CENTRO DE ECONOMÍA APLICADA

INGENIERÍA INDUSTRIAL · UNIVERSIDAD DE CHILE

DOCUMENTOS DE TRABAJO

Serie Economía

Nº 326

ECONOMIC PERFORMANCE, WEALTH DISTRIBUTION AND CREDIT RESTRICTIONS WITH CONTINUOUS INVESTMENT

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Economic Performance, Wealth Distribution and Credit Restrictions with Continuous Investment

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June 10, 2016

Abstract

We study a simple model where entrepreneurs require capital for investment. They have heterogenous wealth and face lending constraints. Agents with little wealth cannot fund their projects, those with intermediate wealth can fund inefficiently sized projects. Only wealthy entrepreneurs attain the efficient firm size. We examine the effects of redistribution. These depend on the aggregate wealth of the economy; in low wealth countries, redistribution reduces credit penetration, efficiency and GDP, while the results are reversed in a wealthy economy. This effect depends on the quality of financial institutions: better institutions reduce the country wealth necessary for redistribution to have positive effects. We add labor as a factor to study the political economy effects of worker protection in bankruptcy and of improvements in credit market legislation.

Keywords: Wealth distribution, firm size, credit market imperfections, bankruptcy

JEL: G30, O15, O16.

*Ronald Fischer acknowledges the support of Fondecyt # 1150063. Fischer and Huerta acknowledge the support of the Instituto Milenio de Sistemas Complejos. The contents of this document, as well as the analysis and conclusions derived from it, are the exclusive responsibility of the authors and does not represent the opinion of the Banco Central de Chile or its Counsellors. We acknowledge useful discussions with Álvaro Aguirre and Gonzalo Castex, and comments in workhops at the Banco Cental de Chile, CEA and FEN of the Universidad de Chile. This is a very revised version of "Economic Performance, Wealth Distribution and Credit Restrictions under variable investment: The open economy", Documento de Trabajo 313, Centro de Economía Aplicada, http://www.dii.uchile.cl/~cea/sitedev/cea/www/download.php?file=documentos_trabajo/ASOCFILE120150504130546.pdf.

1 Introduction

Ever since Kuznets proposed an inverted-U relationship between development and income distribution (Kuznets, 1955), the relation between these two features of a country has been of interest to economists. However, in the context of perfect information used until the mid seventies and the little data on income distribution that existed then it was difficult to develop models of this relationship or to test them. It was only in the late 80's and 90's that a theoretical literature appeared on the topic. The seminal paper of Galor and Zeira (1993) assumed credit restrictions on the acquisition of human capital allowing for a complex relation between the two variables. A rich economy with concentrated wealth would have only a small fraction of educated agents, reducing the growth rate as compared to a country where the same wealth was more equally distributed. In a contemporaneous paper Banerjee and Newman (1993) introduces credit restrictions that determine the choice of activity of agents based on their wealth levels, and the initial distribution of wealth determines the development path of the economy. In this paper we explore another pathway for the connections between between credit restrictions, income distribution and growth. The main relations can be seen in figure 1.

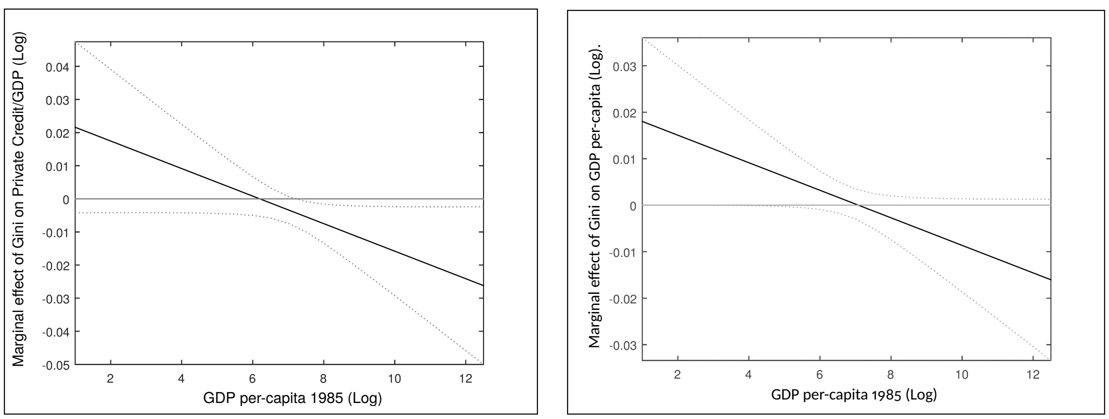

Figure 1: Left: Marginal efect of Gini index on the private credit to GDP ratio as function of 1985 GDP per-capita. Right: Marginal effect of Gini on GDP per-capita as function of GDP per-capita in 1985.

The graph on the right shows that the effect of inequality measured by the Gini index on 2013 GDP per capita depends on the level of 1985 income.[1] The effect of increased inequality is positive for countries that had low 1985 GDP and negative for countries with high initial incomes, a result first observed in Brueckner and Lederman (2015). The graph on the left hand side shows our postulated mechanism to explain this result: for low income countries, more inequality has a positive effect on access to credit, whereas for high income countries, the effect of inequality is to lower access to credit. In the paper, we provide an explanation and linkage between these empirical observations.

