-run discount rate.

Overidentification tests reinforce these conclusions. Only the exponential model is consistently rejected by overidentification tests. Intuitively, the exponential model cannot simultaneously explain high levels of credit card borrowing and high levels of retirement wealth accumulation. By contrast, the quasi-hyperbolic discount function implies that consumers will simultaneously act patiently and impatiently, because consumers have conflicting short-run and long-run discount rates.<sup>4</sup> In theory, low long-run discount rates explain why households accumulate substantial (illiquid) retirement wealth at real interest rates of about 5%, while high short-run discount rates imply that the *same* households borrow regularly on credit cards at real interest rates of 12%. By accumulating wealth in illiquid form, households commit themselves to act patiently in the future (i.e., not to spend down accumulated assets). However, when liquid assets and unused credit card balances are available, households spend when they can and therefore appear impatient.

We conclude the paper by reporting a wide range of robustness checks that reinforce some of our previous findings and identify the limits of our results. We find a robust gap between long and short term discount rates, but our parameter estimates are sensitive to assumptions about the return on illiquid assets and the coefficient of relative risk aversion.

The quasi-hyperbolic time preference parameter estimates in this paper are consistent with the results in other papers that have estimated these parameters with structural models and field data. Paserman (2004) obtains identification from heterogeneity in unemployment durations and reservation wages to find estimates of the short-run annualized discount rate that range from 11% to 91% and a long-run discount rate of only 0.1%. He rejects the exponential discounting null hypothesis for two of three subsamples. Fang and Silverman (2004) estimate models of

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<sup>4</sup>See Angeletos, Laibson, Repetto, Tobacman and Weinberg (2001) and Laibson, Repetto and Tobacman (2003). These predictions rely in part on consumers' access to illiquid assets for long-run saving.

both “sophisticated” and “naive” quasi-hyperbolic discounting. Naive hyperbolic decision-makers incorrectly believe that they will have exponential discount functions in the future.<sup>5</sup> Using data on welfare recipients, they find in the sophisticated case a short-run discount rate of 108% and a long-run discount rate of 13%, and they reject the null hypothesis of exponential discounting. Their results in the naive case are very similar. Finally, Shui and Ausubel (2004) use data from a direct mail credit card interest rate experiment to estimate the parameters of sophisticated and naive quasi-hyperbolic models. They obtain short-run discount rates of 24% in the sophisticated case and 20% in the naive case. In both cases they find a long-term discount rate of 0.01% and they reject the exponential discounting null hypothesis.

When we restrict time preferences to be exponential, we estimate a discount rate of 15%. By contrast, most authors who calibrate exponential discount functions with lifecycle consumption and wealth data have adopted discount rates that are around 5% (Engen, Gale and Scholz 1994, Hubbard, Skinner and Zeldes 1994, Laibson, Repetto and Tobacman 1998, Engen, Gale and Uccello 1999). Our results differ because we ask our model to simultaneously fit wealth accumulation data *and* credit card borrowing data.

The empirical data we use to estimate our model are presented in Section 2. Section 3 summarizes the structural model that we use. We explain the MSM procedure in Section 4. Section 5 presents our results. Section 6 discusses extensions and Section 7 concludes.

## **2 Wealth Accumulation, Credit Card Borrowing, and Consumption-Income Comovement Data**

We estimate exponential and quasi-hyperbolic discount functions by matching moments that characterize wealth accumulation, credit card borrowing, and excess sensitivity of consumption to predictable movements in income. We summarize these statistics in this section. Table 1 reports these moments, and Appendix A contains a detailed description of the data sources and estimation procedures. All of the analysis that we conduct applies to U.S. households whose head has a high

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<sup>5</sup>See Akerlof (1991) and O’Donoghue and Rabin (1999a, 1999b) for analysis of naive hyperbolic discounters. Hyperbolics who *are* aware that they will be hyperbolic in the future are called “sophisticates.” In most consumption models sophisticates and naifs behave similarly (Angeletos et al. 2001). We focus on the sophisticated case. See Section 6 for more discussion.

school degree but not a college degree. These households constitute 59% of the population (U.S. Census Bureau 1995).<sup>6</sup>

The first statistic, % *Visa*, is the fraction of households that borrow on credit cards.<sup>7</sup> Our analysis finds that 67.8% of households *pay interest* on credit card debt in any given month. Specifically, % *Visa* represents the fraction of households that self-report that they did not pay their bill in full at the end of the last month (SCF 1995 and 1998). Though there is considerable heterogeneity among households, credit card borrowing is ubiquitous across the entire distribution of wealth. Table 2 of Laibson et al. (2003) reports the fraction of households borrowing on credit cards by age and by wealth quartile.<sup>8</sup> Among households with a head between ages 20-29 that are in the top wealth quartile for their age group, three-fourths did not repay their credit card bills in full the last time they paid their bills. For households with a head in his or her thirties, over 80% of median wealth-holders had credit card debt. Even among the households with a head between ages 50-59 that are between the 50th and 75th wealth percentiles, 56% borrowed and paid interest on credit card debt in the past month. The typical American household accumulates wealth in the years leading up to retirement and simultaneously borrows using credit cards.

We construct the second statistic, *mean Visa*, by dividing age-specific credit card borrowing by mean age-specific income. We then average this fraction over the lifecycle. The average household has outstanding credit card debt equal to 11.7% of the mean income of its age cohort (SCF and Federal Reserve Board, 1995 and 1998).

The third statistic, *CY*, represents the excess sensitivity of consumption in response to predictable income changes.<sup>9</sup> We estimate that the marginal propensity to consume is 23% of the income change (PSID 1978-1992). This figure is consistent with other analyses in the literature.<sup>10</sup>

The final statistic, *wealth*, averages (weighted) wealth-to-income ratios for households with heads aged 50-59, *excluding* ‘involuntary’ wealth like Social Security and other defined benefit

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<sup>6</sup>Laibson et al. (1998, 2003) examine households in *all* education categories using a calibration framework instead of an estimation framework. They find qualitatively similar results across education categories.

<sup>7</sup>This is the fraction that borrows on any type of card, not just Visa cards.

<sup>8</sup>Note however that the lifecycle patterns in this table are difficult to interpret because they reflect both cohort effects and age effects.

<sup>9</sup>See for example Carroll and Summers (1991), Shea (1995), and Parker (1999).

<sup>10</sup>Most previous work on excess sensitivity has found coefficients between 0 and 0.5. See Deaton (1992) and Browning and Lusardi (1996) for reviews.

pensions. To downweight outliers we apply a scaled arctan function to each ratio before averaging.<sup>11</sup> This weighted *wealth* measure equals 2.60 (SCF 1983-1998). For comparison, the median wealth-to-income ratio for the same sub-sample is 2.16.

### 3 Consumption-Savings Model

Our work extends the numerical simulation literature pioneered by Carroll (1992, 1997), Deaton (1991), Zeldes (1989), Gourinchas and Parker (2002), Hubbard, Skinner and Zeldes (1994, 1995). Our specific analysis is mostly closely related to Gourinchas and Parker (2002) — we adopt their MSM procedure — and Laibson, Repetto and Tobacman (2003) — we adopt their calibrated structural model. In the next section we review the Gourinchas and Parker MSM procedure. In the current section we review the Laibson, Repetto, and Tobacman model.

The calibration parameters for this structural model can be found in Table 2. In the model, economic decision making begins at age 20. Households have an age-dependent survival hazard of  $s_t$  calibrated with data from the U.S. National Center for Health Statistics (1994). Household composition varies deterministically with age as children and adult dependents enter and leave the household.<sup>12</sup> Effective household size  $n_t$  equals the number of adults plus 0.4 times the number of children under 18.<sup>13</sup>

Let  $Y_t$  represent period  $t$  aftertax income from transfers and wages, including labor income, inheritances, private defined benefit pensions, and government transfers including Social Security. During working life  $y_t = \ln(Y_t)$  is modelled as the sum of a cubic polynomial in age, an AR(1), and an iid shock. We approximate the AR(1) with a Markov process, and denote the Markov state  $\zeta$ . During retirement,  $y_t$  is the sum of a linear polynomial in age and an iid shock. Retirement occurs exogenously at age  $T$ . The income process and the retirement age are calibrated from the PSID.

Let  $X_t$  represent liquid asset holdings at the beginning of period  $t$  before receipt of  $Y_t$ . If  $X_t < 0$  then uncollateralized debt — i.e. credit card debt — was held between  $t - 1$  and  $t$ . Households

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<sup>11</sup>The smoothness of arctan (in contrast to, say, the median) implies differentiability of the theoretical moment conditions. Our results are robust to different choices of this transformation. See Appendix A for details on the scaling.

<sup>12</sup>The demographic profiles are estimated parametrically using the PSID.

<sup>13</sup>See Blundell, Browning and Meghir (1994) and Attanasio and Browning (1995).

face a credit limit at age  $t$  of  $\lambda$  times average income at age  $t$ , i.e.,  $X_t \geq -\lambda \bar{Y}_t$ .<sup>14</sup> The model precludes consumers from simultaneously holding liquid assets and credit card debt, though such potentially suboptimal behavior has been documented among a subpopulation of consumers by Gross and Souleles (2002a) and Bertaut and Haliassos (2001).

Positive liquid asset holdings earn a risk-free real aftertax gross interest rate of  $R$ , the average of Moody’s AAA municipal bond yields from 1980-2000 (Gourinchas and Parker 2002). Households pay a gross real interest rate on credit-card borrowing of  $R^{CC}$ . Using data from the Fed, the American Bankruptcy Institute, and the CPI, our estimate of  $R^{CC}$  captures the impact of bankruptcy, default, and inflation, which lower consumers’ effective interest payments.<sup>15</sup>

Let  $Z_t$  represent (net) illiquid asset holdings at the beginning of period  $t$ , with  $Z_t \geq 0, \forall t$ . Illiquid assets include durables, which generate two types of returns: capital gains and consumption flows. For computational tractability, capital gains equal zero (i.e.,  $R^Z = 1$ ) and the annual consumption flow is  $\gamma Z_t = 0.05 \cdot Z_t$ . Hence, the return from holding the illiquid asset is a 5% annual flow of consumption. We also adopt the assumption that  $Z$  can only rise during the owner’s lifetime; i.e., transaction costs are large enough that the  $Z$  asset is never sold until wealth is bequeathed to the next generation. These choices about  $Z$  do not match the properties of a particular illiquid asset though  $Z$  has some of the features of home equity.<sup>16</sup>

Three observations motivate these assumptions about  $Z$ . First, despite increasing financial sophistication many household assets continue to be partially illiquid and were certainly illiquid during our sample period (1978 to 1998). Accessing equity in homes, cars and retirement plans like 401(k)s entails at least small transactions costs and delays. Second, theory (Laibson 1997) and

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<sup>14</sup>The limit is calibrated from the SCF. This is a crude representation of the income-based credit limits that are common in the revolving credit market. Assets are not an important determinant of credit card borrowing limits because large asset classes like retirement accounts (and in some states home equity) can not be seized after a credit card default.

