DECISIONS TO REPLACE CONSUMER DURABLES GOODS: AN ECONOMETRIC APPLICATION OF WIENER AND RENEWAL PROCESSES

Viviana **F**ernandez

SERIE ECONOMIA Nº 87

Agosto, 2000

Centro de Economía Aplicada
Departamento de Ingeniería Industrial
Facultad de Ciencias Físicas y Matemáticas
Universidad de Chile

La Serie de Economía agradece el financiamiento de la Hewlett Foundation.

DECISIONS TO REPLACE CONSUMER DURABLES GOODS: AN ECONOMETRIC APPLICATION OF WIENER AND RENEWAL PROCESSES[1]

Viviana Fernandez[2]

Abstract

Current sales of most consumer durable goods are accounted for by replacements. However, only in recent years has the economic literature provided a more rigorous analysis of replacement purchases by incorporating elements of dynamic programming and of the theory of stochastic processes.

This paper is an empirical study of household replacement decisions modeled as an optimal stopping rule. Using data from the 'Residential Energy Consumption Survey' (RECS) of the U.S. Department of Energy, we conclude that demographic variables, operation and replacement costs, and equipment characteristics may affect ownership spells of appliances such as electric heaters and central air conditioners.

JEL classification: C41, D12; Keywords: Replacement, Wiener process, renewal process

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[1] To be Published in *The Review of Economics and Statistics* 82(3), August 2000, pages 1-11
[2] Assistant Professor, CEA, Department of Industrial Engineering at the University of Chile. Postal: Avenida Republica 701, Santiago-Chile, e-mail: vfernand@dii.uchile.cl.

I. Introduction

The high penetration of consumer durable goods has led current sales to consist mostly of replacement purchases. For example, estimates of the U.S. Industry for 1995 show that, on average, about 65 per cent of appliances sales were accounted for by replacements (‘Appliance’, Dana Chase Publications, September, 1995).[1] Nevertheless, the complexity involved in the timing of replacement purchases of consumer durable goods, and the difficulty of finding appropriate data have made both theoretical and empirical research in this area difficult. In particular, because of the longevity of durable goods, replacement purchases are infrequent, and they usually take place prior to irreparable failure. This makes conventional statistical methods inadequate to analyze replacement decisions. In addition, data suitable for empirical analysis is hard to find. The very few studies conducted to date are based on very specific surveys that are not publicly available.

Only in recent years has the economics theory come up with a more rigorous analysis of the consumer’s decision to replace durable goods. In particular, elements of dynamic programming have been combined with the theory of stochastic processes to give rise to structural models that better capture the dynamic elements involved in replacement decisions (e.g., Rust, 1985, 1987; Ye, 1990; Mauer and Ott, 1995). Moreover, in the field of applied econometrics, duration models have proven to be useful tools to analyze the inter-temporal relationship between the probability of replacement, demographic variables, and characteristics of a particular consumer durable (e.g., Antonides, 1990; Gilbert, 1992; Raymond, Beard and Gropper, 1993).

This article presents an empirical application of a structural replacement model in which the optimal replacement policy is found from the optimal stopping technique of applied statistics (e.g., Rust, 1985, 1987, and Bertsekas, 1976). Our discussion focuses on replacement decisions of heating systems and central air-conditioners utilizing a sample of households from the 'Residential Energy Consumption Survey' (RECS) of the U.S. Department of Energy. We model replacement rates as a function of demographic variables, equipment characteristics, and operation costs. Our results indicate that variables such as the age of the head of the household, natural gas availability, fuel price, and equipment characteristics may affect optimal replacement decisions.

The contribution of this article is threefold. First, it presents the first empirical study on the determinants of appliances replacement for the whole United States. Second, using elements of the theory of stochastic processes, duration analysis, and renewal processes, it derives a testable structural specification that has not been previously presented in the existing literature. By contrast, the empirical studies found to date are reduced-form models that do not come from any optimization process. Third, our empirical findings are potentially relevant to policy making and production planning. In particular, the impact of efficiency improvements—translated into a reduction in operation and maintenance costs—on replacement rates can be quantified by our model.

This article is organized as follows. In section II, we briefly discuss previous empirical studies on the consumer's decision to replace durable goods. In section III, we present a theoretical model for replacement decisions (section 3.1), and derive an econometric specification for it using the concept of the 'first passage time' of the theory of stochastic processes (section 3.2). In section IV, we describe the RECS data (section 4.1), derive a replacement model suitable to these data using the model of section III and basic elements of the renewal theory (section 4.2), and present our estimation (section 4.3). Finally, in section V we present our main conclusions and discuss extensions to pursue in future research.

II. Related Literature

The most recent literature on consumer durable goods replacement provides three empirical studies closely related to ours: Antonides (1990), Gilbert (1992), and Raymond, Beard, and Gropper (1993). These three papers analyze consumer durable goods replacement decisions by utilizing duration models. In particular, Antonides looks at data on replacement of washing machines from a survey in the Netherlands from December 1982 through February 1983. He finds that failure rate is increasing with equipment age, household size, and income, and is decreasing with purchase price.[2] Antonides also concludes that including the date of the most recent equipment repair leads to more efficient parameter estimates, and that expected lifetimes corresponding with duration-dependent hazard rates are more plausible that those corresponding with constant hazard rates.

Gilbert analyzes replacement of automobiles in the United States with panel data for the time period August 1978-December 1984. She considers three different hazard functions: replacing with a new vehicle (h_n), replacing with a used vehicle (h_u), and disposing without replacement (h_d). The author concludes that both h_n and h_d are increasing with income, while the opposite holds for h_u. Race, household size, life stage of the household, education, and car odometer reading also play an important role in replacement decisions. Gilbert also considers some macroeconomic variables in her analysis: interest rate, unemployment rate, new car inflation rate, used car inflation rate, auto maintenance rate, and gasoline inflation rate. Nonetheless, in most cases, these variables turn out to be statistically insignificant. For example, higher inflation rates for gasoline—a measure of higher operation costs—increase h_n but do not significantly affect h_u or h_d.