These features of the data can be explained by Galor and Zeira (1993) and Banerjee and Newman (1993). We propose an complementary explanation for these features, which also allows us to explain

[1]Both graphs were obtained from the baseline regression used by Brueckner and Lederman (2015, page 6). Data from World Development Indicators (WDI) (World Bank, 2015) for 114 countries in the period 1985-2013. The Strength of Legal Rights Index was included as an additional control variable.

observed conflicts between SMEs and big corporations regarding financial market policies, as well as conflicts between workers in these two types of firms regarding worker protection policies. To do this, we develop a model with non-embodied capital (as in Banerjee and Newman, *op.cit.*), where firms can operate at less than optimal levels due to credit restrictions that limit investment or working capital. In this model we can compare the efficiency of an economy depending on the distribution of wealth, quality of the financial system, and on income levels relative to the efficient firm size, and thus explain Figure 1. We add labor as a second factor of production, with agents choosing between becoming entrepreneurs or workers, and firms hiring workers according to their level of operations. In this setting we can examine conflicts between firms of different sizes, as well as between workers in these firms regarding certain policy measures. There is a conflict between small and medium enterprises (SMEs) and large firms regarding improvements in financial markets, as in Shleifer and Wolfenzon (2002) and Rajan and Ramcharan (2011). Other conflicts arise between both small firm owners and workers in these firms with workers and management in large firms on the issue of worker protection.[2]

More precisely, in this paper we study the effects on the performance of an economy of credit restrictions caused by differences in wealth of potential entrepreneurs. Wealth inequality implies that some potential entrepreneurs have no access to credit, while others receive credit for their projects, but the credit is insufficient to attain the efficient firm size. A third group of wealthier potential entrepreneurs face no restrictions on credit and are able to operate efficient and more profitable firms, while a fourth group not only attains the efficient size but can deposit the excess funds.

Unlike most other models which analyze the effect of financial market imperfections on economic performance, this model incorporates non-linear variable investment decisions (see the literature section for references). This allows us to model SME's as firms which have access to credit, but cannot achieve the efficient firm size due to credit constraints. Since we assume that potential entrepreneurs have profitable projects, the fact that they do not receive credit or it is too little to reach the efficient firm size lowers the efficiency of the economy.

In our model there is a continuum of potential entrepreneurs with heterogenous wealth. Capital is combined in variable proportions with one unit of nontangible and unalienable specific capital owned by the agent (an idea for a profitable project, human capital). Banks are competitive and can obtain funds abroad at a fixed rate. There are market imperfections in lending, which leads to credit rationing. In the first sections of the paper, an entrepreneur that invests is always successful, so there is no risk for lenders, except for the risk of the borrower absconding with the funds. In section 5 we extend the model to include labor and the possibility of failure of a firm.

The effects of financial market improvements will have both an effect on the extensive margin – how many potential entrepreneurs can get loans to start their firms– as well as on the intensive margin –entrepreneurs whose credits are inefficiently sized see a relaxation of this constraint and their projects are closer to the efficient size. We study the impacts of a pure wealth redistribution (i.e., with no change in aggregate wealth) in this economy and show that the effects of redistribution on economic activity

[2]The eminent historian of the Labour Party Ross McKibbin observes that for the last hundred years Labour has tried to enroll small businesses, shop owners and small farmers, with no success. *London Review of Books*, 7 January 2016, p4.

depend on the aggregate wealth of the economy (a result mentioned in Banerjee and Duflo (2003)). In wealthy societies, redistribution tends to improve access to credit and therefore GDP, while in the case of poor societies, the effect is reversed. This effect depends on the quality of financial institutions: better institutions reduce the country wealth necessary for redistribution to have positive effects.

Later we add labor and the possibility of firms failing, in order to examine political economy conflicts between owners and workers in small firms against those in large firms regarding various policy measures. First, large firms will oppose measures to improve the performance of financial markets. These measures have two effects: they increase the size of the loans available to restricted firms and also allow loans to previously excluded entrepreneurs. The increased activity raises demand for workers while reducing their supply (because some workers become entrepreneurs), thus raising wages. In small firms, the effects of increased efficiency – due to a larger plant size– compensate for the higher wage, whereas large firms only observe the negative effect of increased wages because they always operate at the efficient plant size. The opposition to financial improvements by entrepreneurs owning large firms is due to this effect on factor prices, which in turn depend on the distribution of wealth. This argument is similar to the explanation given in the empirical study of Rajan and Ramcharan (2011) for the divergence in financial development in early XX century US agricultural counties depending on the distribution of landed wealth. Several papers have hypothesized that the reason for the opposition to financial market reforms in many countries is due to the reduction in expropriation possibilities by managers or controlling shareholders (Rajan and Zingales, 2003; La Porta et al., 2000). This paper provides a complementary explanation for this fact.

A second source of conflicts is preferential treatment of workers in bankruptcy, a very common form of worker protection in civil law countries (for the case of French bankruptcy law, see Davydenko and Franks (2005)). Increased protection for workers in bankruptcy drives a wedge between workers in large and very small firms, even though on average, workers are better off. The reason is that this form of worker protection has a negative effect on small firms, whose access to credit, size and efficiency fall. In turn, demand for labor and wages fall, and for workers in small firms, the negative effect on worker employment counteracts the increased protection in bankruptcy. Workers in larger firms always benefit from increased protection and exert pressure in its favor. In turn, owners of large firms do not face credit restrictions and the higher expected cost due to worker protection is mitigated by lower wages. Hence large firms are less affected by increases in worker protection, and are less opposed to these measures than small firms which suffer from higher labor costs and less credit. Hence, from a political economy point of view, workers in large firms tend to be aligned with their employers regarding these measures, while employees and owners of small firms are opposed.