<sup>15</sup>We omitted two considerations that would lower the *effective* credit card rate. First, without declaring bankruptcy, households might be able to default on their credit card debt. Second, consumption may be unusually low in the bankruptcy/default state, which reduces the cost of borrowing since repayment only occurs in “good states.” On the other hand, we also omitted two considerations that raise the effective credit card rate. The model does not account for the stigma associated with bankruptcy (Gross and Souleles 2002b) or for the cost of future exclusion from credit markets. Robustness checks — using a wide range of credit card interest rates — are provided in section 5.3.

<sup>16</sup>Consider a consumer who owns a house of fixed real value  $H$  and derives annual consumption flows from the house of  $\gamma H$ . Suppose the consumer has a mortgage of size  $M$  and home equity of  $H - M$ . The real cost of the mortgage is  $\eta M$ , where  $\eta = i \cdot (1 - \tau) - \pi$  is the nominal mortgage interest rate adjusted for inflation and the tax deductibility of interest payments. If we assume  $\eta \approx \gamma$ , the net benefit to the homeowner is  $\gamma H - \eta M \approx \gamma(H - M) = \gamma Z$ . Section 6 discusses enriching the modelling of  $Z$ .



simulations (Laibson et al. 2003) have shown that for the quasi-hyperbolic case, small transactions costs have the same impact on consumers as complete illiquidity. Third, illiquidity of  $Z$  mimics the optimal savings mechanisms that have recently been derived for quasi-hyperbolic consumers (Amador, Werning and Angeletos 2004). Finally, assuming that  $Z$  is illiquid increases computational tractability by limiting the choice set of the consumer. In Subsection 5.3 we evaluate the robustness of our set-up by making  $Z$  more attractive, and we highlight the potential drawbacks of assuming that  $Z$  is illiquid when we discuss extensions of the model in Subsection 6.5.

Let  $I_t^X$  represent net investment into the liquid asset  $X$  during period  $t$ , and let  $I_t^Z$  represent net investment into the illiquid asset  $Z$  during period  $t$ . Then the dynamic budget constraints are given by,

$$X_{t+1} = R^X (X_t + I_t^X) \quad (1)$$

$$Z_{t+1} = R^Z (Z_t + I_t^Z). \quad (2)$$

Since the interest rate on liquid wealth  $R^X$  depends on whether the consumer is borrowing or saving in her liquid accounts,

$$R^X = \begin{cases} R^{CC} & \text{if } X_t + I_t^X < 0 \\ R & \text{if } X_t + I_t^X \geq 0 \end{cases}$$

Denote  $r^{CC} = R^{CC} - 1$ . The static budget constraint is:

$$C_t = Y_t - I_t^X - I_t^Z$$

The state variables  $\Lambda_t$  at the beginning of period  $t$  are liquid wealth ( $X_t + Y_t$ ), illiquid wealth ( $Z_t$ ), and the value of the Markov process ( $\zeta_t$ ). The non-redundant choice variables are  $I_t^X$  and  $I_t^Z$ . Consumption is calculated as a residual.

The consumer has constant relative risk aversion and a quasi-hyperbolic discount function. For  $t \in \{20, 21, \dots, 90\}$ , self  $t$  has instantaneous payoff function

$$u(C_t, Z_t, n_t) = n_t \cdot \frac{\left(\frac{C_t + \gamma Z_t}{n_t}\right)^{1-\rho} - 1}{1-\rho}$$

and continuation payoffs given by

$$\beta \sum_{i=1}^{90-t} \delta^i \left( \prod_{j=1}^{i-1} s_{t+j} \right) [s_{t+i} \cdot u(C_{t+i}, Z_{t+i}, n_{t+i}) + (1 - s_{t+i}) \cdot B(X_{t+i}, Z_{t+i})].$$

Here  $\rho$  is the coefficient of relative risk aversion, and  $B(\cdot)$  represents the payoff in the death state, which incorporates a bequest motive.<sup>17</sup> The first expression in the bracketed term is the utility flow that arises in period  $t + i$  if the household survives to age  $t + i$ . The second expression is the termination payoffs in period  $t + i$  which arises if the household dies between period  $t + i - 1$  and  $t + i$ . The quasi-hyperbolic discount function  $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, \dots\}$  corresponds to a short-run discount rate of  $-\ln(\beta\delta)$  and a long-run discount rate of  $-\ln(\delta)$ .<sup>18</sup> When  $\beta = 1$  the consumer has time consistent preferences — i.e., exponential discounting.

Following Strotz (1955) we model behavior as an intra-personal game among selves  $\{20, 21, \dots, 90\}$ . Taking the strategies of other selves as given, self  $t$  picks a strategy at time  $t$  that is optimal from the perspective of time  $t$ . This strategy is a map from the Markov state  $\Lambda_t = \{t, X + Y, Z, \zeta\}$ , to the choice variables  $\{I^X, I^Z\}$ . An equilibrium is a fixed point in the strategy space, such that all strategies are optimal given the strategies of other players. We solve for equilibrium strategies using numerical backwards induction.

Let  $V_{t,t+1}(\Lambda_{t+1})$  represent the time  $t + 1$  continuation payoff function of self  $t$ . Then self  $t$ 's objective function is

$$u(C_t, Z_t, n_t) + \beta\delta E_t V_{t,t+1}(\Lambda_{t+1}). \quad (3)$$

Self  $t$  chooses  $\{I^X, I^Z\}$  in state  $\Lambda_t$  to maximize this expression. The sequence of continuation payoff functions is defined recursively

$$V_{t-1,t}(\Lambda_t) = s_t[u(C_t, Z_t, n_t) + \delta E_t V_{t,t+1}(\Lambda_{t+1})] + (1 - s_t)E_t B(\Lambda_t). \quad (4)$$

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<sup>17</sup>Liquidated bequeathed wealth is consumed by heirs as an annuity (Laibson et al. 2003). Specifically, if  $\bar{n}$  is average effective household size over the life-cycle,  $\bar{y}$  is average labor income over the life-cycle, and  $u_1(\bar{y}, 0, \bar{n})$  is the partial derivative of instantaneous utility  $u(C, Z, n)$  with respect to consumption, we assume,  $B(X, Z) = (R - 1) \cdot \max\{0, X + \frac{2}{3}Z\} \cdot \frac{u_1(\bar{y}, 0, \bar{n})}{1 - \delta}$ . Often liquidating bequeathed wealth entails large transactions costs, so we multiply bequeathed illiquid wealth by two-thirds. The rest of the expression follows because the total consumption of the bequest recipient approximately equals  $\bar{y}$ , and on average the effective household size of the bequest recipient equals  $\bar{n}$ .

<sup>18</sup>See Phelps and Pollak (1968) and Laibson (1997).

The induction continues in this way.

We simulate lifecycle choices for  $J_s = 5000$  individual households. We generate  $J_s$  independent streams of income realizations. Households make equilibrium decisions conditional on their state variables. From the simulated profiles of  $C$ ,  $X$ ,  $Z$ , and  $Y$ , we calculate the moments used in the MSM estimation procedure. Note that the simulated profiles, and hence the summary moments, depend on the parameters of the model. Since the model cannot be solved analytically, its quantitative predictions are derived from the simulated lifecycle profiles. Variability arising from (finite sample) simulation error is addressed in the estimation procedure.

## 4 Method of Simulated Moments Procedure

We estimate the parameters of the model’s discount function in the second stage of a Method of Simulated Moments procedure, closely following the methodology of Gourinchas and Parker (2002). MSM allows us to evaluate the predictions of our model, to formally test the nested null hypothesis of exponential discounting,  $\beta = 1$ , and to perform specification tests. We use MSM rather than GMM because the model cannot be solved analytically and because MSM provides a way of accounting for additional uncertainty from simulation error.<sup>19</sup> The current section describes our procedure. Appendix B presents derivations and some technical details.

The MSM procedure has two stages. In the first stage, nuisance parameters,  $\hat{\chi}$ , are estimated using standard GMM techniques (see Table 2). We take these  $N_\chi = 28$  nuisance parameters and their associated variances,  $\Omega_\chi$ , from Laibson et al. (2003).<sup>20</sup> Some authors describe this first-stage as the “calibration” stage. These first-stage estimates match those of numerous other researchers.<sup>21</sup>

Given  $\hat{\chi}$  and  $\Omega_\chi$ , the second stage uses additional data and more of the model’s structure

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<sup>19</sup>See McFadden (1989), Pakes and Pollard (1989), and Duffie and Singleton (1993) for the first formulations of MSM, and Stern (1997) for a review of simulation-based estimation techniques.

<sup>20</sup>Included in  $\chi$  are seven pre-retirement income level coefficients, three pre-retirement income variability coefficients, the retirement age, five post-retirement income coefficients, one post-retirement income variability coefficient, six effective household size coefficients, the credit limit, the coefficient of relative risk aversion, and three interest rates.

<sup>21</sup>Hubbard et al. (1994) report an almost identical process for after-tax non-asset income. Attanasio and Weber (1995) report similar family size profiles over the life cycle. Bernheim, Skinner and Weinberg (2001) provide similar estimates of the retirement age and income replacement rates. Ausubel (1991) reports credit card interest rates that are similar to our estimates before we correct for personal bankruptcy.

to estimate  $N_\theta$  additional parameters  $\theta$ .<sup>22</sup> The second stage, taking the first stage parameters fixed at  $\hat{\chi}$ , chooses  $\theta$  to minimize the distance between the empirical and the simulated moments. Specifically, we use the data from Section 2 on wealth accumulation, credit card borrowing, and excess sensitivity to estimate  $\theta = (\beta, \delta)$  in the second stage. MSM differs from a calibration exercise followed by a one-stage estimation in that it propagates uncertainty in the first stage parameters into the standard errors of the second stage parameter estimates. In other words  $\Omega_\theta$ , the variance matrix of  $\hat{\theta}$ , depends on  $\Omega_\chi$ . For three of the model's parameters that are not pinned down precisely by available data,  $r^{CC}$ ,  $\gamma$ , and  $\rho$ , we perform additional robustness checks in Subsection 5.3.