Replacement of main heating equipment for the state of Alabama was studied by Raymond, Beard, and Gropper in 1990. The authors' results indicate that the probability of equipment replacement depends negatively on the age of the head of the household and the availability of natural gas, and positively on equipment age, and higher than expected household energy usage.[3] Other factors included in the hazard specification are income, a poor credit rating dummy variable, a urban location dummy, and house square footage. None of these variables, however, turn out to be statistically significant. As we will see in section 4.2, Raymond, Beard, and Gropper's work is particularly relevant to us because it deals with replacement of one of the appliances we analyze. Indeed, in selecting the relevant variables to be included in our replacement model for home heaters, we consider those economic variables used by the authors as well as other factors that seemed relevant given the national scope of our data set.

Before closing this section, we should point out that the three empirical studies described above use reduced-form duration models that are not derived from any optimization process. By contrast, the testable specification presented in sections III and IV comes from a structural duration model that prescribes an optimal replacement policy based on the optimal stopping technique (e.g., Bertsekas, 1976, or Rust, 1985 and 1987). In particular, given that operation and maintenance cost of a consumer durable good is increasing with time, we hypothesize that its dynamics can be described by a Brownian motion with drift. To date, as far as we are aware of, nobody has tested this assumption empirically.

III. A Structural Duration Model for Replacement of Consumer Durable Goods

Ye (1990) considers a replacement problem where the state of a piece of equipment is described by the instantaneous maintenance and operation cost, which is assumed to increase stochastically with equipment physical deterioration. The author assumes that, at any given time, the household faces two choices: i) continuing to pay rising maintenance and operation cost for the deteriorating durable asset; or ii) paying a fixed cost to purchase a new machine with a guaranteed low initial maintenance and operation cost (secondary markets are neglected). The household's objective function is the expected total discounted cost of maintenance and operation as well as of purchasing, and the optimal replacement rule is defined as a stopping barrier.

The econometric specification derived below is an extension of Ye (1990)'s replacement model. In particular, we resort to the concept of the 'first passage time' for a stochastic process to give Ye's specification an econometric form that can be fitted to our data set.

3.1 The Optimal Replacement Policy Modeled as an Optimal Stopping Barrier

Let us assume that, in order to obtain a fixed level of service from a durable good, households must incur an instantaneous maintenance and operating cost, x(t). This cost may be viewed as an indicator of the state of the piece of equipment. In particular, a higher x indicates a more deteriorated piece of equipment. The evolution of x(t) is described by a Wiener process with constant drift and variance rate parameters, b and σ^2, respectively:

$$dx = bdt + \sigma dw, \qquad (1)$$

where w(t) represents a standard Wiener process. The expected discounted total cost of obtaining the required service from this piece of equipment is given by:

$$K(x) = E\left[\int_0^\infty e^{-rs} x(s)ds \mid x(0) = x\right], \qquad (2)$$

where x(s) evolves according to (1), r is the discount rate, and x(0) represents the state of the piece of equipment at time zero. The latter does not necessarily coincide with the state of a new piece of equipment, x^*. For example, it may be the case that our current piece of equipment was a second-hand purchase or that, when we moved into our current housing unit, equipment was already installed. The replacement cost is implicitly included in the cost function K(x). If replacement occurs at time t, x(t) will be at some state, $\bar{x}$ at time t^{-} and it will jump back to x^* in time t^{+}. By assumption, the installation cost of new equipment is a fixed amount, C, and the scrap value of the previous equipment is zero. When x reaches $\bar{x}$, the upper barrier, the total operating and maintenance cost is given by $K(\bar{x})$. But when that happens, replacement will take place and then:

$$K(\bar{x})=C+K(x^*). \tag{3}$$

Equation (3), which is called the 'value matching condition',[4] gives an upper boundary for the state variable, x. It basically states that the total cost right before the jump, $K(\bar{x})$, equals the total cost after the jump, $K(x^*)$, plus the cost of installing a new piece of equipment, C.

There is no lower bound for x, but K(x) is assumed to be bounded:

$$\lim_{x\to -\infty} \left| K(x) \right| < \infty. \tag{4}$$

Although this assumption is not necessarily realistic because it allows the instantaneous operating and maintenance cost to become negative, it avoids the problem of explosive behavior.

The total cost at time t can be expressed as the sum of the instantaneous operating and maintenance cost over the time interval (t, t+dt) and the continuation region beyond t+dt. That is, $K(x)=xdt+E[K(x+dx)e^{-rdt}]$. Using this fact and Ito's lemma, a second-order differential equation for K(x) can be obtained:

$$\frac{\sigma^2}{2} K''(x)+bK'(x)-rK(x)= -x, \tag{5}$$

where $K''(x)$ and $K'(x)$ stand for the second and first derivatives of K(x) with respect to x, respectively. Equation (5) is an non-homogeneous second-order differential equation with constant coefficients, which can be solved with the two boundary conditions (3) and (4). The use of standard techniques for solving this type of differential equation leads to the following solution for K(x):

$$K(x)=\frac{e^{\lambda x}}{e^{\lambda\bar{x}}-e^{\lambda x^*}}\left(C+\frac{x^*-\bar{x}}{r}\right)+\frac{x}{r}+\frac{b}{r^2}\,, \tag{6}$$

where λ is the positive root of the characteristic equation $(1/2)\sigma^2p^2+bp-r=0$.

The optimal upper barrier, $\bar{x}$, can be found by the 'smooth-pasting condition' which states that, if $\bar{x}$ is optimally chosen, then it should solve $K'(\bar{x})=0$. That is,

$$1+\lambda(rC+x^*-\bar{x})=e^{l\,(x^*-\bar{x})}\,. \tag{7}$$

Given that both functions $f_1(\bar{x})\equiv 1+\lambda(rC+x^*-\bar{x})$ and $f_2(\bar{x})\equiv\exp[\lambda(x^*-\bar{x})]$ are monotonically decreasing in $\bar{x}$ and $f_1'(.)<f_2'(.)$, they can intersect only once. This implies that $\bar{x}$ is unique. Moreover, by the envelope theorem, the optimal barrier is increasing in the rate of deterioration, b, and in the variance, σ^2, and is decreasing in the discount rate when b is sufficiently small (for further details, see Ye (1990)).