Various implications of the model are verified by empirical research, as we have mentioned and show in the literature review below. Other predictions of the model have not been studied empirically (as far as we have been able to ascertain). An example is the result that if the countries are either very wealthy or very poor, the effects of improvements of credit protection parameter on the efficiency of the economy are larger in more unequal countries. The impacts are reversed for economies with intermediate levels of average wealth.

1.1 Literature Review

Unlike many previous theoretical models, which have analyzed the effect of financial market imperfections on the performance of an economy using fixed investment choices see Hoshi et al. (1993), Holmstrom and Tirole (1997) and Repullo and Suarez (2000), this model incorporates continuous investment with non-linear effects on productivity. While there are papers that examine financial market imperfections using investments of this type, this is done in the context of a single firm, as in Burkart and Ellingsen (2004). Tirole (2006) analyzes the case of variable investment with a constant productivity of investment, thus all firms are equally efficient.

Our modelling allows us to define a class of firms which have access to credit, but are inefficient because they cannot achieve the efficient size due to credit constraints. Many studies have documented the high returns to capital in SME's, which suggests that credit constraints reduce the efficiency of these firms. Beck and Demirgüç-Kunt (2008), for example, present evidence of reduced productivity due to credit restrictions in small and medium enterprises (SMEs).[3]

This paper examines the impact of financial market improvements on the performance of the economy by considering the relaxations on the constraints facing SME's. Studies show that movable collateral and centralized registries improve financial markets and that credit bureaus also help to reduce the adverse selection and moral hazard problems facing borrowers. These measures reduce credit market imperfections, see Japelli et al. (2005), Miller, ed (2003). Djankov et al. (2007) show that these improvements increase the ratio of private credit to GDP. The efficiency of insolvency regimes also improves access to credit as shown in the 2014 GFDR. Levine (2005) collects the literature on finance and growth and concludes that financial markets that work better improve growth by easing financial constraints. Fracassi et al. (Forthcoming) show that small business loans for small firms increase their probability of success.

The paper also examines the effects of changes in the wealth distribution on the performance of the economy through the action of the credit constraints on the efficiency of firms.[4] Banerjee (2009) studies the effect of wealth inequality on economic performance. In his model, financial market imperfections reduce the efficiency of firms by not allowing efficient entrepreneurs to start firms and by implicitly subsidizing the prices of factors. Galor and Zeira (1993) study the effect of inequality and credit constraints on the acquisition of human capital, leading to reduced growth. Benabou (1996) examines the effect of inequality on growth acting through capital taxation in response to political pressures. Galor (2009) provides an overview of the relationship between income distribution and development.

Empirically the evidence is varied. Forbes (2000) finds that inequality is positively related to growth, while Barro (1999) finds a U-shaped relationship, i.e., higher inequality retards growth in poor countries,

[3] Many related studies are collected in the Global Financial Development Report (GFDR) 2014 and also Demigürç-Kunt, ed (2014, p.116 ff). See also Beck et al. (2008).

[4] Balmaceda and Fischer (2010) obtained similar results in a model with a fixed investment size. Note that Tirole (2006, p. 474) describes similar effects in the simpler case of two levels of wealth.

and it accelerates growth in richer countries. Easterly (2007) found that inequality causes underdevelopment while Brückner et al. (2014) find that growth reduces inequality. The review article Cunha Neves and Tavares Silva (2014) shows that the contradiction in empirical results is still unresolved. More recently, Brueckner and Lederman (2015) use a panel of 104 countries for the period 1970-2010 to study the effect of inequality on growth. They find that the effect of inequality on GDP depends on the level of development, so higher income inequality has a positive effect on GPD per capita in poor countries. Perotti (1996) examines the nexus between income distribution and political institutions.

Finally, there are papers that point out that the interests of small and large businesses are at odds in policy measures. We have already mentioned Balmaceda and Fischer (2010) and we must add Shleifer and Wolfenzon (2002) and the results that are reviewed in Morck et al. (2005).

Section 2 describes the model. In Section 3 we analyze the comparative statics. Section 4 shows the effects of wealth restribution in different countries. Section 5 extends the model to add labour and bankruptcy and Section 6 concludes.

2 The model

We examine a static model of an open economy with heterogeneous agents and variable-investment decisions. The single period is divided into four stages (see Figure 2). In the first stage, a continuum of agents indexed by $z \in [0, 1]$ are born, each endowed with one unit of inalienable specific capital (an idea, an ability or a project) that cannot be transferred or sold. Each entrepreneur is also born with different amounts of observable wealth or mobile capital K_z. The cumulative wealth distribution among the population of agents is given by $\Gamma(\cdot)$, which has a continuous density and full support.

During the second stage, agents go to the credit market to either deposit their mobile capital or to borrow funds for their projects. In the third stage, agents who receive a loan either invest in a firm or abscond, committing *ex-ante* fraud. As in Burkart and Ellingsen (2004), if an agent absconds with a loan, a fraction $1 - \phi$ of the loan is recovered by the legal system. Therefore, $1 - \phi$ represents the degree of *ex-ante* creditor protection or the *loan recovery rate*. Agents who do not need a loan always invest in their project. Agents who are unable to obtain loans may choose to deposit their wealth in a bank, losing the contribution of their specific capital. In the last stage, deposits are repaid and payoffs are realized.

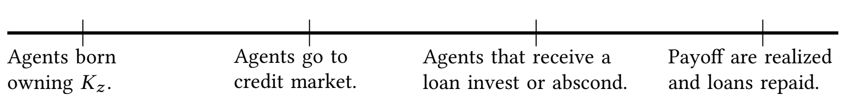

Figure 2: Time line.