Denote the empirical vector of  $N_m$  second stage aggregate moments by  $\bar{m}_{J_m}$ . Let  $J_m$  be the numbers of empirical observations used to calculate the elements of  $\bar{m}_{J_m}$ .<sup>23</sup> Denote the theoretical population analogue to  $\bar{m}_{J_m}$  by  $m(\theta, \chi)$  and let  $m_{J_s}(\theta, \chi)$  be the simulation approximation to  $m(\theta, \chi)$ . Let  $g(\theta, \chi) \equiv [m(\theta, \chi) - \bar{m}_{J_m}]$  and  $g_{J_s}(\theta, \chi) \equiv [m_{J_s}(\theta, \chi) - \bar{m}_{J_m}]$ . The moment conditions imply that in expectation

$$Eg(\theta_0, \chi_0) = E[m(\theta_0, \chi_0) - \bar{m}_{J_m}] = 0,$$

where  $(\theta_0, \chi_0)$  is the true parameter vector. Define derivatives of the moment functions with respect to the parameters by  $G_\theta \equiv \frac{\partial g(\theta_0, \chi_0)}{\partial \theta}$  and  $G_\chi \equiv \frac{\partial g(\theta_0, \chi_0)}{\partial \chi}$ . Let  $\Sigma_g$  be the variance-covariance matrix of the second stage moments in the population. Let  $\Omega_g \equiv E[g(\theta_0, \chi_0)g(\theta_0, \chi_0)']$  be the variance of the second stage moment estimates  $\bar{m}_{J_m}$ , which is calculated directly and consistently from sample data.<sup>24</sup>

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<sup>22</sup>In principle  $\theta$  and  $\chi$  could be estimated simultaneously. We separate the task for three reasons (Gourinchas and Parker 2002). First, lacking the relevant panel data variables, we use population aggregates to identify  $\theta$ . By contrast, in the first stage, we are able to use longitudinal household data on income and family characteristics to identify  $\chi$ . Second, most of the data we use to identify  $\theta$  and  $\chi$  come from separate datasets and are therefore uncorrelated. Exceptions are the credit limit and *CY*. Covariances between the second stage moments and the credit limit are approximately zero and *CY*'s large standard error means that it has little weight in our second-stage estimation anyway. (In particular, our results don't qualitatively change when we omit *CY* from the list of moments.) Third, with current technology it is computationally infeasible to dramatically increase the number of parameters estimated in our MSM procedure.

<sup>23</sup>Though the main text does not discuss it, the procedure accounts for the fact that  $J_m$  differs for different moments. Appendix B contains details.

<sup>24</sup>If the same number of empirical observations  $\bar{J}_m$  were available to calculate all of the second stage moments, then we would have  $\Omega_g = \Sigma_g / \bar{J}_m$ .

Let  $W$  be a positive definite  $N_m \times N_m$  weighting matrix. Define

$$q(\theta, \chi) \equiv g_{J_s}(\theta, \chi) \cdot W^{-1} \cdot g_{J_s}(\theta, \chi)' \quad (5)$$

as a scalar-valued loss function, equal to the weighted sum of squared deviations of simulated moments from their corresponding empirical values. Then our procedure is to fix  $\chi$  at the value of its consistent first-stage estimator, minimize the loss function  $q(\theta, \hat{\chi})$  with respect to  $\theta$ , and define the estimator as<sup>25</sup>

$$\hat{\theta} = \arg \min_{\theta} q(\theta, \hat{\chi}). \quad (6)$$

Pakes and Pollard (1989) demonstrate that under regularity conditions satisfied here  $\hat{\theta}$  is a consistent estimator of  $\theta_0$ , and  $\hat{\theta}$  is asymptotically normally distributed. As shown in Appendix B,

$$\Omega_{\theta} = \text{Var}(\hat{\theta}) = (G'_{\theta} W G_{\theta})^{-1} G'_{\theta} W [\Omega_g + \Omega_g^s + G_{\chi} \Omega_{\chi} G'_{\chi}] W G_{\theta} (G'_{\theta} W G_{\theta})^{-1}, \quad (7)$$

where  $\Omega_g^s = \frac{J_m}{J_s} \Omega_g$  is the simulation correction.

This equation is used to calculate standard errors for our estimates of  $\theta$ . All derivatives are replaced with consistent numerical analogues, which we calculate using the model and simulation procedure.<sup>26</sup> We estimate  $\Omega_g$  and  $\Omega_{\chi}$  consistently from sample data. After obtaining estimates using the weighting matrix  $W = \Omega_g^{-1}$ , we can construct the optimal weighting matrix  $W_{opt} = [\Omega_g + \Omega_g^s + G_{\chi} \Omega_{\chi} G'_{\chi}]^{-1}$ . Many authors (Altonji and Segal 1996, West, Wong and Anatolyev 2004, for example) have found optimally-weighted GMM procedures lead to biased estimates in small samples, so our baseline estimates use the simple weighting matrix  $W = \Omega_g^{-1}$ . In robustness checks we find that our qualitative conclusions are not affected by adoption of either of these weighting matrices.

To interpret the expression for  $\Omega_{\theta}$ , first consider the simulation correction  $\Omega_g^s$ . As the size of the simulated population  $J_s$  relative to the size of the sample  $J_m$  goes to infinity, the simulation correction approaches zero. Intuitively, as the simulation becomes an ideal approximation for the

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<sup>25</sup>We perform this minimization with Matlab's Nelder-Mead simplex algorithm. This algorithm is slower but more robust than derivative-based methods, and here it is preferred because of the nonconvexities in quasi-hyperbolic policy functions.

<sup>26</sup>We take numerical derivatives on both sides of the optimum and accept the derivative that has the most conservative implications for  $\Omega_{\theta}$ .

true population, the simulation correction disappears. Next examine the first stage correction  $G_\chi \Omega_\chi G'_\chi$ . This correction increases with the uncertainty  $\Omega_\chi$  in our estimates of the first-stage parameters; note that  $\Omega_\chi$  itself is increasing in the underlying population variance of  $\chi$  and decreasing in the number of observations we use to estimate  $\hat{\chi}$ . The first-stage correction also increases with the sensitivity of the second-stage moments to changes in the first-stage parameters,  $G_\chi$ .

When neither the simulation correction nor the first stage correction matter, we obtain,

$$\Omega_\theta = (G'_\theta W G_\theta)^{-1} G'_\theta W \Omega_g W G_\theta (G'_\theta W G_\theta)^{-1}.$$

In the benchmark case where we assume  $W = \Omega_g^{-1}$ , this becomes the standard GMM variance formula:  $\Omega_\theta = (G'_\theta \Omega_g^{-1} G_\theta)^{-1}$ .

MSM also allows us to perform specification tests. If the model is correct,

$$\begin{aligned} \xi(\hat{\theta}, \hat{\chi}) &\equiv g_{J_s}(\hat{\theta}, \hat{\chi}) \cdot W_{opt} \cdot g'_{J_s}(\hat{\theta}, \hat{\chi}) \\ &= g_{J_s}(\hat{\theta}, \hat{\chi}) \cdot [\Omega_g + \Omega_g^s + G_\chi \Omega_\chi G'_\chi]^{-1} \cdot g'_{J_s}(\hat{\theta}, \hat{\chi}) \end{aligned}$$

will have a chi-squared distribution with  $N_m - N_\theta$  degrees of freedom. This test statistic equals  $q(\hat{\theta}, \hat{\chi})$  in the optimal-weighting case.

## 5 Results

In this section we discuss the paper's three sets of findings. We report estimates for the discount factors  $\beta$  and  $\delta$ , including the special case in which we impose  $\beta = 1$  (leaving  $\delta$  as the only free parameter). Second, we evaluate the statistical fit of the estimated models, using both t-tests and overidentification tests. Finally, we ask whether these models accurately predict key empirical regularities in the lifecycle literature.

The coefficient of relative risk aversion  $\rho$ , the return on illiquid assets  $\gamma$ , and the credit card interest rate  $r^{CC}$  affect the quantitative results and these parameters are difficult to pin down empirically. As benchmarks we adopt the assumptions  $\gamma = 5\%$ ,  $r^{CC} = 11.52\%$ , and  $\rho = 2$ , but we also examine the robustness of our findings to changes in these parameters in subsection 5.3. Sensitivity to the model's other parameters is already accounted for by the first-stage correction in

the MSM procedure. Unless otherwise specified we study estimates based on the robust weighting matrix  $W = \Omega_g^{-1}$ , but we include some representative results using the efficient weighting matrix.

## 5.1 Identification

Identification of  $\beta$  and  $\delta$  depends on the way the simulated moments  $m_{J_s}(\theta, \hat{\chi})$  vary as functions of  $\beta$  and  $\delta$ . The pre-retirement weighted wealth-to-income ratio, *wealth*, increases in both  $\beta$  and  $\delta$ . All of the other moments decrease in  $\beta$  and  $\delta$ . Hence,  $\beta$  and  $\delta$  are substitutes. Nevertheless, the model is identified since  $\beta$  and  $\delta$  are not *perfect* substitutes.

Identification of  $\beta$  and  $\delta$  can be visualized in Figure 1, which plots  $q(\theta, \hat{\chi})$ . Recall that  $q$  is a weighted sum of squared deviations of the simulated moments from their empirical analogs. Smaller values of  $q$  reflect a closer fit between the simulated model and the data. In Figure 1,  $q$  resembles an upward-opening paraboloid. Intermediate values of  $\beta$  and  $\delta$  — neither zero nor one — minimize the distance between the simulated moments and the empirical moments.

Figure 1 exhibits an extended valley in the plot of  $q$ , traversing from high  $\delta$  and low  $\beta$  to low  $\delta$  and high  $\beta$ . The orientation of this valley implies that  $\beta$  and  $\delta$  are partial substitutes; when  $\delta$  is high, low values of  $\beta$  best match the empirical facts, and vice-versa. If the valley had a flat bottom,  $\beta$  and  $\delta$  would be perfect substitutes and the model would not be identified.

The lowest point in the valley is  $(\beta, \delta) = (0.703, 0.958)$ , and the paraboloid rises steeply as  $\beta$  rises. As  $\beta$  approaches 1 the model does very poorly; near  $\beta = 1$ , matching credit card data requires a low value of  $\delta$  (e.g.,  $\delta \approx 0.85$ ), but with  $\delta$  this low, wealth accumulation vanishes. Figure 2 displays a higher-resolution plot, which highlights the fact that the model cannot match the data when  $\beta$  is close to 1.

The figures reflect the key intuition that *low* long-term discount rates are necessary to match observed levels of retirement wealth. A household will only accumulate illiquid wealth that has a return of about 5% if the household's long-term discount rate is not much greater than 5%. If the long-term discount rate is pinned down in this way, it is necessary to have a  $\beta$  value below one to match the data on credit card borrowing.

## 5.2 Benchmark Estimates

We report our benchmark estimates in Table 3. In the unconstrained case (Column 1) the MSM procedure yields an estimate of  $\hat{\beta} = 0.703$ , with a standard error (s.e.(i) in the Table) of 0.109. For this specification  $\hat{\beta}$  lies significantly below 1; the  $t$ -stat for the  $\beta = 1$  hypothesis test is  $t = \frac{1-0.703}{0.109} = 2.72$ . The MSM procedure yields an estimate of  $\hat{\delta} = 0.958$ , with a standard error of 0.007. The estimated values of  $\beta$  and  $\delta$  imply a short-run discount rate of  $-\ln(0.703 * 0.958) = 39.5\%$  and a long-run discount rate of  $-\ln(0.958) = 4.3\%$ .

At the estimated parameter values, the quasi-hyperbolic model generates the moment predictions reported in Column 1 of Table 3. We can compare these simulated moments with the sample moments  $\bar{m}_{J_m}$ , which are reproduced in Column 5. Qualitatively, the model predicts both active borrowing on credit cards and accumulation of midlife wealth. Quantitatively, the model predicts a fraction borrowing three standard errors from the sample value, a level of borrowing that differs by five standard errors, and a consumption-income comovement coefficient and measure of wealth accumulation that are both off by about one standard error. However, such comparisons are not meaningful, since the first-stage parameters of the model are estimated with uncertainty. Once this uncertainty is formally taken into account — which is implicit in the MSM procedure — the model is consistent with the data. The (inverse) goodness-of-fit measure  $\xi(\hat{\theta}, \hat{\chi}) = 3.01$  compares favorably to the 5% critical value of 5.99 for a chi-squared distribution with two degrees of freedom. For the benchmark case, we fail to reject the overidentification test.