3.2. Finding the Probability Density Function of Replacement Times

In the replacement model just described, we look at the trajectory of maintenance and operation cost from installation of a new piece of equipment until an upper barrier—which determines the optimal timing of replacement—is reached. This suggests that we can give the model of section 3.1 an econometric specification by finding the probability density function of the time elapsed until replacement takes place. That is, the first passage time of the instantaneous operation and maintenance costs, x, from x^* to $\bar{x}$.[5]

Consider the Wiener process, {x(t)} of (1) when the upper barrier, $\bar{x}$, is determined by (7). From the theory of stochastic processes (see Cox and Miller, 1965, pp. 219-221), the transition probability density function (p.d.f.) of {x(t)}, p(x, t), must be the solution to the differential equation:

$$\frac{1}{2}s^2\frac{\partial p}{\partial x^2}-b\frac{\partial p}{\partial x}=\frac{\partial p}{\partial t}\quad (x<\bar{x}), \tag{8}$$

subject to the boundary conditions:

$$p(x, 0)=\delta(x-x^*), \tag{9}$$

$$p(\bar{x}, t)=0 \ (t>0), \tag{10}$$

where $\bar{x}$ is defined by the implicit function $H(\bar{x}, b, \sigma^2)\equiv 1+\lambda(rC+x^*-\bar{x})-\exp[\lambda(x^*-\bar{x})]=0$.

Equation (8)—the Kolmogorov forward equation for a Wiener process with drift and variance parameters b and σ^2, respectively—describes the evolution of the p.d.f. p(x, t) over time. Condition (9) states that, at time t=0, p(x, t) is located entirely at the point $x=x^*$, where $\delta(.)$ represents the Dirac delta function.[6] Condition (10) states that p(x, t) must vanish at $x=\bar{x}$ for all t. That is, the process is terminated if $\bar{x}$ is ever reached (i.e., when replacement is optimal).

The p.d.f. of the first passage time for the Wiener process {x(t)}, T, can be obtained once we find the solution for the density p(x, t) from (8), (9) and (10) (see Cox and Miller, pp. 219-225, or Lancaster, 1990, pp. 118-121):

$$g_T(t|b, \sigma, \bar{x}, x^*)=-\frac{d}{dt}\int_{-\infty}^{\bar{x}} p(x, t)dx=\frac{\bar{x}-x^*}{\sigma\sqrt{2\pi t^3}}\exp\left(\frac{-(\bar{x}-x^*-bt)^2}{2\sigma^2 t}\right), \qquad t\geq 0. \tag{11}$$

Its survivor function, $G_T(t)$, is given by:

$$G_T(t|b, \sigma, \bar{x}, x^*)=\Phi\left(\frac{\bar{x}-x^*-bt}{\sigma\sqrt{t}}\right)-\exp\left(\frac{2(\bar{x}-x^*)b}{\sigma^2}\right)\Phi\left(\frac{-(\bar{x}-x^*)-bt}{\sigma\sqrt{t}}\right), \tag{12}$$

where $\Phi(.)$ represents the cumulative distribution function of a standard normal, and $\bar{x}$ is given by the implicit function $H(\bar{x}, b, \sigma^2)=0$.

The moment generating function of T is given by:

$$g_T^*(t|b, \sigma, \bar{x}, x^*)=\exp\left[\frac{\bar{x}-x^*}{\sigma^2}(b-\sqrt{b^2-2t\sigma^2})\right]. \tag{13}$$

Given that by assumption b>0, all the moments of T exist and can be obtained from $E(T^k|b, \sigma, \bar{x}, x^*)=(-1)^k\left(\frac{\partial^k g^*(t)}{\partial t^k}\right)_{t=0}$. In particular, the first two moments of T around the origin are given by:

$$E(T|b, \sigma, \bar{x}, x^*)=\frac{\bar{x}-x^*}{b}, \qquad V(T|b, \sigma, \bar{x}, x^*)=\frac{(\bar{x}-x^*)s^2}{b^3}. \tag{14}$$

The econometric specification for replacement times just derived can be interpreted as a duration model. Indeed, in this case 'failure' represents the event of first touching the upper barrier (i.e., the event of replacing the current piece of equipment), and 'duration' is the time elapsed until this event takes place—if it ever does.[7] In addition, this specification is structural because the upper barrier is not arbitrarily chosen but derived from an optimization problem.

IV Econometric Implementation of the Structural Replacement Model

In this section, we focus on the econometric implementation of the above duration model for the data contained in the 'Residential Energy Consumption Survey' (RECS) of the U.S. Department of Energy. As explained below, this data set only provides information on ages of a set of selected appliances the sampled households currently own. However, no information on ages at which the sampled households have replaced previous equipment is recorded. This implies that the model parameters cannot be estimated directly from the probability density function for replacement times in equation (11). Instead, we need to find the probability density function of equipment ages. As we will see below, the theory of renewal processes provides an asymptotic approximation to such distribution that can be derived from the probability density function of replacement times.

4.1 Description of the Data

Our econometric application focuses on replacement of two home appliances: electric heating equipment and central air-conditioning equipment. Our sample was taken from the ‘Residential Energy Consumption Survey’ (RECS) 1990. The RECS is a national sample survey for the United States that has been conducted triennially by the U.S. Department of Energy since 1984. The universe of the RECS comprises all housing units occupied as a primary residence in the 50 states and District of Columbia. The two major parts by which the RECS is conducted are the Household Survey and the Energy Suppliers Survey. The Household Survey gathers information regarding the housing unit through personal interviews with the selected households. The Energy Suppliers Survey collects data regarding actual energy consumption from household billing records maintained by the fuel suppliers. The data are gathered by questionnaires mailed to all suppliers for the selected households.

The Household Survey covers questions on type of the housing unit, year the housing unit was constructed, space-heating fuels and equipment, water-heating fuels and equipment, air-conditioning fuels and equipment, cooking fuels and equipment, number, type, age, and size of refrigerators, inventory of appliances, and demographic characteristics of the occupants of the housing unit. The information provided by the RECS about the ages of home appliances refers only to equipment the sampled households currently own. No information is provided about the age at which previous equipment has been replaced. Purchase prices of the sampled appliances–proxy for equipment quality—are not recorded either.

Equipment ages are recorded in intervals. To illustrate, consider the question of age of air-conditioning equipment presented below. The questions about the ages of the other sampled appliances are analogous.