There is only one good in this economy, with $f(\cdot)$ its production function such that $f'(\cdot) > 0$, $f''(\cdot) < 0$, $f'(0) = +\infty$ and $f(0) = 0$. Thus the model incorporates the assumption of decreasing returns to scale to capital investment. Agents are assumed to be price takers in the credit and output market. We normalize the price of the single good.

Agents who operate a firm try to maximize their total utility from consumption given by:

$$U(C_z) = U(K_z, D_z) = \begin{cases} f(K_z + D_z) - (1+r)D_z - \theta & \text{if the agent forms a firm} \\ (1+r)K_z & \text{if the agent deposits her wealth with a bank.} \end{cases} \tag{1}$$

Here θ is a sunk startup cost of a firm, D_z is the amount loaned or borrowed by entrepreneur z, $(1+r)K_z$ is the return on wealth in the competitive banking system with $r = \rho$ being the competitive interest rate paid by domestic banks on deposits, with ρ the international rate, because of our assumption of a small open economy.

The profit of a firm is:

$$\pi(K_z + D_z) = f(K_z + D_z) - (1+r)(K_z + D_z) - \theta \tag{2}$$

Using this definition, the utility function can be rewritten as:

$$U(K_z, D_z) = \pi(K_z + D_z) + (1+r)K_z \tag{3}$$

Without credit market imperfections, all agents, no matter how small their initial capital stock, would have access to the credit market. Thus, all entrepreneurs would be able to borrow as much as they wanted at the interest rate r, and therefore, would be able to operate their firms at the profit maximizing capital level K^*:

$$f'(K^*) = 1 + r \tag{4}$$

However, not all entrepreneurs will be able to reach the optimal capital level, because there are market imperfections and loans are limited by moral hazard. The borrower may decide to abscond in order to finance non-verifiable personal consumption. Thus, we assume that investment decisions are non-contractible, and that loans used to finance personal benefits are only repaid to the extent that creditor rights are enforced. Since the legal system is able to recover only a fraction $1-\phi$ of the amounts loaned, an increase in $1-\phi$ is an improvement in *ex-ante* creditor protection or in the loan recovery rate.

In contrast, those entrepreneurs who decide to invest all their borrowed capital plus their initial wealth in a firm, enjoy returns only after repaying their obligations, i.e. output and sales revenue are verifiable and can be pledged to investors. Furthermore, all these agents would like to operate their firms at the optimal capital level K^*, but due to moral hazard and credit market imperfections, some agents will have partial access to credit market and may decide to operate their business using a lower amount than optimal capital stock. Moreover, poorer agents may not have access to the credit market. In other words, there is credit rationing: a rationed borrower may be willing to pay a higher interest rate to lenders in order to get a loan or a higher loan, but investors do not want to grant such a loan, because they cannot trust the borrower.

Therefore, the model characterizes two types of constrained entrepreneurs: those that do not have enough capital stock to access to the credit market and that may decide to deposit their wealth instead of forming a firm (see proposition 1) , and those agents who have partial access to credit market who get a loan that allows them to operate their firms, but at a sub-optimal level. On the other hand, there are two types of unconstrained agents: those who have enough capital stock to get a loan that allows them to operate efficiently, and those richer entrepreneurs who own more than the optimal capital level, who form an efficient firm and decide after to loan their surplus capital. In summary, the model distinguishes between four types of agents.

The demand for credit originates in agents who own less than the optimal capital stock K^*. Note that two types of agents deposit money: agents who do not have access to credit and decide to not form a firm, and by those richer entrepreneurs who own more than the optimal capital level K^*.

Because of competition in the banking market, banks have losses if they lend to agents who commit fraud. In order to assure that fraudulent behavior never occurs in the equilibrium, borrowers must satisfy the following incentive compatibility constraint to receive a loan:

$$f(K_z + D_z) - (1 + r)D_z - \theta \geq \phi D_z \tag{5}$$

Condition (5) assures that the utility received by an agent who receives a loan D_z if she decided to not abscond, is at least the same that she would obtain if she did. In addition, this inequality implies that the marginal return for getting a loan is at least $1 + r + \phi$, i.e. returns for borrowers are between this value and $1 + r$. Additionally, the following breakeven constraint or participation constraint must be satisfied:

$$\pi(K_z, D_z) \geq 0 \tag{6}$$

Condition (6) ensures that the profit of the firm is not negative. Note that this condition is the same as asking that the utility of the entrepreneur for operating a firm is at least what she will obtain from loaning all her capital:

$$U(K_z, D_z) = f(K_z + D_z) - (1 + r)D_z - \theta \geq (1 + r)K_z. \tag{7}$$

2.1 Critical capital levels

In order to study the behavior of entrepreneurs we need to define several regimes that are clearly differentiated by the capital levels of entrepreneurs. There will be critical capital levels such that agents that belong to the intervals between these capital levels behave and are treated similarly by banks. The first critical capital level K_d is the lowest level of capital required to receive a loan, so agents with $K_z < K_d$ are excluded from the capital markets and do not form firms, as we show below.

We define the capital level K_r as the level that allows an entrepreneur to borrow $K^* - K_r$, a loan large enough to set up an efficient firm. Those entrepreneurs with capital $K_z \in [K_d, K_r)$ receive loans, but they are too small to operate efficiently, while those with more capital than K_r attain the efficient firm size. Those entrepreneurs with more than K^* in capital deposit the surplus. It is now possible to define Small and Medium Enterprises (SMEs) endogenously:

Definition 1 *An entrepreneur z forms an SME if $K_z \in [K_d, K_r)$.*

Figure 3 shows the behavior of entrepreneurs z for different levels of K_z.