We also estimate  $\delta$  alone, imposing the restriction  $\beta = 1$ . This exponential discounting case yields the results in Column 2. We find  $\hat{\delta} = 0.846$  — implying a discount rate of 16.7% — and a standard error of 0.025. At these point estimates, the empirical facts about credit card borrowing and excess sensitivity are matched quite well. However, with such a high discount rate the model cannot account for observed wealth data. Instead, it predicts  $wealth = -0.05$ ;  $wealth$  loses in the tug of war between fitting  $wealth$ , which requires a low discount rate, and fitting the credit card variables % *Visa* and *mean Visa*, which requires a high discount rate.<sup>27</sup> The best fit

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<sup>27</sup>The empirical value of  $wealth$  is 2.6, twenty standard errors from its simulated value of  $wealth(\hat{\theta}, \hat{\chi}) = -0.05$ . However, matching the empirical value of  $wealth$  would require % *Visa* to approach 0, forty standard errors from its empirical value. If the returns to illiquid wealth (i.e.,  $\gamma$ ) were high enough, an exponential model could more successfully match the facts simultaneously. The results from Case B in Subsection 5.3.2 provide suggestive evidence.



available under an exponential model predicts that typical households have negative total assets in their peak accumulation years. Goodness of fit naturally suffers:  $\xi(\beta = 1, \delta = 0.846, \hat{\chi}) = 217 \gg 3.01 = \xi(\beta = 0.703, \delta = 0.958, \hat{\chi})$ . With the exponential restriction we estimate only one parameter, but  $\xi(\beta = 1, \delta = 0.846, \hat{\chi})$  compares unfavorably to even the 1% critical value of 11.34 for a chi-squared distribution with three degrees of freedom. Recall that above we compared  $\xi(\beta = 0.703, \delta = 0.958, \hat{\chi})$  to a chi-squared distribution with only two degrees of freedom. This difference accounts for the degree of improvement in goodness-of-fit possible merely by adding a free parameter. The  $p$ -value represents the probability that the benchmark model could have generated the observed data, so the overidentification test rejects the exponential model at the 1% level. The  $p$ -value for the unconstrained model exceeds the  $p$ -value for the constrained model by many orders of magnitude.

The standard errors reported as “s.e.(i)” in Table 3 incorporate corrections for the first stage estimation and for the simulation error. For comparison, we also report standard errors without these corrections: s.e.(ii) only includes the first stage correction, s.e.(iii) only includes the simulation correction, and s.e.(iv) includes neither.

Comparing s.e. (i) and (ii) reveals that if the simulation were infinitely large, so that the simulation exactly captured the properties of the theoretical population, the standard error on  $\beta$  would fall only from 0.1093 to 0.1090. Evidently the simulation correction matters little.

However, the standard errors are dramatically affected by the first stage correction. Comparing s.e (i) and (iii), if the first stage parameters were known with certainty the standard error on  $\beta$  would shrink from 0.1093 to 0.0170. In other words, consistent estimates of the variance-covariance matrix  $\Omega_\theta$  depend strongly on including the first stage correction. If we had not incorporated the first stage, our standard errors would have been biased down by a factor of six.

Using the optimal weighting matrix largely preserves the pattern of the benchmark results. Our optimal-weights findings are reported in Columns 3 and 4. The quasi-hyperbolic results with the optimal weighting matrix are similar to those with  $W = \Omega_g^{-1}$ . The estimated  $\hat{\beta}$  and  $\hat{\delta}$  are slightly higher, the standard error on  $\hat{\beta}$  is lower, and the standard error on  $\hat{\delta}$  is higher. In the exponential case,  $\hat{\delta}$  is found to be substantially larger than in the benchmark; now  $\hat{\delta}$  is selected by the estimation procedure to match the data on wealth at the expense of matching borrowing facts.

Uncertainty in all of the first stage parameters except  $\gamma$ ,  $r^{CC}$ , and  $\rho$  has been incorporated into

the standard errors reported above.<sup>28</sup> However,  $\gamma$ ,  $r^{CC}$ , and  $\rho$  are difficult to pin down empirically so in the next subsection we explore the robustness of our findings to changes in those parameters.

## 5.3 Robustness

### 5.3.1 Parameter Perturbations

We perturb the return on the illiquid asset ( $\gamma$ ), the credit card interest rate ( $r^{CC}$ ), and the coefficient of relative risk aversion ( $\rho$ ), one-by-one from their benchmark values (i.e.,  $\gamma = 5\%$ ,  $r^{CC} = 11.52\%$ , and  $\rho = 2$ ). We report the resulting estimates of  $\beta$  and  $\delta$  in Table 4.

In Column 1 we reproduce the benchmark results as a reference. In Column 2 we set  $\gamma = 3.38\%$ , corresponding to the average tax- and inflation-adjusted mortgage interest rate from 1980-2000, as calculated from Freddie Mac's historical series of nominal mortgage interest rates and the CPI-U, assuming a marginal tax rate of 25%. Intuitively, this choice for  $\gamma$  reflects the interest savings resulting from paying off a dollar of mortgage debt. We interpret 3.38% as being at the low end of a range of possible assumptions about returns to the illiquid asset  $Z$ . In Column 2 we estimate a lower  $\hat{\beta}$  and a higher  $\hat{\delta}$  than in the benchmark case. Intuitively, when the return on the illiquid asset is relatively low, consumers will only invest in the illiquid asset if they have a low long-run discount rate. Lowering the long-run discount rate makes the consumer more patient generally, so the MSM procedure generates a greater short-run discount rate (i.e., a lower value of  $\beta$ ) to offset this effect and thereby maintain high rates of credit card borrowing and consumption-income comovement. As a net result of these changes, the over-identifying restrictions are now rejected.

In Column 3, we consider the case  $\gamma = 6.59\%$ . Flavin and Yamashita (2002) calculate this as the *average* real aftertax return to housing, including capital gains, use-value, maintenance costs, and taxes.<sup>29</sup> We view this figure as falling toward the upper end of a range of possible returns to the illiquid asset in our model. If our model distinguished between average and marginal returns we would want to use the marginal return not the average return on housing.<sup>30</sup> Moreover, even the average return to housing may be lower than 6.59% because of transaction costs of buying/selling

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<sup>28</sup>Our measure of  $r^{CC}$  is constructed from aggregate data, so its true variability is underestimated in the first stage.

<sup>29</sup>Flavin and Yamashita (2002) assume that use-value is equal to the real return on financial assets plus housing depreciation plus property taxes.

<sup>30</sup>Because of transaction costs, at the margin it may be optimal to pay off the mortgage rather than buying a larger house. The marginal return to paying off a mortgage is 3.38% as we argue above.

real estate.

Using  $\gamma = 6.59\%$  we obtain a higher estimate of  $\hat{\beta}$  and a lower estimate for  $\hat{\delta}$  than in the benchmark case. As  $\gamma$  approaches the credit card interest rate, the model can more easily accommodate simultaneous illiquid wealth accumulation and credit card borrowing. Despite the increased estimate for  $\hat{\beta}$ , the  $\beta = 1$  hypothesis is still rejected at the 99% confidence level. We also report a borderline rejection of the over-identifying restrictions for this case.

The large effects resulting from changes in  $\gamma$  contrast with the small effects we now report arising from changes in the credit card real interest rate,  $r^{CC}$ . In Column 4 we assume  $r^{CC} = 10\%$  and find that  $\hat{\beta}$  rises and  $\hat{\delta}$  falls relative to the benchmark case. Column 5 shows similar effects in the opposite direction for  $r^{CC} = 13\%$ . We introduce these perturbations for two reasons. First, formal incorporation of uncertainty in  $r^{CC}$  through the first stage correction only accounts for variation in population average interest rates. Additional tests in Columns 4 and 5 could capture individual-level variation. In addition, these changes correspond to different perspectives on how bankruptcy matters for the cost of credit card borrowing. Our benchmark value for  $r^{CC}$  equals the debt-weighted average interest rate from the Fed, minus inflation, minus the personal bankruptcy rate. This ignores the fact that (i) the marginal utility of consumption may be especially high in the bankruptcy state, implying that our correction is too small, and (ii) bankruptcy carries stigma (Gross and Souleles 2002b), implying that our correction is too large. We favor the middle specification as our benchmark, but observe that changes in  $r^{CC}$  of about 150 basis points reported in Columns 4 and 5 have little quantitative effect on the time preference estimates. However, these perturbations do lead to a rejection of the overidentifying restrictions.

Finally, we examine the effect of varying the coefficient of relative risk aversion  $\rho$ . Economists disagree about how to calibrate  $\rho$ . In order to account for the equity premium puzzle, the consumption CAPM requires  $\rho > 10$  (Kocherlakota 1996). But most lifecycle consumption models assume  $\rho \in [0.5, 5]$ , consistent with typical introspective choices about hypothetical large gambles (Mankiw and Zeldes 1991). Using a structural approach, Gourinchas and Parker (2002) identify  $\rho$  from lifecycle consumption profiles. For different specifications they find  $\rho$  between 0.1 and 5.3, with a benchmark estimate of 0.51. When liquidity constraints do not bind and preferences are time-separable,  $\rho$  is the inverse of the elasticity of substitution. Euler Equation estimates of the EIS range roughly between 0 and 1 (Hall 1988). Further complicating the picture, recent theo-

retical work casts doubt on the prevailing approach to modeling risk attitudes. Kahneman and Tversky (1979) and others propose and use models of loss aversion that imply first-order risk aversion. Rabin (2000) argues that seemingly-reasonable attitudes toward small gambles imply totally unreasonable attitudes toward larger gambles in an expected utility model with second-order risk aversion. Chetty (2004) proposes that consumption commitments could cause different local and global levels of risk aversion.

Recall that we adopted  $\rho = 2$  for our benchmark. We now examine the effect of adopting  $\rho = 1$  and  $\rho = 3$ . Column 6 of Table 4 reports the effect of assuming  $\rho = 1$ . We find that  $\hat{\beta}$  and  $\hat{\delta}$  both rise relative to their benchmark estimates. With less curvature in the utility function consumers are more willing to consume early in life and retire in relative poverty (and thereby more willing to borrow on their credit cards and less willing to accumulate assets). Raising  $\beta$  and  $\delta$  offsets these effects. In this specification,  $\hat{\beta}$  is only marginally significantly different from 1. Finally, assuming  $\rho = 1$  generates a rejection of the overidentifying restrictions.