[Table 1]

The RECS 1990 contains approximately 5,100 households, out of which 3,398 are homeowners. Tables 2 presents a statistical summary of some variables contained in the RECS 1990 for homeowners—the ones considered in our estimation for being the most likely to replace their home appliances.

[Table 2]

4.2 Modeling the RECS Data: An Application of Renewal Theory

Following the literature of renewal theory (e.g., Cox, 1962, pp. 61-65, or Ross, 1996, pp. 98-114), let us consider a population of components—e.g., durable assets—whose failure-time, Y, is a continuous non-negative random variable with distribution, F, such that F(0)=P{Y=0}<1. The terms components and failure-time can be given different interpretations depending on the particular problem under study. In particular, in our model failure is understood as the event in which a piece of equipment is replaced because its operation and maintenance costs have reached the threshold $\bar{x}$.

Suppose that we start with a new component at time zero. This component fails at time Y_1, and it is immediately replaced by a new component with failure time, Y_2. Consequently, the second failure will occur after a total time of Y_1+Y_2. This process continues in the same fashion, so that the failure-time of the τ-th component used is Y_τ and the τ-th failure takes place at time S_τ:

$$S_\tau = Y_1+...+Y_\tau, \qquad \tau \geq 0,\ S_0=0. \tag{15}$$

If $\{Y_1, Y_2, ...\}$ are independent identically distributed (i.i.d.) non-negative random variables, with common distribution F, this system is called an ordinary renewal process.

From the previous section, we see that equipment failure-time is not observable from the RECS because this only provides information on ages of equipment currently held by the sampled households. Therefore, the concept of backward recurrence-time or time since the last replacement is key to find a model specification for our data. If we let U_t be the age of the component in use at t (see Figure 1, where '*' denotes equipment replacement) then, by virtue of the renewal theorem, as $t \to \infty$ the p.d.f. of U_t approaches

$$f_U(u) = \frac{1 - F(u)}{h}, \qquad u \geq 0, \tag{16}$$

where η represents the expected value of Y_i assuming an ordinary renewal process. This is a proper p.d.f. because is non-negative and it integrates to one. The latter result holds because η equals the integral of the survivor function of Y_i,1-F(.), over $[0,\infty)$.[8]

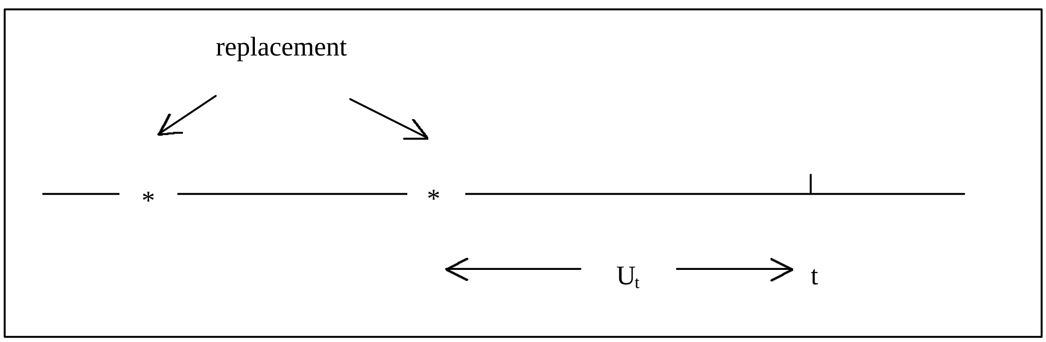

Figure 1. Backward Recurrence-Time, U_t

A renewal process for which Y_1 has a p.d.f. given by (16) is called an equilibrium (or stationary) renewal process. Indeed, if a simple renewal process starts in the past ($t \to -\infty$) remote from the time origin (t=0) and it begins to be observed at t=0, the time of the first failure will be distributed as (16). In other words, an equilibrium renewal process represents an ordinary renewal process in which the system has been running for a long time before it is first observed.

Now consider the replacement model of the previous section, and let T be the time at which the upper barrier $\bar{x}$ is first hit. And, suppose that we have a sequence of hitting or replacement times, $\{T_n, n=1, 2, ...\}$. Under the assumption that equipment lifetimes are independently and identically distributed, the T_n's can be thought of as being the times at which events occur in an ordinary renewal process (Ross, 1970, pp. 186).[9] Hence, an asymptotic approximation to the p.d.f. of equipment age, U, can be obtained from (12), (14), and (16):

$$f_U(u|b, \sigma, \bar{x}, x^*) = \frac{b}{\bar{x}-x^*}\left[\Phi\left(\frac{\bar{x}-x^*-bu}{\sigma\sqrt{u}}\right) - \exp\left(\frac{2(\bar{x}-x^*)b}{\sigma^2}\right)\Phi\left(\frac{-(\bar{x}-x^*)-bu}{\sigma\sqrt{u}}\right)\right], \quad u \geq 0, \tag{17}$$

where $\bar{x}$ is the solution to $H(\bar{x}, b, \sigma^2)=0$.

The cumulative distribution function of equipment age or elapsed duration is given by:[10]

$$F_U(u|b, \sigma, \bar{x}, x^*) = 1 + \frac{-(\bar{x}-x^*)+bu}{\bar{x}-x^*}\Phi\left(\frac{\bar{x}-x^*-bu}{\sigma\sqrt{u}}\right) - \frac{\bar{x}-x^*+bu}{\bar{x}-x^*}\exp\left(\frac{2(\bar{x}-x^*)b}{\sigma^2}\right)\Phi\left(\frac{-(\bar{x}-x^*)-bu}{\sigma\sqrt{u}}\right). \tag{18}$$

In order to incorporate household characteristics into our analysis, we assume that for household 'i' the expression $(\bar{x}-x^*)_i/\sigma_i$ takes the form:

$$\frac{(\bar{x}-x^*)_i}{\sigma_i} = \exp(\beta' \mathbf{z}_i), \tag{19}$$

where β is a vector of parameters, and $\mathbf{z}_i$ represents a vector of household characteristics. This specification ensures that $(\bar{x}-x^*)_i/\sigma_i$ is non-negative. For simplicity, we take the ratio b_i/σ_i to be constant across households, and equal to b/σ. Under these extra assumptions, an asymptotic approximation to the likelihood function of U_i, age of current appliance of household 'i', is given by:

$f_{U_i}(u_i | \tilde{b}, \beta, \mathbf{z}_i)=$

$$\tilde{b}\exp(-\beta'\mathbf{z}_i)\left[\boldsymbol{\Phi}\left(\frac{\exp(\beta'\mathbf{z}_i)-\tilde{b}u_i}{\sqrt{u_i}}\right)-\exp\left(2\tilde{b}\exp(\beta'\mathbf{z}_i)\right)\boldsymbol{\Phi}\left(\frac{-\exp(\beta'\mathbf{z}_i)-\tilde{b}u_i}{\sqrt{u_i}}\right)\right], \qquad (20)$$

where $\tilde{b} \equiv b/\sigma$.