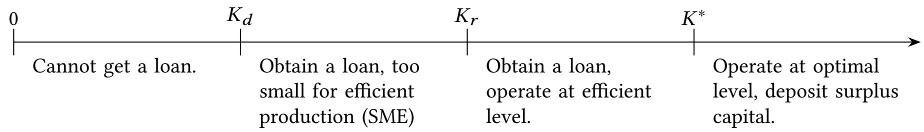

Figure 3: Entrepreneurs' decisions.

We define the following auxiliary function:

$$\psi(K_z, D_z) \equiv f(K_z + D_z) - (1 + r + \phi)D_z - \theta, \tag{8}$$

a concave function which will allow us to define the critical capital levels. We begin by noting that $\psi(K_z, D_z) = 0$ defines the debt D_z such that a entrepreneur with wealth K_z is indifferent between operating a firm and absconding with the loan and committing *ex ante* fraud.

Given the value of K_d, there is an associated debt level that maximizes the utility of the entrepreneur at K_d subject to the auxiliary function being nonnegative, so there is no absconding. This is equivalent to maximizing the auxiliary function (compare equations (8) and (3)) subject to no absconding. In addition, the minimum capital stock K_d is defined as smallest capital such that its owner is able to get a loan (i.e., does not abscond). Therefore, the pair (K_d, D_d) is determined as the solution to the following *minimax* problem:

$$\min_{K \geq 0} \max_{D \geq 0} \psi(K, D) \geq 0$$

To simplify the problem, note that the solution to the problem of minimizing $\psi(K, D)$ satisfies $\psi(K, D) = 0$, which is feasible and negative values violate the incentive compatibility constraint. Thus we can rewrite the *minimax* problem as:

$$\max_{D \geq 0} \psi(K_d, D)$$
$$\text{s.t. } \psi(K_d, D) = 0.$$

In this problem, the objective function is continuous and concave. Since the function $\psi(K_d, \cdot)$ is maximized at D_d and the incentive compatibility constraint is binding, the entrepreneur cannot obtain a larger loan, and a smaller loan violates the incentive compatibility constraint. Taking the Lagrangian leads to the following definition:

Definition 2 *The minimum debt $D_d \geq 0$ and the minimum capital stock $K_d \geq 0$ are defined by the following two conditions*[5]*:*

$$\psi(K_d, D_d) = f(K_d + D_d) - (1 + r + \phi)D_d - \theta = 0 \tag{9}$$

$$\psi_D(K_d, D_d) = f'(K_d + D_d) - (1 + r + \phi) = 0 \tag{10}$$

From (10), the marginal return to investment of the first agent with access to capital is $1 + r + \phi$. Thus, this is the highest return to investment and as K_z increases, the return falls, eventually to $1 + r$.

In order to determine the critical capital level K_r that allows for an efficient plant size, we impose the condition that the maximum debt for an entrepreneur to K_r allows the firm to attain exactly the capital level K^*. Therefore, the incentive compatibility constraint binds, and the maximum debt of an entrepreneur who owns K_r is $D_r \equiv K^* - K_r$.

Definition 3 *The critical capital level K_r of the first agent who is able to invest in the optimal plant size is defined by:*

$$\psi(K_r, K^* - K_r) = f(K^*) - (1 + r + \phi)(K^* - K_r) - \theta = 0 \tag{11}$$

2.2 The optimal choices of entrepreneurs

Agents that do not have access to loans could potentially use their own capital to create a firm. The next result shows that this is not optimal:

Proposition 1 *Agents with $K_z \in [0, K_d]$ do not form firms. They prefer to deposit their capital.*

Proof: We have that $\psi(K_d, D_d) = 0$ from (9) and also that $d\psi(K_d, D)/dD > 0$ for $D < D_d$ by concavity of f and (10). Thus, $\psi(K_d, D) < 0$ for $D < D_d$. Therefore $\psi(K_d, 0) < 0$, which implies that $\psi(K, 0) < 0$ for $K < K_d$ because f is increasing. ∎

The general problem of agents who have access to the credit market ($K_z > K_d$) is to maximize their utility while satisfying the participation and incentive constraints. They solve the problem:

$$\begin{aligned} \max_{I_z} \; & U(K_z, D_z) \\ \text{s.t. } & \psi(K_z, D_z) \geq 0 \\ & \pi(K_z, D_z) \geq 0 \end{aligned} \tag{12}$$

It is easy to solve this problem using Lagrangians, but we obtain more insight by a more intuitive approach. First, note also that agents with wealth above K^* do not want to invest more than K^* in their projects, since the return on the additional investment is lower than $1 + r$, which they would obtain by

[5] We assume that the minimum capital stock to get a loan is positive ($K_d > 0$). If $\theta > 0$, $K_d > 0$.

depositing the excess above K^*. Second, for those agents in the range $[K_r, K^*)$, which can get a loan big enough to invest the efficient amount, any bigger loan means they pay more for the loan than the profits from the additional investment. Similarly, investing less than K^* means that their returns fall by more than the cost of the additional investment. In the case of agents with wealths in the range $[K_d, K_r)$, any additional debt they can achieve generates more profits than its cost, so they get the largest loan they can and are constrained by the incentive constraint.