Column 7 of Table 4 reports the effect of assuming  $\rho = 3$ . As one would predict from the previous experiment, we now find that  $\hat{\beta}$  and  $\hat{\delta}$  both fall relative to their benchmark estimates. As a result the null hypothesis of  $\beta = 1$  is rejected with a t-statistic of 3.2. Finally, assuming  $\rho = 3$  does not lead to a rejection of the overidentifying restrictions.

In the bottom panel of the table we estimate the model with the restriction  $\beta = 1$ . In every case, the restricted model is rejected by the overidentification test.<sup>31</sup>

### 5.3.2 Extreme Cases

We also consider two extreme cases to evaluate the consequences of letting  $\gamma$ ,  $r^{CC}$ , and  $\rho$  move together. In Case A, we consider the effect of simultaneously assuming  $\gamma = 3.38\%$ ,  $r^{CC} = 13\%$ , and  $\rho = 3$ , and in Case B we assume  $\gamma = 6.59\%$ ,  $r^{CC} = 10\%$ , and  $\rho = 1$ . We consider both sets of assumptions to be extreme, but we present estimates for them to indicate rough upper and lower bounds on estimates of  $\beta$  and  $\delta$ .

Case A combines the three perturbations of first stage parameters that we discussed above and

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<sup>31</sup>In columns 2 and 3, the exponential results are nearly identical to the benchmark case; the point estimates don't differ at all. This occurs because simulated exponential households accumulate zero illiquid wealth at the estimated  $\hat{\delta}$  in both the benchmark case and in the perturbed cases.

reported in Table 4. Each of those perturbations lowered estimates of  $\beta$ . In Column 1 of Table 5 we report that their combined effect results in estimates of  $\hat{\beta} = 0.375$  and  $\hat{\delta} = 0.972$ . When we restrict  $\beta$  to equal 1 (Column 2) we estimate  $\hat{\delta} = 0.770$ . With such a low  $\hat{\delta}$ , the exponential discounting model predicts credit card borrowing and excess sensitivity quite well, but predicts approximately no wealth accumulation.

In Case B we combine the three other perturbations from Table 4 to estimate  $\hat{\beta} = 0.908$  and  $\hat{\delta} = 0.943$  (Column 3 of Table 5). This estimate carries a small standard error, implying that even under aggressive assumptions about  $\gamma$ ,  $r^{CC}$ , and  $\rho$ , we still find that  $\hat{\beta}$  is significantly less than 1. In Column 4 we report estimates for the restricted exponential model and find that  $\hat{\delta} = 0.936$ . This is the most successful case for the exponential model among all of the simulations that we perform. Nevertheless, this simulation as well as all of the other simulations reported in Table 5 reject the overidentification restrictions.

## 6 Extensions

This paper’s findings suggest several directions for future work.

### 6.1 The coefficient of relative risk aversion

Given the sensitivity of the model’s quantitative results to the value of  $\rho$ , it would be natural to estimate  $\rho$  simultaneously with  $\beta$  and  $\delta$ . We can anticipate the results of such an exercise by considering the limit case of linear utility. Under the maintained assumption of  $\rho = 0$ , the Euler Equation for the exponential model would only be satisfied with a discount rate at least as great as the credit card interest rate. This discount rate would be too high to generate observed levels of wealth accumulation. Higher short-term than long-term discount rates would then still be needed to reproduce both the wealth and credit card moments. Low values of  $\rho$  also worsen the model’s performance on specification tests at the estimated discounting parameters. This occurs because risk neutral consumers who borrow at all will borrow a great deal, leaving no liquidity buffer for the following period. At the estimated parameter values % *Visa* is underpredicted and *mean Visa* is overpredicted.

It is not clear what large values of  $\rho$  would imply, though the gradient that we observe in our

simulations suggests that raising the value of  $\rho$  lowers the estimated value of  $\beta$ .

## 6.2 Naivete

Strotz (1955), Akerlof (1991) and O’Donoghue and Rabin (1999a, 1999b) propose that decision makers with dynamically inconsistent preferences make current choices under the “naive” belief that later selves will act in the interests of the current self. Angeletos et al. (2001) find that naive and sophisticated quasi-hyperbolic agents behave similarly in consumption models like the one discussed in this paper.

However, two puzzles remain that perhaps a model of naivete could address. First, the sophisticated quasi-hyperbolics in these simulations would be better off if they had no access to credit cards throughout their lifecycles. Specifically, according to a comparison of value functions, at age 20, sophisticated quasi-hyperbolics would be willing to pay \$2000 to get rid of their credit cards immediately and *never* have access to them in the future. This begs the question of why only a tiny fraction of consumers cut up their credit cards.

Second, the spread between the cost of funds and the credit card interest rate is “too high.” As Ausubel (1991) has pointed out, the spread cannot be accounted for by standard explanations like default probabilities; instead consumers seem to pick their credit card under the naive belief that they will not borrow in the future. Teaser rates may also be explained by a model of naive quasi-hyperbolic discounting (Shui and Ausubel 2004).<sup>32</sup>

## 6.3 Further generalization of intertemporal preferences

Standard models explore the one-parameter exponential discount function and we have explored a two-parameter generalization. We may find that richer representations better describe discounting patterns. It is also possible that the nature of discounting changes over the course of the lifecycle,<sup>33</sup> providing another possible framework for explaining patterns of wealth accumulation and credit card borrowing.

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<sup>32</sup>See also the theoretical contribution of DellaVigna and Malmendier (2004).

<sup>33</sup>Our assumption that marginal utility varies with demographics (and thus with age) is a special case of the general class of age-specific discounting studied by Attanasio and Weber (1995) and Attanasio et al. (1999).

## 6.4 Heterogeneity

Another natural direction for future work is to relax the assumption of homogeneous preferences. Specifically, one might wonder whether two groups of exponential consumers, one patient and the other impatient, could account for the facts. To us, the data suggest that there is substantial heterogeneity in the population, but that the heterogeneity does not explain why the median household both borrows on its credit cards and invests in illiquid assets. Laibson et al. (2003) find that credit card borrowing is pervasive across the entire wealth distribution. Nevertheless, a model with heterogeneous preferences may resolve some of the empirical tensions discussed in this paper.

## 6.5 Institutional assumptions

The assets in the model are stylized and it would be natural to make our institutional assumptions more realistic. In a richer model households would be able to declare bankruptcy, default on their credit card debt, sell their  $Z$  assets, borrow against their  $Z$  assets, and engage in many other types of financial transactions. We are most interested in exploring changes in assumptions about  $Z$ , since the  $Z$  asset plays a central role in driving our results. For example, if the  $Z$  asset were perfectly liquid our model would be unable to explain credit card borrowing (since consumers with liquid retirement assets would spend down these liquid assets rather than borrowing on a high-interest credit card). Hence, the illiquidity of  $Z$  is critical for the predictive power of the model. Illiquidity of  $Z$  also mimics the optimal mechanisms that have recently been derived for quasi-hyperbolic consumers (Amador et al. 2004).

Other work has explored the consequences of making  $Z$  less illiquid (i.e., by introducing a transaction cost for liquidating  $Z$ ).<sup>34</sup> This does not change the model's qualitative predictions. However, such transaction cost models may be too crude to capture the implications of collateralized borrowing against  $Z$ . For example, home equity lines of credit are rapidly gaining popularity in the U.S. and such instruments make heretofore illiquid home equity relatively easy to access. If home equity lines of credit provide immediate liquidity with little or no transactions costs, then  $Z$  is really not an illiquid asset. However, if applying for a home equity line of credit generates

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<sup>34</sup>Laibson et al. (2003) report simulations in which liquidating part of the illiquid asset generates a fixed cost of \$10,000 and a proportional cost of 10%.

immediate effort costs as well as bureaucratic delays, then quasi-hyperbolic consumers will prefer to use their credit cards rather than applying for such credit lines (Laibson 1997).

To explore these issues, researchers should develop high frequency models in which the units of time are days and not years (and  $\beta$  multiplies all utility flows starting tomorrow). In such models the nature of the effort costs and the bureaucratic delays will play an important role. For example, if an application for a home equity credit line yields immediate effort costs and only delayed liquidity (say in a week), such credit may *not* be tempting to quasi-hyperbolic consumers. However, if a home equity application has vanishingly small effort costs and yields immediate liquidity, then such a loan will be appealing to quasi-hyperbolic consumers and the model will then predict that splurges will *not* be funded from credit cards.

Such high frequency models are beyond the scope of the current paper. As home equity lines of credit and other technologies for making illiquid assets liquid become increasingly popular (only 7.8% of homeowners in the 1998 SCF had such loans), it will become more important for economists to study the micro-structure and micro-timing of the loan application process. Though the parameters of this paper's model are identified off data from a time period when home equity was more illiquid than today, if consumers sharply devalue rewards that are delayed by only a few days, models of the credit application process will need to reflect these high frequency preferences.

## 6.6 Consumption shocks

In addition to income uncertainty, consumers also face stochastic shocks to preferences and consumption needs (Amador et al. 2004). Expenses for car repairs and health care, for example, often come unexpectedly. In principle this additional volatility could generate higher levels of (illiquid) wealth accumulation and credit card borrowing by promoting saving after unusually good shocks and borrowing after unusually bad shocks. We examine this possibility qualitatively by proxying for consumption shocks with increased income uncertainty. Specifically, we re-estimate  $\beta$  and  $\delta$  after doubling the variances of the parameters of our calibrated income process.<sup>35</sup> This yields

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<sup>35</sup>We consider this a plausible upper bound, as it increases the conditional and unconditional standard deviations of income by about 50%. Hubbard, Skinner and Zeldes (1995) and Palumbo (1999) estimate dynamic processes of health expenditures, finding conditional and unconditional standard deviations equivalent to roughly one-third and one-fifth, respectively, of the conditional and unconditional standard deviations in income that we document in the PSID.



results very similar to the benchmark run discussed above: the estimate of  $\beta$ , 0.62, is significantly less than 1; the specification test on the quasi-hyperbolic model is borderline; and the specification test on the exponential model, when  $\beta$  is restricted to equal 1, is rejected. Though these results support the intuition that volatility can't explain why the same time-consistent consumers would both accumulate illiquid assets and borrow on credit cards, subsequent analyses might formally calibrate and incorporate stochastic shocks to the marginal utility of consumption.

## 6.7 Second stage moment sets

Estimates of discount functions should be cross-validated by identifying the model from different sets of empirical moments. For example, one could identify the parameters with wealth-to-income ratios at every age, instead of using just the pre-retirement wealth stock.

We could also analyze the consumption drop at retirement documented by Banks, Blundell and Tanner (1998) and Bernheim et al. (2001). However, both the exponential and quasi-hyperbolic benchmark models, evaluated at the estimated parameters, predict similar consumption drops around retirement, so this test is unlikely to distinguish the two models.

Illiquidity of investments provides another source of identifying data. Households invest very little of their wealth in liquid form (SCF). Even with an expansive definition of liquid assets, only 18.6% of total US household wealth is liquid.<sup>36</sup> The exponential model overpredicts this share, while the quasi-hyperbolic model overpredicts it by a smaller – but still large – amount.<sup>37</sup>

The calibrated quasi-hyperbolic model predicts a small consumption boom early in life when credit cards are acquired. One could empirically test this prediction. Finally, many facts about the behavior of the elderly remain unexplained, particularly the pattern of anomalously slow decumulation of assets. Illiquid asset holding may help to explain these facts.