For a sample of n independent observations, the likelihood function of equipment age is given by:

$f_{U_1, U_2, ..., U_n}(u_1, u_2, ..., u_n | \tilde{b}, \beta, \mathbf{z}_1, ..., \mathbf{z}_n)=$

$$\prod_{i=1}^{n}\tilde{b}\exp(-\beta'\mathbf{z}_i)\left[\boldsymbol{\Phi}\left(\frac{\exp(\beta'\mathbf{z}_i)-\tilde{b}u_i}{\sqrt{u_i}}\right)-\exp\left(2\tilde{b}\exp(\beta'\mathbf{z}_i)\right)\boldsymbol{\Phi}\left(\frac{-\exp(\beta'\mathbf{z}_i)-\tilde{b}u_i}{\sqrt{u_i}}\right)\right]. \qquad (21)$$

Estimates of $(\bar{x}-x^*)_i$, b_i, σ_i can be obtained from the smooth-pasting condition for household ‘i’ once we have obtained estimates for $\tilde{b}$ and β:

$$1+\lambda_i(rC_i+(x^*-\bar{x})_i)=e^{l_i(x^*-\bar{x})_i}, \qquad (22)$$

where λ_i represents the positive root of the characteristic equation $½\sigma_i^2p^2+b_ip-r=0$, i=1, .., n.

However, the likelihood function in (21) cannot be fitted to the RECS data because we do not observe the equipment ages. Instead, we are given only the intervals into which the age of each household’s appliance falls. Therefore, the likelihood function to be fitted to the data of Table 1 takes the form:

$$L=\prod_{i=1}^{n}[F_U(u_1 | \beta, \tilde{b}, \mathbf{z}_i)]^{d_1}\prod_{i=1}^{n}\prod_{j=2}^{4}[F_U(u_j | \beta, \tilde{b}, \mathbf{z}_i)-F_U(u_{j-1} | \beta, \tilde{b}, \mathbf{z}_i)]^{d_j}$$

$$.\prod_{i=1}^{n}[1-F_U(u_4 | b, \tilde{b}, \mathbf{z}_i)]^{1-d_1-d_2-d_3-d_4}, \qquad (23)$$

where $F_U(.)$ is given by (18), d_j=1 if age category = j, with j=1, 2, 3, 4 (for instance, d_1=1 if age is less than two years old, and 0 otherwise), and u_1=2, u_2=5, u_3=10, and u_4=20. The u_i’s were obtained by noting that, if the underlying distribution of equipment age is continuous and times are grouped into unit intervals so that the discrete observed part is V=[U], with [U] the 'integer part of U'; then, $P(V=v)=P(u \leq U<u+1)=F_U(u+1)-F_U(u)$. For instance, the probability that a piece of equipment is between two and four years old is given by $P(2\leq V\leq 4)=P(V=2)+P(V=3)+P(V=4)$, which equals $P(2\leq U<3)+P(3\leq U<4)+P(4\leq U<5)=F_U(5)-F_U(2)$.

As before, estimates of $(\bar{x}-x^*)_i$, b_i, σ_i can be obtained from the smooth-pasting condition for household 'i' once we have obtained estimates for $\tilde{b}$ and β.

Before closing this section, it is important to point out that, since we allow equipment to be second-hand, duration is interpreted not as a household decision time, but as equipment lifetime. And, replacement in turn refers not to replacement of several pieces of equipment, but replacement of the same piece of equipment by different households.[11]

4.3 Model Estimation and Results

In our econometric application, we focus on replacement of electric heaters and central air-conditioning equipment. Following Raymond, Beard, and Gropper, we consider only those sampled households who own their homes, and for whom these are their primary residence.[12] The estimation is also carried out conditional on owning an electric heater (main heating equipment) or a central air conditioner,[13] and on knowing the equipment age (i.e., categories 96 and 99 in Table 1 are disregarded). After these adjustments, the sample sizes for electric heaters and central air conditioners are 505 and 1,245, respectively.

Data on replacement costs were obtained from the 'National Construction Estimator' (1990), 'Consumer Reports' (1992), and the RECS (1990). In particular, \$675 and \$992 were taken as average prices of new electric main heating system and central air-conditioning equipment in 1990, respectively. [14] Estimates of annual operation costs of new equipment were obtained as the average annual usage of electricity in 1990 dollars for the ‘less-than-two-year old’ category from the RECS: $326 per year for electric heaters and $249 per year for central air conditioners. Given that we were not able to find good proxies for annual maintenance costs, they were neglected in our calculations. For simplicity, we took the real interest rate to be the annual effective real yield of saving deposits in the United States in 1990: 5.84 per cent (‘Statistical Abstract of the United States’, 1996).

For electric main heaters, we modeled $(\bar{x}-x^{*})_i/\sigma_i$ as an exponential function of a constant term, age of the head of the household (per ten years), nominal monthly income (per $10,000), home square footage (per thousand square feet), dummy variables for urban location (=1 if urban), natural gas availability (=1 if available), and poor credit rating (=1 if poor credit rating),[15] price of electricity (cents per kWh) and heating degree days (per thousands, base = 65 Fahrenheit degrees). Our estimates were obtained by the method of maximum likelihood using the ‘ML’ routine of the statistical package ‘Time Series Processor’ (‘TSP’) 4.4.