Given this behavioral pattern, it is convenient to think of firms owned by agents with $K_z > K^*$ as Large firms, with sufficient resources to achieve the optimal plant size and invest their surplus in the credit market. Those entrepreneurs with $K_z \in [K_r, K^*)$ can be identified with Larger Medium sized enterprises, which can produce efficiently. Entrepreneurs in the range $K_z \in [K_d, K_r)$ are not efficient producers and can be associated to small and medium sized enterprises (SMEs). The remaining agents do not form enterprises. Thus we have shown that there are four categories of agents:

1. Agents that do not form firms; with $K_z \in [0, K_d)$
2. Agents with inefficient firms (SMEs), with $K_z \in [K_d, K_r)$
3. Agents that borrow up to the efficient size (Large SMEs): $K_z \in [K_r, K^*]$
4. Agents that form efficient firms and deposit their surplus assets (Large firms): $K_z > K^*$.

The characteristics of the debt function associated to SMEs is described by the following result:

Proposition 2 *The effective debt curve $D_z(K_z)$ of SMEs satisfies the following properties:*

1. $\frac{\partial D_z(K_z)}{\partial K_z} > 1$ *if* $K_z \in (K_d, K_r]$.
2. $D_z(K_z) > K_z$ *if* $K_z \in (K_d, K_r]$.
3. $D_z(K_z)$ *is concave in* K_z.

Proof: See appendix.

The loan size jumps from 0 to a positive value $D_d(K_d) > K_d$ at K_d and it is larger than K_z until K_r, i.e., the entrepreneur with sufficient capital to attain the optimal size through a loan, so all SMEs have leverage ratios larger than 1. Leverage ratios increase as SMEs grow in size and fall for entrepreneurs who attain the efficient size. Note also that the inefficiency and lower productivity of SMEs due to credit limitations is consistent with the literature (see Banerjee (2009) or Demigürç-Kunt, ed (2014), for a recent review of the evidence).

With the results of proposition 2 we can depict the optimal loans as a function of the capital of the entrepreneur as shown in figure 4. By proposition 1, entrepreneurs with $K_z < K_d$ do not form firms and prefer to deposit their small capital. Similarly, agents with $K_z > K^*$ deposit their excess capital.

Associated to this optimal loan function, there is a utility function associated to each level of entrepreneurial capital. Figure 5 shows this. In particular, there is a jump in entrepreneurial utility at K_d, when entrepreneurs can obtain loans and form firms. The slopes for low wealth ($K_z < K_d$) and wealthy entrepreneurs ($K_z > K^*$) are the same and grow at the rate $(1 + r)$.

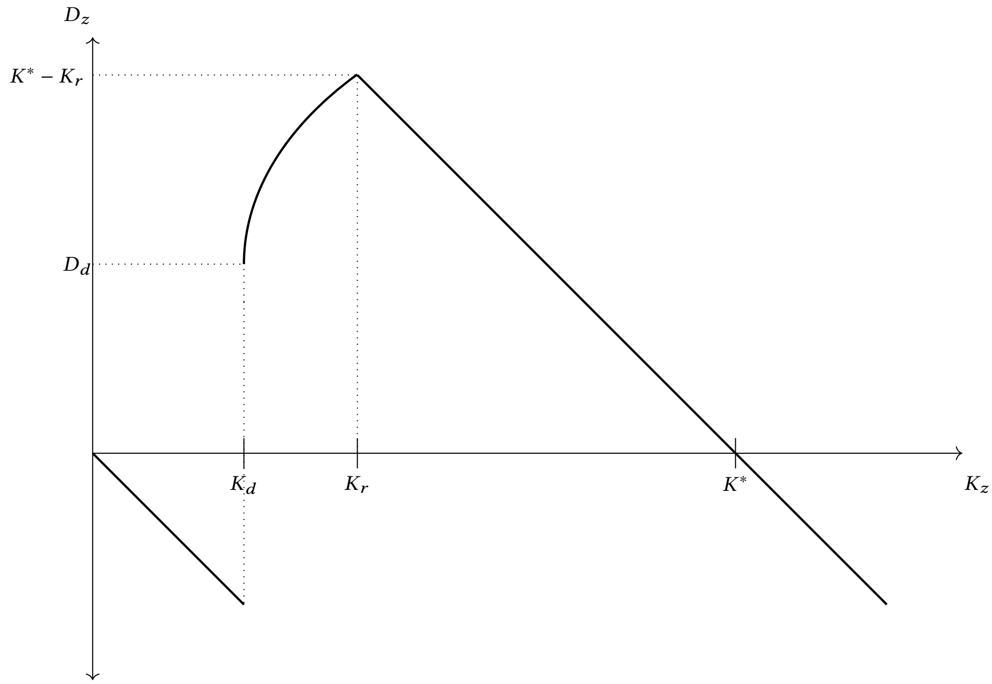

Figure 4: Effective loan curve.

3 Comparative statics

In this section we examine the effects of changes in the fundamentals of the model: improvements in creditor protection ($\phi\downarrow$), reductions in the fixed costs of forming a firm (θ) and changes in the international interest rate. Note that in the small open economy, the adjustments require inflows or outflows of capital.