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<sup>36</sup>The methodology for calculating the share is analogous to that for calculating % *Visa*, *mean Visa*, and *wealth* described in Appendix A. The definition includes cash, checking and savings accounts, money market accounts, call accounts, CDs, bonds, stocks and mutual funds.

<sup>37</sup>The calibrated hyperbolic model predicts a share equal to 37%.

## 7 Conclusion

This paper estimates time preferences using a structural model and field data. U.S. households accumulate large stocks of wealth before retirement, borrow actively on credit cards, and exhibit excess sensitivity of consumption to predictable movements in income. To explain these phenomena the MSM procedure estimates  $\beta = 0.703$  and  $\delta = 0.958$ . Intuitively, the implied low long-term discount rate ( $-\ln \delta = 4.2\%$ ) accounts for observed levels of (illiquid) wealth accumulation. The high short-term discount rate ( $-\ln \beta\delta = 39.5\%$ ) explains the observed frequency and levels of credit card borrowing and excess sensitivity of consumption. Our benchmark specification fails to reject the overidentification restrictions. The MSM procedure does reject the restriction to exponential discounting ( $\beta = 1$ ).

Our parameter estimates are sensitive to the calibration choices, and some calibrations lead to a failure of the overidentification tests. In addition, our economic environment is stylized and future work should enrich the realism of our modelling framework. However, the evidence reported here suggests that consumption-savings models will better match field data when the models incorporate discount rates in the short run that exceed discount rates at longer horizons. Structural estimation using field data is likely to be a useful complement to laboratory studies that measure time preferences.

## A Second-Stage Moments Appendix

We now discuss the procedures we use to construct the second-stage moments. We use the SCF to derive *wealth*, % *Visa* and *mean Visa*, and the PSID to construct *CY*. Procedures are very similar for the share of liquid assets. All quantities are deflated to 1990 dollars.

### A.1 SCF Moments

We use the 1983, 1989, 1992, 1995, and 1998 SCFs to compute the *wealth* moment. We derive % *Visa* and *mean Visa* from the 1995 and 1998 SCFs. We control for cohort effects, household demographics, and business cycle effects to make the characteristics of the population and the simulated data fully analogous. We assign to households in our simulations the mean empirical cohort, demographic, and business cycle effects. We adopt the following procedures.

For each variable of interest  $x$  we first use weighted least squares, applying the SCF population weights, to estimate

$$x_i = FE_i + BCE_i + CE_i + AE_i + \xi_i \quad (8)$$

Here  $FE_i$  is a family size effect that consists of three variables, the number of heads, the number of children, and the number of dependent adults in the household.  $BCE_i$  is a business cycle effect proxied by the unemployment rate in the household’s region of residence. In 1983, the unemployment rate is the rate in the state of residence. In 1992, 1995, and 1998, it is the rate in the Census Division. In 1989 the nationwide rate was used because information on household location is not available in the public use data set.  $CE_i$  is a cohort effect that consists of a full set of five-year cohort dummies,  $AE_i$  is an age effect that consists of a full set of age dummies, and  $\xi_i$  is an error term.

Next, we define the “typical” household to be identical to the simulated household (i.e. with head and spouse, exogenous age-varying numbers of children and adult dependents, an average cohort effect, and an average unemployment effect<sup>38</sup>). Then for each variable we create a new variable  $\hat{x}_i$  that captures what  $x_i$  would have been had household  $i$  been typical. For example, if household  $i$  is identical to the “typical” household except for having more children, we set

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<sup>38</sup>These averages are the means used in the calibration of the income process, which is based on the PSID. Refer to Table 3 and Laibson et al. (2003) for details.

$\hat{x}_i = x_i + \beta(\overline{nkids} - nkids_i)$ , where  $\beta$  is the coefficient for number of kids in the regression above and  $\overline{nkids}$  is the average number of children in a household as a function of the head's age. All moments were estimated using  $\hat{x}_i$ .

For *wealth*, we restrict the sample to households with heads between ages 50-59. We include all real and financial wealth (e.g., home equity and CDs) as well as all claims on defined contribution pension plans (e.g., 401(k)). The measure does not include Social Security wealth and claims on defined benefit pension plans, since these flows appear in our calibrated income process. If a household had a negative net balance in any illiquid asset, we set the balance equal to zero (e.g., we set home equity equal to the max of 0 and the value of home minus outstanding mortgages minus used portion of home equity lines of credit). Since there is no separate information on the amount borrowed against home equity lines of credit in the 1983 SCF, we assume that in that year no household had an outstanding home equity line balance.<sup>39</sup>

Let  $\kappa = 10 \cdot \frac{2}{\pi}$ . Then *wealth* is the mean of  $\kappa \cdot \arctan\left(\frac{\hat{x}_i}{\kappa}\right)$  in the sample, applying the SCF population weights. We use this arctan scaling in order to downweight outliers. This function has noteworthy properties. First, it is symmetric around the origin. Second, it is approximately linear in a neighborhood of the origin. Third, as  $\hat{x}_i$  gets very large, it asymptotes to 10. We compute the standard error of *wealth* directly from the sample values of  $\kappa \cdot \arctan\left(\frac{\hat{x}_i}{\kappa}\right)$ .

To construct % *Visa* we create a dummy variable *hasdebt* equal to one for household  $i$  if  $i$  has a positive outstanding credit card balance in the SCF. We correct *hasdebt* to generate  $\hat{x}_i$ . We then regress  $\hat{x}_i$  on a full set of age dummies. % *Visa* is a linear combination of the estimated coefficients on the age dummies, where the weights are derived from the same conditional survival probabilities we use in the simulations. The standard error is computed directly from the weights and the standard errors on the age dummy estimates.

Construction of *mean Visa* is complicated by the fact that aggregate credit card borrowing data from the Fed indicate that 1995 and 1998 SCF borrowing magnitudes are biased downward by a factor of three. We correct for this bias as follows. First we compute average outstanding interest-bearing balances. According to the Fed, aggregate debt outstanding at year-end 1995 and 1998 were \$443 billion and \$561 billion, respectively. From these figures we subtract an upper

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<sup>39</sup>In the 1983 SCF, 1.7% of homeowners with a high school degree reported having a credit line secured by home equity.

bound on the float (the balances that are still in their one-month grace period, which do not accrue interest). This upper bound is obtained by dividing total purchase volume, approximately \$1 trillion in 1998, by 12. We then divide by the number of U.S. households with credit cards, using Census Bureau data on total households and SCF data on the percentage of households with cards. We obtain average household borrowing conditional on having a card of \$5115 in 1995 and \$6411 in 1998. These figures are consistent with those from a proprietary account-level data set analyzed by Gross and Souleles (2002a, 2002b).

In our simulations we focus on households headed by people with high school degrees, so next we use the SCF data on borrowing to scale the Fed average borrowing figure for just the high school educated group. In particular, we define  $\alpha$  such that

$$debt_{all}^{Fed} = \alpha \cdot (w_{nhs}debt_{nhs}^{SCF} + w_{hs}debt_{hs}^{SCF} + w_{coll}debt_{coll}^{SCF})$$

with weights  $w_{nhs}$ ,  $w_{hs}$ , and  $w_{coll}$  defined by the proportion of educational categories in the population (0.25, 0.5, 0.25, respectively) and  $debt_{educ}^{source}$  equal to the average debt reported by *source* for educational group *educ*. Focusing now exclusively on the HS educational group, let  $debt_i^{SCF}$  be the level of credit card debt reported in the SCF for household  $i$ . Let  $debt_i = \alpha \cdot debt_i^{SCF}$  be the corrected credit card debt. Calculate age specific income means ( $\bar{y}_t$ ) and create  $debtinc_i$  as  $debt_i/\bar{y}$ <sup>40</sup>. Then, we correct  $debtinc_i$ , creating  $\hat{x}_i$ , and regress  $\hat{x}_i$  on a full set of age dummies. The moment *mean Visa* is a linear combination of the estimated coefficients on the age dummies, again using the weights derived from the conditional survival probabilities used in the simulations. Again, the standard error is computed directly from the weights and the standard errors on the age dummy estimates.

Covariances between the SCF moments were constructed by jointly estimating the above means.

## A.2 PSID Moment

We use PSID data from 1978 to 1992 to estimate the *CY* moment. In the data, we define consumption to include food, rent, and utilities (the most general definition available in the PSID).

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<sup>40</sup>When calculating the age-specific income means we group together ages 20-21, 70-74, 75-79, and 80 and over because we have very few observations at those ages.

The rental value of an owner-occupied home is assumed to be 5% of the value of the home. If the household neither owns nor rents, rent is the self-reported rental value of the home if it were rented.

We construct the  $CY$  moment by using 2SLS to estimate

$$\Delta \ln(C_{it}) = \alpha E_{t-1} \Delta \ln(Y_{it}) + X_{it} \beta + \varepsilon_{it},$$

where  $C_{it}$  is just food, rent, and utilities. We assume an MA(1) process for the error term and instrument for  $E_{t-1} \Delta \ln(Y_{it})$  with  $\ln Y_{it-3}$  and  $\ln Y_{it-4}$ . The overidentification test does not reject this specification. The vector  $X_{it}$  includes age, cohort, and business cycle effects, the change in effective family size, the mortality rate, and lagged wealth. Since wealth is observed in the PSID only in 1984 and 1989, in the other years we estimate wealth using the intertemporal budget constraint and a projected value of total consumption. Total consumption was projected from the PSID's partial measure using the CEX: in the CEX we regress total consumption on food, rent and utilities consumption, and then we use the coefficients to infer total consumption from the available PSID measure.

## B MSM Procedure Appendix

Since  $m(\theta; \hat{\chi})$  is difficult to evaluate we replace it with an unbiased simulator, calculated by first taking  $J_s$  draws of the initial distribution and then constructing the corresponding simulated expectations. Define  $m_{J_s}(\theta; \hat{\chi})$  as the vector of simulated moments. Now we can find the vector  $\hat{\theta}$  that minimizes  $g'_{J_s}(\theta; \hat{\chi}) W g_{J_s}(\theta; \hat{\chi})$ , where  $g_{J_s}(\theta; \hat{\chi}) = \bar{m}_{J_s} - m_{J_s}(\theta; \hat{\chi})$ .

The first order condition for the second stage (incorporating the use of simulation) is given by

$$g'_{J_s \theta}(\hat{\theta}; \hat{\chi}) W g_{J_s}(\hat{\theta}; \hat{\chi}) = 0.$$

where  $g_{J_s \theta}(\hat{\theta}; \hat{\chi}) = \partial g_{J_s}(\hat{\theta}; \hat{\chi}) / \partial \theta'$ .