[Table 3]

Table 3 shows our estimates. As we see, natural gas availability and age of the head of the household are positively correlated with replacement times—the function $(\bar{x}-x^{*})_i/\sigma_i$ is increasing in these two variables. This implies that the older the head of the household the less likely is that he/she will replace his/her electric heating system, and that when natural gas is available in the household’s neighborhood replacement is also less likely to happen (i.e., the duration between replacements—replacement time—is larger). The same conclusions were reached by Raymond et al. Regarding age of the head of the household, the authors do not attempt to find the reason-why for such a relationship. We think that two plausible explanations are the following. It is possible that preferences of older heads of households change more slowly. Alternatively, older heads of households may have higher implicit discount rates.

Regarding natural gas availability, Raymond et al. think that the positive association of this variable and replacement time may be due to differentials in the lifetimes of electric versus natural gas powered systems. More likely, we think that such a relationship holds because of differentials in the operation costs of gas versus electric powered equipment. Indeed, electricity is much more expensive than natural gas: according to the RECS, in 1990 the average price of natural gas was 0.57 cents/thousand BTU versus 2.19 cents/ thousand BTU for the average price of electricity. Those households without gas service in their neighborhood cannot switch from an electric to a gas powered system and, hence, they are more likely to replace electric equipment.

Table 3 also shows that the variable heating degree days is positively correlated with replacement time. It is likely that this covariate is capturing some equipment characteristics such as quality. In particular, electric equipment for colder regions may be more expensive to replace.[16] Unfortunately, the RECS does not provide any information on heating equipment characteristics other than fuel type. As expected, higher income is associated with a higher probability of replacement. However, this covariate is not statistically relevant at the standard levels of significance. This is also the case for the urban location dummy and the price of electricity. A higher standardized parameter of equipment physical deterioration, b/σ, leads to earlier replacement on one hand, because the higher the standardized drift, the earlier the upper cost threshold for replacing equipment is reached. On the other hand, from Ye's set-up we know that a marginal increase in the (standardized) drift leads to a marginal increase in the upper threshold, which delays replacement. Consequently, the overall impact of an increase of b/σ on duration is ambiguous.

Table 4 shows the summary of our estimates of $\bar{x}$-x^*, b, σ, equipment lifetime, and total discounted costs, K(x), for all households. Estimates of $\bar{x}$-x^*, b, and σ for household 'i' (with i=1, ...,n) are computed from the smooth-pasting condition (22) using the estimates of $(\bar{x}-x^*)_i/\sigma_i$ and b/σ obtained from Table 2. The sample means of $\bar{x}$-x^*, b, and σ are calculated from the estimates of these parameters averaging over all households. The estimated equipment lifetime for each household is computed as the ratio of the estimates of $(\bar{x}-x^*)_i$ and b_i (see equation 14). The sample mean of equipment lifetime is obtained by averaging over the estimates for all households. The estimate of total cost, K(x), for household 'i' is computed from equation (6) by setting $x=x^*$, and plugging in the estimates of $\bar{x}_i$, b_i, and σ_i previously calculated. The parameter x^* is taken to be identical for all households and equal to our estimate of annual operation costs of new equipment. Our estimate of the replacement cost, C, is also taken to be equal for all households. As before, the sample mean of total cost is obtained by averaging over the estimates for all households.

As we can see, on average a piece of electric heating equipment is replaced when $\bar{x}$-x^* is approximately \$99 per year, with a standard deviation of \$16 per year across households. The estimated drift and instantaneous variance parameters have a mean of \$5 and \$6 per year, respectively, with corresponding standard deviations of \$2 and \$1.8 per year. The mean of expected lifetime equals 21 years with a standard deviation of 5 years. This estimate is plausible when compared with figures given by the industry in 1992 (see Table 8 at the end of this section). In addition, the expected total discounted cost is about \$6,538 with a standard deviation of \$255. The figures labeled as 'minimum' and 'maximum' denote the extreme values observed for the estimates of the table across the sampled households.

[Table 4]

For central air-conditioning equipment, we modeled $(\bar{x}\text{-}x^*)_i/\sigma_i$ as an exponential function of a constant term, age of the head of the household (per ten years), nominal monthly income (per \$10,000), home square footage (thousand square feet), air-conditioner cooling capacity (thousand BTU/hour), dummy variables for urban location (=1 if urban) and poor credit rating (=1 if poor credit rating), price of electricity (cents per kWh) and cooling degree days (in thousands, base = 65 Fahrenheit degrees). As above, our estimates were obtained by the method of maximum likelihood using the 'ML' routine of TSP 4.4.

Table 5 shows our estimates for central air-conditioning equipment. Important factors for replacement are age of the head of the household, cooling capacity, and price of electricity. As in our previous estimation, duration (replacement time) is positively associated with increases in age of the head of the household. Increases in cooling capacity also reduce the likelihood of replacement. This result may arise from the fact that replacing with more efficient units is more costly. The variable cooling degree days does not have much of an impact on replacement decisions. This result can be contrasted with our conclusions for the previous appliance. Indeed, the price effect being (possibly) captured now by cooling capacity was picked up by heating degree days in the case of electric main heaters. The price of electricity is statistically significant at the 10 per cent level, and has the expected sign. That is, a higher price of electricity leads to earlier equipment replacement due to its impact on operation costs. Our conclusion about the standardized drift holds in this case as well.

In Table 6 we present the summary of estimates of $\bar{x}\text{-}x^*$, b, σ, equipment lifetime, and total discounted costs for central-air conditioning equipment. These figures can be interpreted in the same fashion as those of Table 4.

[Table 5]

Table 7 shows the goodness-of-fit of our replacement models for electric heaters and central air conditioners. We report the age counts observed in the RECS data and those predicted by the model with their corresponding standard errors. T-ratios are calculated as the difference of the observed count and the fitted count over the standard error of the fitted count for each age category. Assuming that these t-ratios are asymptotically distributed as standard normal, we conclude that there is no statistical difference between the observed and fitted age counts for electric heaters. However, our asymptotic approximation is not that accurate for central air conditioners. As we see, there exists a statistically difference at the 5 per cent level between fitted and observed counts for the age categories ‘2-4 years old’ and ‘5-9 years old’. The model tends to overestimate slightly the number of appliances in these two categories. Hence, assuming identical renewal processes for the sampled central air conditioners may be a worse approximation than it is for electric heaters.