3.1 Effects of changes in ϕ, θ and r.

We can easily show that:

Lemma 1 *I a small open economy, an improvement in* ex ante *protection ($\phi\downarrow$), a reduction in fixed costs θ or a fall in the interest rate r lead to a reduction in K_d, an increase in the maximum loan D_z, for $K \in [K_d, K_r]$. The improvement in protection or the reduction of fixed costs lead to a fall in K_r.*

Proof: From (9) and (10) we obtain:

$$\frac{\partial K_d}{\partial \phi} = \frac{D_d}{1+r+\phi} > 0$$

$$\frac{\partial K_r}{\partial \phi} = \frac{\overbrace{K^* - K_r}^{D_r}}{1+r+\phi} > 0$$

$$\frac{\partial D_z}{\partial \phi} = \frac{D_z}{f'(K_z + D_z) - (1+r+\phi)} < 0$$

The proof for θ and r is similar. ■

The lemma shows that the minimum capital required to obtain a loan is smaller as the credit recovery rate improves so the mass of agents with access to loans increases.[6] Moreover, the minimum capital K_r needed for a loan that allows the efficient investment K^* is also smaller. Using the lemma it is easy to see that the difference between K_r and K_d shrinks as the credit recovery rate $1-\phi$ improves. This means that the range of capitals that give rise to SMEs is smaller, and conversely, the range of capitals that gives rise to efficient firms is larger.

Another consequence of increased loan recovery rates is that credit constrained SMEs become more efficient, as the loan sizes increase and they get closer to the efficient plant size. Large firms do not benefit from the improved financial system. As an immediate corollary of these results, a reduction in the costs of setting up a firm or an improvement in *ex ante* protection translates into an influx of funds into the economy, as expected. The next step is to examine the effects on the aggregate variables of this economy.

Definition 4 *We define GDP as follows:*

$$GDP = \int_{K_d}^{K_r} [f(K_z + D_z) - (1+r)D_z - \theta]\partial\Gamma(K_z) + \int_{K_r}^{K^*} [f(K^*) - (1+r)(K^* - K_z) - \theta]\,\partial\Gamma(K_z) + (f(K^*) - \theta)(1 - \Gamma(K^*)) \quad (13)$$

Total investment is:

$$I = \int_{K_d}^{K_r} (K_z + D_z)\partial\Gamma(K_z) + K^*(1 - \Gamma(K_r)) \quad (14)$$

Gross Output is:

$$GO = \int_{K_d}^{K_r} f(K_z + D_z)d\Gamma(K_z) + f(K^*)(1 - \Gamma(K_r)) \quad (15)$$

[6]Fabbri and Padula (2004) show empirically, that in Italy, as the quality of legal enforcement of debt contracts improves, the probability of obtaining a loan increases, other things equal.

Credit Penetration (the measure of entrepreneurs that receive loans) is:

$$CP \equiv 1 - \Gamma(K_d), \tag{16}$$

We are led to the following result:

Proposition 3 *In a small open economy, an improvement in ex-ante protection ($\phi\downarrow$) leads to an increase in the following macroeconomic variables: i) Gross Output and GDP,ii) Total investment,iii) Total debt and iv) credit penetration. Similar results apply to reductions in the fixed cost of setting up a firm θ.*

Proof: We prove the result only for GDP, as the others cases are simple. Differentiating GDP defined by (13) in terms of ϕ:

$$\frac{\partial GDP}{\partial \phi} = \int_{K_d}^{K_r} \left(\overbrace{[f'(K_z + D_z) - (1+r)]}^{>0} \frac{\partial D_z}{\partial \phi} \right) \partial\Gamma(K_z) \\ - \overbrace{[f(K_d + D_d) - (1+r)D_d - \phi]}^{>0} \frac{\partial K_d}{\partial \phi} \gamma(K_d) < 0 \tag{17}$$

where we have used the fact that $\frac{\partial D_z}{\partial \phi} < 0$, $\frac{\partial K_d}{\partial \phi} > 0$ and $\partial K^* / \partial \phi = 0$.

■

Observation: The same theorem and results apply to increases in the fixed costs θ.

The effects of an improvement in *ex ante* protection for loans on GDP, investment and total debt have two sources: first, there is an inframarginal effect as those agents that received loans that were not large enough to attain the efficient investment size now receive larger loans and become more efficient producers, and there is a marginal effect, because additional agents receive loans.

The results of this proposition are consistent with the empirical results in La Porta et al. (1997) (and more recent papers, such as Djankov et al. (2007) and La Porta et al. (2008)), who found that improved for creditor rights increased lending in the economy.

We now show that an improvement in the loan recovery rates ($\phi\downarrow$) leads to a better distribution of wealth in the Generalized Lorenz sense, which means that the new distribution is "better" in a well defined sense.

Definition 5 *(Shorrocks, 1983) The Generalized Lorentz (GL) Curve is defined as:*

$$GL(K_z) = \int_0^{K_z} U(K_z, D_z) \partial\Gamma(K_z) \tag{18}$$

The GL curve induces an ordering among distributions of income (or utility in this case) that satisfies reasonable welfare properties. Consider two distributions of income F, G. If the GL curve associated to F lies above and does not cross G, then F GL-dominates G, and this ordering is equivalent to second order stochastic dominance. In turn, this means that if F GL-dominates G, F is preferred by all symmetric utilitarian welfare functionals with increasing and concave utility (Foster and Shorrocks, 1988; Thistle, 1989).

Figure 5 shows the effect of the improvement in loan recovery rates on the utility of the different agents. The primed variables show the new values on the axis, while the dark curves show the displacements from the original (lighter) utility. The next result shows that the Generalized Lorentz curve with improved loan recovery rates lies above the original Lorenz curve, and thus leads to an unequivocal improvement in social welfare.