Following Gourinchas and Parker (2002) and Newey and McFadden (1994), an expansion of

$g_{J_s}(\hat{\theta}; \hat{\chi})$  around  $\theta_0$  to first order leads to

$$g'_{J_s\theta}(\hat{\theta}; \hat{\chi}) W \left[ g_{J_s}(\theta_0; \hat{\chi}) + g_{J_s\theta}(\theta_0; \hat{\chi}) (\hat{\theta} - \theta_0) \right] = 0.$$

Rearranging terms and defining  $\hat{J}_m$  as the (scalar) rate of convergence of  $\hat{\theta}$ ,

$$\sqrt{\hat{J}_m} (\hat{\theta} - \theta_0) = - \left[ g'_{J_s\theta}(\hat{\theta}; \hat{\chi}) W g_{J_s\theta}(\theta_0; \hat{\chi}) \right]^{-1} g'_{J_s\theta}(\hat{\theta}; \hat{\chi}) W \sqrt{\hat{J}_m} g_{J_s}(\theta_0; \hat{\chi}).$$

Let  $\Pi \equiv \left[ g'_{J_s\theta}(\hat{\theta}; \hat{\chi}) W g_{J_s\theta}(\theta_0; \hat{\chi}) \right]^{-1} g'_{J_s\theta}(\hat{\theta}; \hat{\chi}) W$ . Expanding  $g_{J_s}(\theta_0; \hat{\chi})$  around  $\chi_0$ ,

$$\sqrt{\hat{J}_m} (\hat{\theta} - \theta_0) = -\Pi \left[ \sqrt{\hat{J}_m} g_{J_s}(\theta_0; \chi_0) + \sqrt{\hat{J}_m} g_{J_s\chi}(\theta_0; \chi_0) (\hat{\chi} - \chi_0) \right]. \quad (9)$$

To evaluate Equation 9, first note that

$$\begin{aligned} \sqrt{\hat{J}_m} g_{J_s}(\theta_0; \chi_0) &= \sqrt{\hat{J}_m} [\bar{m}_{J_m} - m_{J_s}(\theta_0; \chi_0)] \\ &= \sqrt{\hat{J}_m} [\bar{m}_{J_m} - m(\theta_0; \chi_0)] + \sqrt{\hat{J}_m} [m(\theta_0; \chi_0) - m_{J_s}(\theta_0; \chi_0)] \end{aligned}$$

The two bracketed terms represent independent sets of draws from the same population. The first term equals  $\sqrt{\hat{J}_m} g(\theta_0; \chi_0)$ , which is asymptotically normally distributed:  $\sqrt{\hat{J}_m} g(\theta_0; \chi_0) \rightarrow N(0, \Sigma_g)$ . We estimate  $\Omega_g = \frac{\Sigma_g}{J_m} = E[g(\theta_0; \chi_0) g(\theta_0; \chi_0)']$  directly from its sample counterpart.<sup>41</sup> The second term represents the simulation error. At the true value of  $\theta$ , the simulated moments were generated from a finite number of random draws from the true population. Therefore, the second term is also asymptotically normal (as the size of the simulated sample goes to infinity) with mean 0 and variance  $\hat{J}_m \frac{\Sigma_g}{J_s}$ . Finally, since variation in the simulation and the data are independent,  $\sqrt{\hat{J}_m} g_{J_s}(\theta_0; \chi_0) \rightarrow N\left(0, \left(1 + \frac{\hat{J}_m}{J_s}\right) \Sigma_g\right)$ . To operationalize this expression for the variance, given the different numbers of observations  $J_m$  in the sample, we conservatively use the pairwise maximum numbers of observations,  $\max(J_{ma}, J_{mb})$ , to weight the  $(a, b)$ 'th cell of  $\Sigma_g$  in the simulation correction.

Now turn to the second term of Equation 9. In the main text we have defined the variance of

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<sup>41</sup>In fact, the  $(a, b)$ 'th cell is  $\Omega_g(a, b) = \frac{\Sigma_g(a, b)}{\min(J_{ma}, J_{mb})}$ .

the first stage parameter estimates  $\hat{\chi}$  as  $\Omega_\chi = E [(\hat{\chi} - \chi_0) (\hat{\chi} - \chi_0)']$ .

Thus,  $\sqrt{\hat{J}_m} g_{J_s \chi}(\theta_0; \chi_0) (\hat{\chi} - \chi_0) \rightarrow N(0, \hat{J}_m G_\chi \Omega_\chi G_\chi')$ , and  $\sqrt{\hat{J}_m} (\hat{\theta} - \theta_0) \rightarrow N(0, \Sigma_\theta)$ , where Equation 9 implies

$$\Sigma_\theta = (G'_\theta W G_\theta)^{-1} G'_\theta W \left[ \left( 1 + \frac{\hat{J}_m}{J_s} \right) \Sigma_g + \hat{J}_m \cdot G_\chi \Omega_\chi G_\chi' \right] W G_\theta (G'_\theta W G_\theta)^{-1}, \quad (10)$$

by the asymptotic normality of  $\hat{\chi}$  and  $g(\cdot)$  and by the Slutsky theorem, assuming zero covariance between the first and second stage moments. Dividing by  $\hat{J}_m$  we obtain our key equation,

$$\Omega_\theta = Var(\hat{\theta}) = (G'_\theta W G_\theta)^{-1} G'_\theta W [\Omega_g + \Omega_g^s + G_\chi \Omega_\chi G_\chi'] W G_\theta (G'_\theta W G_\theta)^{-1}.$$

Standard errors reported in the text and tables equal the square roots of the diagonal elements of  $\Omega_\theta$ .



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TABLE 1  
SECOND-STAGE MOMENTS

Description and Name	$\bar{m}_{J_m}$	$se(\bar{m}_{J_m})$
% Borrowing on Visa: “% Visa”	0.678	0.015
Mean (Borrowing <sub>t</sub> / mean(Income <sub>t</sub> )): “mean Visa”	0.117	0.009
Consumption-Income Comovement: “CY”	0.231	0.112
Average weighted $\frac{wealth}{income}$ : “wealth”	2.60	0.13

Source: Authors’ calculations based on data from the Survey of Consumer Finances, the Federal Reserve, and the Panel Study on Income Dynamics. Calculations pertain to households with heads who have high school diplomas but not college degrees. The variables are defined as follows: % Visa is the fraction of U.S. households borrowing and paying interest on credit cards (SCF 1995 and 1998); mean Visa is the average amount of credit card debt as a fraction of the mean income for the age group (SCF 1995 and 1998, weighted by Fed aggregates); CY is the marginal propensity to consume out of anticipated changes in income (PSID 1978-92); and wealth is the weighted average wealth-to-income ratio for households with heads aged 50-59 (SCF 1983-1998).

TABLE 2  
FIRST STAGE ESTIMATION RESULTS

<p>Demographics</p> <p><i>Number of children</i></p> $k = \beta_0 * \exp(\beta_1 * \text{age} - \beta_2 * (\text{age}^2)/100) + \varepsilon$ <table style="margin-left: 40px;"> <tr> <td><math>\beta_0</math></td> <td><math>\beta_1</math></td> <td><math>\beta_2</math></td> </tr> <tr> <td>0.006</td> <td>0.324</td> <td>0.005</td> </tr> <tr> <td>(0.001)</td> <td>(0.005)</td> <td>(0.007)</td> </tr> </table> <p><i>Number of dependent adults</i></p> $a = \beta_0 * \exp(\beta_1 * \text{age} - \beta_2 * (\text{age}^2)/100) + \varepsilon$ <table style="margin-left: 40px;"> <tr> <td><math>\beta_0</math></td> <td><math>\beta_1</math></td> <td><math>\beta_2</math></td> </tr> <tr> <td>8.0e-9</td> <td>0.727</td> <td>0.007</td> </tr> <tr> <td>(0.000)</td> <td>(0.016)</td> <td>(0.016)</td> </tr> </table>	$\beta_0$	$\beta_1$	$\beta_2$	0.006	0.324	0.005	(0.001)	(0.005)	(0.007)	$\beta_0$	$\beta_1$	$\beta_2$	8.0e-9	0.727	0.007	(0.000)	(0.016)	(0.016)	<p>Liquid assets and noncollateralized debt</p> <p><i>Credit limit <math>\lambda</math></i></p> <p style="text-align: center;">0.318 (0.017)</p> <p><i>Return on positive liquid assets <math>R</math></i></p> <p style="text-align: center;">1.0279 (0.024)</p> <p><i>Credit card interest rate <math>R^{cc}</math></i></p> <p style="text-align: center;">1.1152 (0.009)</p>																														
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<p>Illiquid Assets</p> <p><i>Consumption flow as a fraction of assets <math>\gamma</math></i></p> <p style="text-align: center;">0.05</p> <p style="text-align: center;">-</p>	<p>Preference Parameter</p> <p><i>Coefficient of relative risk aversion <math>\rho</math></i></p> <p style="text-align: center;">2</p> <p style="text-align: center;">-</p>																																																
<p>Income from transfers and wages</p> <p><i>Income process - In the labor force</i></p> $y = \ln(Y) = \beta_0 + \beta_1 * \text{age} + \beta_2 * (\text{age}^2/100) + \beta_3 * (\text{age}^3/10000) + \beta_4 * \text{Nheads} + \beta_5 * \text{Nchildren} + \beta_6 * \text{Ndep.adults} + \xi^W$ $\xi^W_t = \eta_t + \nu_t = \alpha \eta_{t-1} + \varepsilon_t + \nu_t$ <table style="margin-left: 40px;"> <tr> <td><math>\beta_0</math></td> <td><math>\beta_1</math></td> <td><math>\beta_2</math></td> <td><math>\beta_3</math></td> <td><math>\beta_4</math></td> <td><math>\beta_5</math></td> <td><math>\beta_6</math></td> <td><math>\alpha</math></td> <td><math>\sigma^2_\varepsilon</math></td> <td><math>\sigma^2_\nu</math></td> </tr> <tr> <td>7.439</td> <td>0.118</td> <td>-0.201</td> <td>0.081</td> <td>0.548</td> <td>-0.033</td> <td>0.170</td> <td>0.782</td> <td>0.029</td> <td>0.026</td> </tr> <tr> <td>(0.340)</td> <td>(0.021)</td> <td>(0.050)</td> <td>(0.035)</td> <td>(0.019)</td> <td>(0.005)</td> <td>(0.008)</td> <td>(0.017)</td> <td>(0.008)</td> <td>(0.011)</td> </tr> </table> <p><i>Income Process - Retired</i></p> $y = \ln(Y) = \beta_0 + \beta_1 * \text{age} + \beta_2 * \text{Nheads} + \beta_3 * \text{Nchildren} + \beta_4 * \text{Ndep.adults} + \xi^R$ <table style="margin-left: 40px;"> <tr> <td><math>\beta_0</math></td> <td><math>\beta_1</math></td> <td><math>\beta_2</math></td> <td><math>\beta_3</math></td> <td><math>\beta_4</math></td> <td><math>\sigma^2_{\xi^R}</math></td> </tr> <tr> <td>8.433</td> <td>-0.002</td> <td>0.554</td> <td>0.199</td> <td>0.204</td> <td>0.051</td> </tr> <tr> <td>(0.849)</td> <td>(0.013)</td> <td>(0.084)</td> <td>(0.172)</td> <td>(0.102)</td> <td>(0.013)</td> </tr> </table> <p style="text-align: right;"><i>Retirement age <math>T</math></i></p> <p style="text-align: right;">63 (0.730)</p>		$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	$\beta_6$	$\alpha$	$\sigma^2_\varepsilon$	$\sigma^2_\nu$	7.439	0.118	-0.201	0.081	0.548	-0.033	0.170	0.782	0.029	0.026	(0.340)	(0.021)	(0.050)	(0.035)	(0.019)	(0.005)	(0.008)	(0.017)	(0.008)	(0.011)	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\sigma^2_{\xi^R}$	8.433	-0.002	0.554	0.199	0.204	0.051	(0.849)	(0.013)	(0.084)	(0.172)	(0.102)	(0.013)
$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	$\beta_6$	$\alpha$	$\sigma^2_\varepsilon$	$\sigma^2_\nu$																																								
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Source: Authors' estimation, exactly following Laibson, Repetto, and Tobacman (2003), based on data from the PSID, SCF, FRB, and American Bankruptcy Institute, for households with heads who have high school diplomas but not college degrees.