[Tables 6, 7, and 8]

V Conclusions and Topics for Future Research

In this empirical article, we have modeled replacement rates of electric heaters and central air-conditioning equipment as a function of demographic variables, equipment characteristics, and operation costs using the RECS 1990. Our estimation shows that the age of the head of the household appears as an important factor to replacement decisions. In particular, older heads of households tend to replace their heating and cooling equipment later than younger ones because they may have a higher implicit discount rate. Variables such as income, living in an urban area, poor credit rating do not appear as statistically significant at the conventional significance levels. By contrast, operations costs may affect replacement rates. For example, a higher price of electricity may increase the replacement rates of electric equipment. Equipment characteristics seem also relevant. Indeed, a higher cooling capacity may delay replacement of central air conditioners. Finally, climatic differences across regions of the United States may also affect households replacement decisions.

As we see, in general, these findings are intuitive and have economic content. In addition, our estimates of equipment lifetimes are plausible and within the ranges given by the U.S. industry in 1992 for heaters and air-conditioners lifetimes. Hence, we conclude that our analysis may be helpful to policy making and production planning. In particular, one interesting application of our approach is to look at the relationship between energy-efficiency increases (i.e., decreases in operation costs) and replacement rates. In addition, our model can be useful to sales forecasting by constructing series of replacement sales from our estimates of equipment survival probabilities (Fernandez, 1997 and 1998, present some examples on this subject).

Other important point to make is that the econometric specification presented in this article is obtained from a dynamic optimization process. This can be contrasted with the reduced-form models found in the existing literature. However, we are aware that our econometric model has some limitations. First, the drift and variance parameters of the Wiener process describing the evolution of operation and maintenance costs are constant. This assumption could be relaxed to consider the case where these parameters depend on the evolution of maintenance and operation costs over time (e.g., Mauer and Ott, 1995). Second, the replacement models for heating and air-conditioning equipment were estimated separately ignoring the fact that a household's replacement rates may be correlated across different appliances.[17] Then, a more realistic specification should take that into account and postulate a model where the Wiener processes describing the evolution of instantaneous costs of durable goods are correlated.

Finally, the probability density function of equipment age used in our estimation is an asymptotic approximation. Indeed, the renewal theorem is an adequate assumption for relatively old households whose equipment has been replaced several times. The RECS provides the age of each sampled housing unit so, in principle, it is possible to resort to estimation by simulation (e.g., Gourieroux and Monfort, 1996) to find a 'non-asymptotic' distribution of equipment age. In particular, we can simulate a renewal process for each household assuming the date of housing construction as its beginning.[18] So far, we have explored this possibility unsuccessfully due to computational problems. However, we think this is an interesting idea that we could continue to pursue in the future.

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Tables

Table 1. About How Old is Your Central Air-Conditioning Equipment?

	Age Category
1	Less than two years old
2	2-4 years old
3	5-9 years old
4	10-19 years
5	20 years old or older
96	does not know
99	not applicable

Table 2. Some Demographic Indicators for Homeowners in the RECS 1990

Variable	Mean	Standard Deviation
Age of head of household (years)	51.5	16.7
Monthly income ($)	3,024.4	1,959.3
Price of electricity (cents/kWh)	81.9	22.5
Cooling degree days (1)	1,279.5	926.4
Heating degree days (2)	4,292.9	2,034.9
Home area (square feet)	2,211.4	1,166.6
Home age (years)	15.6	12.9
Family size (number)	2.7	1.5

Number of observations=3,398.

Notes (1) Cooling degrees days (CDD) is the number of degrees the average daily temperature is above the base temperature from January 1990 to December 1990. The average daily temperature (ADT) is calculated as the arithmetic average of the highest and lowest temperatures recorded on a given day. That is, CDD = ADT-base temperature (65 Fahrenheit degrees). (2) Heating degrees days (HDD) is the number of degrees the average daily temperature is below the base temperature from January 1990 to December 1990. That is, HDD = base temperature (65 Fahrenheit degrees) –ADT. Both CDD and HDD are recorded in the RECS.

Table 3. Parameter Estimates of the Asymptotic Approximation to the Probability Distribution of Age of Electric Heaters

Covariate	Parameter Estimate	Standard Error (*)	P-value
Constant term	2.362	0.309	0.000
Age head of household (10 years)	0.076	0.022	0.000
Monthly income ($10,000)	-0.217	0.206	0.293
Urban area dummy	-0.092	0.080	0.255
Natural gas availability	0.213	0.069	0.002
Home area (1,000 square .feet)	0.038	0.036	0.288
Heating degree days (1,000)	0.065	0.018	0.000
Price of electricity (cents/kWh)	-0.233e-2	0.193e-2	0.228
Poor credit rating dummy	0.056	0.132	0.670
Standardized parameter of physical deterioration $(b/\sigma)_i$	0.832	0.150	0.000

Log of likelihood function at convergence = -726.6
Number of observations = 505
(*): Standards errors computed from the covariance of analytic first derivatives using the Berndt-Hall-Hall-Hausman (BHHH) algorithm (see Greene, 1996).

Table 4. Summary Statistics for the Estimates of $\bar{x}$ -x^*, b, σ, Lifetime and Total Discounted Cost for Electric Heaters

Estimates	Mean	Standard Deviation	Minimum	Maximum
$\bar{x}$ -x* in $	98.831	16.342	65.592	150.089
b_i in $ per year	5.132	2.084	1.644	13.085
σ_i in $ per year	6.168	1.756	1.976	15.723
Expected lifetime (years)	21.251	5.383	11.47	39.895
$K(x)_i$ in $	6,538.52	255.247	6,010.807	7,321.749

Table 5. Parameter Estimates of the Asymptotic Approximation to the Probability Distribution of Age of Central Air Conditioners

Covariates	Parameter estimate	Standard error (*)	P-value
Constant term	0.969	0.217	0.000
Age head of household (10 years)	0.086	0.017	0.000
Monthly income ($10,000)	0.005	0.142	0.970
Urban area dummy	0.027	0.058	0.638
Home area (1,000 sq. feet)	0.030	0.024	0.203
Cooling capacity (1,000 BTU/hr)	0.112	0.010	0.000
Cooling degree days (1,000)	0.026	0.028	0.353
Price of electricity (cents/kWh)	-0.226e-2	0.135e-2	0.095
Poor credit rating dummy	-0.061	0.166	0.711
Standardized parameter of physical deterioration $(b/\sigma)_i$	0.634	0.048	0.000

Log of likelihood function at convergence = -1,838.79
Number of observations = 1,245
(*): Standards errors computed from the covariance of analytic first derivatives using the Berndt-Hall-Hall-Hausman (BHHH) algorithm (see Greene, 1996).