Note: Standard errors in parentheses. The constant of the deterministic component of income includes a year of birth cohort effect and a business cycle effect proxied by the unemployment rate.

The dynamics of income estimation includes a household fixed effect.

Illiquid asset consumption flows and the coefficient of relative risk aversion are assumed to be exactly known in the context of the first stage. We examine sensitivity to these parameters in Subsection 5.3 on Robustness.

This table only reports standard errors, but the full covariance matrix is used in the second-stage estimation.



TABLE 3  
BENCHMARK STRUCTURAL ESTIMATION RESULTS

	(1)	(2)	(3)	(4)	(5)
	Hyperbolic	Exponential	Hyperbolic Optimal Wts	Exponential Optimal Wts	Data
Parameter estimates $\hat{\theta}$					
$\hat{\beta}$	0.7031	1.0000	0.7150	1.0000	-
s.e. (i)	(0.1093)	-	(0.0948)	-	-
s.e. (ii)	(0.1090)	-	-	-	-
s.e. (iii)	(0.0170)	-	-	-	-
s.e. (iv)	(0.0150)	-	-	-	-
$\hat{\delta}$	0.9580	0.8459	0.9603	0.9419	-
s.e. (i)	(0.0068)	(0.0249)	(0.0081)	(0.0132)	-
s.e. (ii)	(0.0068)	(0.0247)	-	-	-
s.e. (iii)	(0.0010)	(0.0062)	-	-	-
s.e. (iv)	(0.0009)	(0.0056)	-	-	-
Second-stage moments					
<i>% Visa</i>	0.634	0.669	0.613	0.284	0.678
<i>mean Visa</i>	0.167	0.150	0.159	0.049	0.117
<i>CY</i>	0.314	0.293	0.269	0.074	0.231
<i>wealth</i>	2.69	-0.05	3.22	2.81	2.60
Goodness-of-fit					
$q(\hat{\theta}, \hat{\chi})$	67.2	436	2.48	34.4	-
$\xi(\hat{\theta}, \hat{\chi})$	3.01	217	8.91	258.7	-
<i>p-value</i>	0.222	<1e-10	0.0116	<2e-7	-

Source: Authors' calculations.

Note on standard errors: (i) includes both the first stage correction and the simulation correction, (ii) includes just the first stage correction, (iii) includes just the simulation correction, and (iv) includes neither correction.

TABLE 4  
ROBUSTNESS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Benchmark	$\gamma = 3.38\%$	$\gamma = 6.59\%$	$r^{CC} = 10\%$	$r^{CC} = 13\%$	$\rho = 1$	$\rho = 3$
Hyperbolic							
Parameter Estimates $\hat{\theta}$							
$\hat{\beta}$	0.7031	0.5071	0.8024	0.7235	0.6732	0.8186	0.5776
s.e. (i)	(0.1093)	(0.0441)	(0.0614)	(0.1053)	(0.1167)	(0.0959)	(0.1339)
$\hat{\delta}$	0.9580	0.9731	0.9425	0.9567	0.9595	0.9610	0.9545
s.e. (i)	(0.0068)	(0.0188)	(0.0093)	(0.0071)	(0.0045)	(0.0037)	(0.0096)
Goodness-of-fit							
$q(\hat{\theta}, \hat{\chi})$	67.2	108.4	49.7	64.1	70.7	63.0	67.7
$\xi(\hat{\theta}, \hat{\chi})$	3.01	16.79	5.27	12.09	10.97	7.97	1.85
$p$ -value	0.222	0.0002	0.0717	0.0024	0.0041	0.0186	0.3965
Exponential							
Parameter Estimates $\hat{\theta}$							
$\hat{\beta}$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
s.e. (i)	-	-	-	-	-	-	-
$\hat{\delta}$	0.8459	0.8459	0.8459	0.8520	0.8354	0.8924	0.7841
s.e. (i)	(0.0249)	(0.0249)	(0.0250)	(0.0267)	(0.0262)	(0.0204)	(0.0357)
Goodness-of-fit							
$q(\hat{\theta}, \hat{\chi})$	435.6	435.6	435.6	434.7	436.6	438.1	435.5
$\xi(\hat{\theta}, \hat{\chi})$	217	217	263	177	339	349	310
$p$ -value	<1e-10	<1e-10	<1e-10	<1e-10	<1e-10	<1e-10	<1e-10

Source: Authors' calculations.

Note: The benchmark assumes  $\gamma=5\%$ ,  $r^{CC}=11.52\%$ , and  $\rho=2$ . Columns (3) through (8) perturb parameters one at a time.

TABLE 5  
EXTREME CASES

	Case A		Case B	
	(1) Hyperbolic	(2) Exponential	(3) Hyperbolic	(4) Exponential
Parameter Estimates $\hat{\theta}$				
$\hat{\beta}$	0.3750	1.0000	0.9075	1.0000
s.e. (i)	(0.4859)	-	(0.0285)	-
$\hat{\delta}$	0.9717	0.7695	0.9434	0.9359
s.e. (i)	(0.0228)	(0.0262)	(0.0059)	(0.0071)
Second-stage moments				
<i>% Visa</i>	0.650	0.680	0.643	0.506
<i>mean Visa</i>	0.188	0.153	0.155	0.097
<i>CY</i>	0.504	0.297	0.230	0.141
<i>wealth</i>	2.55	-0.06	2.62	2.52
Goodness-of-fit				
$q(\hat{\theta}, \hat{\chi})$	106.1	436.1	38.9	145.2
$\xi(\hat{\theta}, \hat{\chi})$	16.06	319.5	7.52	19.68
<i>p</i> -value	0.0003	<1e-10	0.0233	0.0002

Source: Authors' calculations.

Note: Case A assumes  $\gamma=3.38\%$ ,  $r^{CC}=13\%$ , and  $\rho=3$ . Case B assumes  $\gamma=6.59\%$ ,  $r^{CC}=10\%$ , and  $\rho=1$ . The benchmark assumes  $\gamma=5\%$ ,  $r^{CC}=11.52\%$ , and  $\rho=2$ .

Figure 1: q versus beta and delta

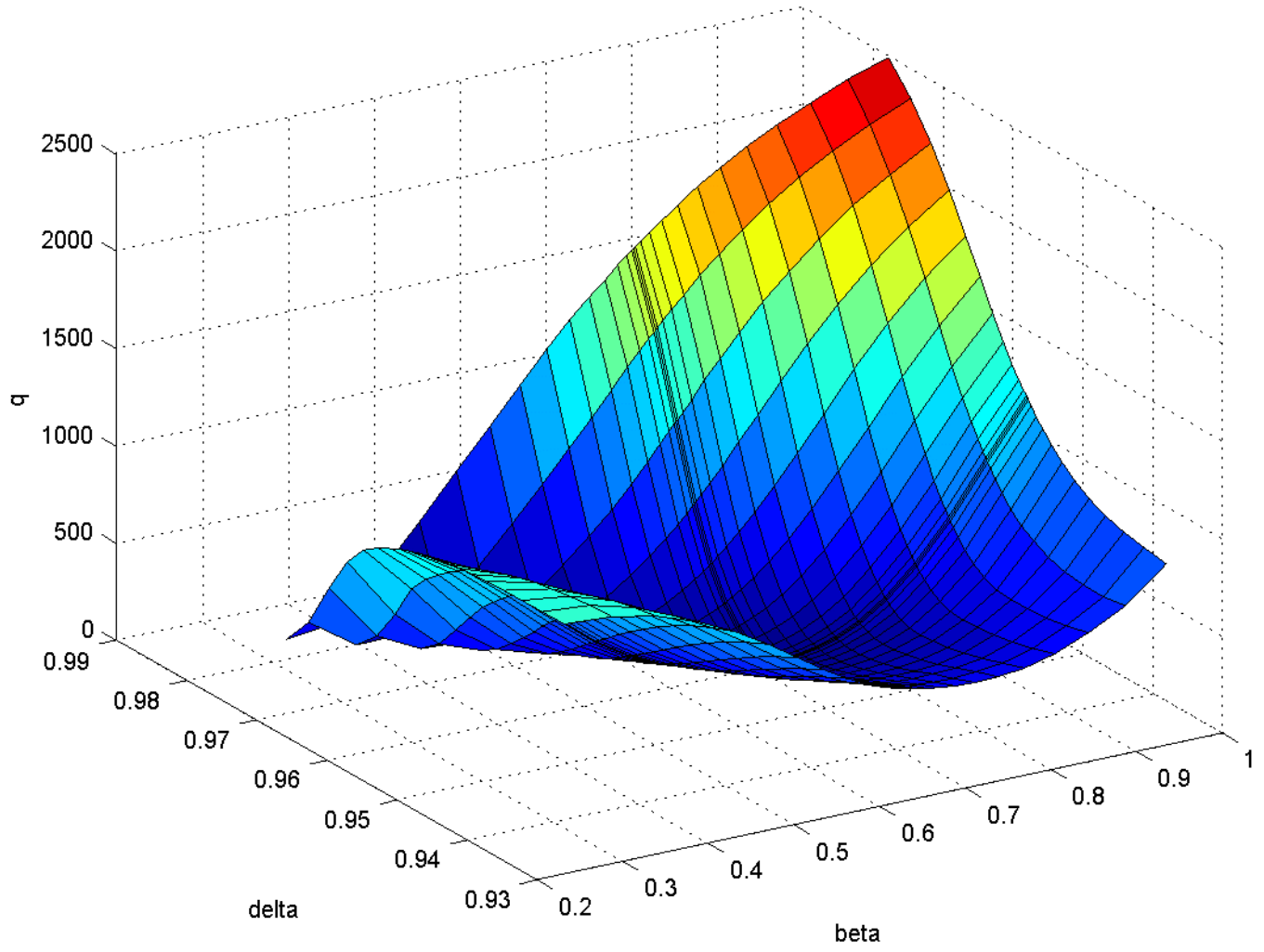


Figure 2: Projected, zoomed q versus beta and delta