Table 6. Summary Statistics for the Estimates of $\bar{x}-x^*$, b, σ, Lifetime and Total Discounted Cost for Central Air Conditioners

Estimates	Mean	Standard Deviation	Minimum	Maximum
$(\bar{x}-x^*)_i$ in $	184.824	60.618	120.317	410.941
b_i in $ per year	14.266	12.451	4.394	72.013
σ_i in $ per year	22.509	19.645	6.934	113.618
Expected lifetime (years)	16.729	4.879	5.706	27.379
$K(x)_i$ in $	6,134.688	792.562	5,227.106	8,854.271

Table 7. Actual and Fitted Counts of Electric Heaters and Central Air Conditioners

Age category	Actual Counts Electric Heaters	Fitted Counts Electric Heaters	t-ratio	Actual Counts Central Air Conditioners	Fitted Counts Central Air Conditioners	t-ratio
Less than 2 years old	44	50.00 (7.074)	-0.848	172	158.11 (8.518)	-1.631
2-4 years old	75	74.74 (7.079)	0.036	258	236.60 (8.523)	2.511
5-9 years old	130	123.72 (7.092)	0.886	303	358.56 (8.630)	6.438
10-19 years old	183	183.82 (10.818)	0.076	394	371.01 (12.964)	1.774
20 years old or more	73	72.20 (7.652)	0.105	118	122.01 (9.172)	-0.437

Notes: 1) Standard errors between parenthesis. Each age count, n_k, has a binomial distribution with probability, p_k, k=1, 2, 3, 4, 5, so that its variance is given by $np_k(1-p_k)$, with n the sample size. The p_i for each category was first calculated for each household and then averaged over all households. 2) The t-ratios are calculated as (observed count-fitted count)/standard error fitted count.

Table 8. Life Expectancy (Years) Given by the Industry in 1992

Comfort Conditioning Appliances	Low	High	Average
Warm-Air Electric Furnace	10	20	16
Unitary Air-Conditioners	5	19	12

Source: "A Portrait of the U.S. Appliance Industry 1992." Appliance, September 1992. Dana Chase Publications.

Endnotes

[1] This estimate considers major appliances (e.g., washing machines, refrigerators, water heaters) and comfort conditioning appliances (e.g., heating equipment, central air conditioners).
[2] The author points out that family size and price can be regarded as proxies for frequency of use and product quality, respectively.
[3] This variable is measured as $(u-\hat{u})/\hat{u}$, where u is actual consumption of electricity (average kWh/month), and $\hat{u}$ represents the fitted value from a linear regression of u on household's stock of energy-using durable goods and exogenous factors such as house square footage and housing unit type.

[4] See Dixit and Pyndick (1994) for a comprehensive discussion of these concepts.

[5] Let T_α be the functional denoting the first passage time of a stochastic process X to a level $\alpha \in \mathbb{R}$, set of real numbers. T_α is defined as $T_\alpha=\inf\{t\geq 0; X_t=\alpha\}$ (see Karatzas and Shreve, 1997).

[6] For simplicity, we assume that the household starts with a new piece of equipment whose operation and maintenance cost is given by x^*.

[7] For b>0, $\lim_{t\to\infty} G_T(t|b, \sigma, \bar{x}, x^*)= 0$. For b<0, there is a probability greater than zero that the upper barrier is never reached. Specifically, in that case, $\lim_{t\to\infty} G_T(t|b, \sigma, \bar{x}, x^*)=1-\exp\{2(\bar{x}-x^*)b\}/\sigma^2$.

[8] This can be readily checked by integration by parts.

[9] We implicitly assume that pieces of equipment when new are all identical. This, of course, neglects the possibility of technological change over time. For the two particular cases we will analyze, this is an acceptable assumption

[10] This can be checked by differentiation: $F_U'(u|a, \tilde{b})=f_U(u|a, \tilde{b})$, where $a\equiv(\bar{x} - x^*)/\sigma$ and $\tilde{b}\equiv b/\sigma$.

[11] One referee made me see that I should make this point clear.

[12] It is unlikely that tenants replace equipment of the housing unit they rent.

[13] We do not attempt to model the behavior of potential first-time buyers of electric heaters and central air conditioners. Hence, marginal changes in household's characteristics are meant to quantify changes in the likelihood of replacement only. See Poirier and Ruud (1981)'s article on the appropriateness of endogenous switching.

[14] This corresponds to C, the cost of installing a new piece of equipment.

[15] Those people who received aid in terms of food stamps, unemployment benefits or income from AFDC (Aid to Families with Dependent Children) during the 12 months prior to the conduction of the survey were classified as having a poor credit rating.

[16] We do not have information on equipment purchase prices, which would allow us to control for quality heterogeneity.

[17] Using the technique of generalized residuals (Gourieroux and Monfort, 1987), we found that the residuals of the replacement models for heating and air-conditioning equipment were correlated. That means that the timing of replacing a heating system may be correlated with that of replacing a cooling system for a given household.

[18] We draw data from the cumulative distribution function of equipment lifetime to infer the age of current equipment, using as starting values the parameter estimates from the asymptotic model. In order to infer the equipment current age, we utilize the information on housing unit age provided by the RECS. In particular, we simulate n equipment lifetimes for each household so that their sum is greater or equal than housing unit age. That is, $\sum_{i=1}^{n} l_i \geq a_{hu}$, where l_i and a_{hu} stand for the lifetime of equipment 'i' and housing unit age, respectively. If strict inequality holds, then an approximation to age of current equipment is calculated as the difference between a_{hu} and $\sum_{i=1}^{n-1} l_i$, the sum of the n-1 simulated lifetimes. New parameters estimates can be found by minimizing the difference between the number of appliances observed in each age category and that predicted by the simulations.