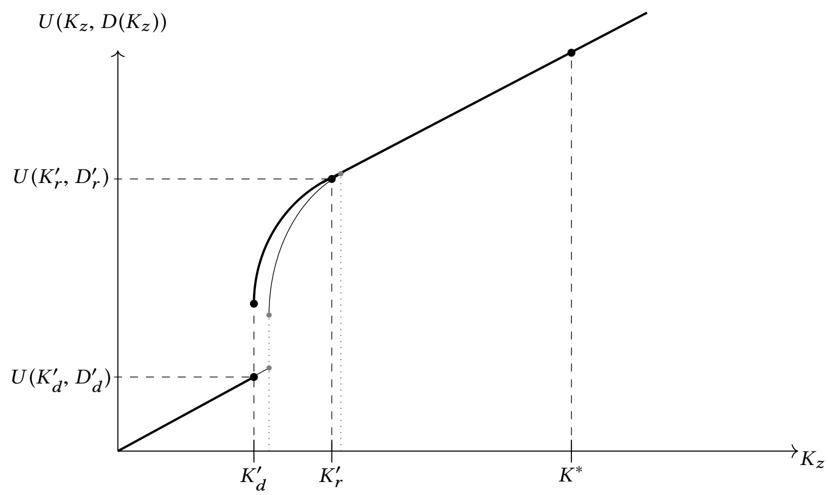

Figure 5: Shift in the utility function due to the change in ϕ.

Proposition 4 *Consider two open economies A and B with the same initial wealth distribution, but which differ in their ex-ante protection parameter ϕ_A and ϕ_B respectively. If $\phi_A < \phi_B$ then $GL(K_z, \phi_A) \geq GL(K_z, \phi_B), \forall K_z$. Thus F is preferred by all symmetric utilitarian welfare functionals with increasing and concave utility.*

Proof: See appendix. ■

4 Changes in the wealth distribution

In this section we analyze the effects of changes in the distribution of wealth on the performance of an economy. In order to isolate the effects due to wealth distribution, independent of any real wealth effects, we consider Mean Preserving Spreads (MPS) of the original wealth distribution. As two distributions, the second being a MPS of the first, have the same mean, any effects we derive are due solely to the increase in wealth inequality in the second distribution. Recall that a MPS of any distribution implies a single-crossing property at the mean of the distribution.

Definition 6 *Consider two distributions $\Gamma_1(K_z)$ and $\Gamma_2(K_z)$ with the same expected value. The distribution $\Gamma_1(K_z)$ is said to be a MPS of the initial wealth distribution $\Gamma_0(K_z)$, if the following conditions are satisfied: $\Gamma_1(K_z) > \Gamma_0(K_z)$ if $K_z < E(K_z)$ and $\Gamma_1(K_z) < \Gamma_0(K_z)$ if $K_z \geq E(K_z)$.*

To simplify the proofs in the appendix, we impose the following condition on the two distributions:[7]

Assumption 1 (Double crossing condition) *The density functions associated to the two distributions cross at only two points.*

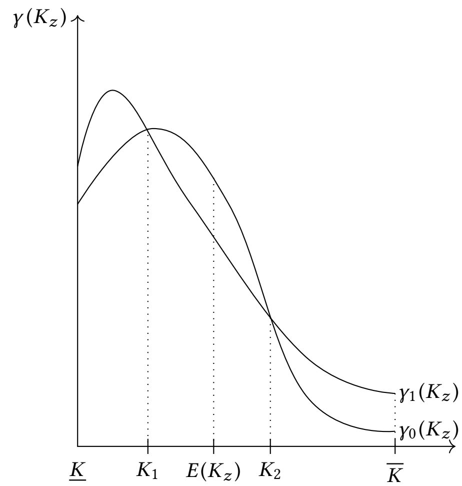

Figure 6: Densities associated to two MPS Distributions that satisfy the double crossing condition.

In figure 6, the densities γ_0, γ_1 are associated to the distributions Γ_0 and Γ_1. They have the same expectation and cross at only two points. At K_1, the positive difference $\Gamma_1(K_z) - \Gamma_0(K_z)$ is maximized, while at K_2 it is minimized (and is negative). The points $K_1, E(K_z), K_2$ define 4 intervals which will be useful in the proofs, which we denote as the first, second, third and fourth intervals. Most common distributions, including the lognormal, satisfy these conditions for appropriate MPS.[8]

To proceed, we define $\Gamma_\lambda = \lambda\Gamma_1 + (1-\lambda)\Gamma_0$, where $\lambda \geq 0$ and Γ_1 is a MPS of Γ_0. As λ increases we obtain a sequence of riskier (i.e., more unequal) distributions that transform Γ_0 continuously into Γ_1. The following result describes the effects of an increase in wealth inequality on various measures of the performance of an economy.

We begin by noting the effects of redistribution on credit penetration. We first refine the concept of credit penetration defined in (16). First, we have the previous definition: the fraction of potential entrepreneurs that have access to credit, i.e., $1 - \Gamma(K_d)$. Second, we can consider the fraction of agents that do not face credit restrictions, i.e., $1 - \Gamma(K_r)$. We obtain the following corollary.

[7]The results continue to hold if the assumption does not hold.

[8]Two distributions defined on the same range and having the same expected value necessarily cross at least twice. For other applications of the double crossing condition, Benassi et al. (2002). Note that two Pareto distributions with the same expectation have different ranges, so the condition is not applicable.

Corollary 1 (Credit penetration)

1. *If* $K_d < (>) E(K_z)$*, then* $1 - \Gamma(K_d)$ *increases (decreases) if inequality decreases (increases).*

2. *If* $K_r < (>) E(K_z)$*, then* $1 - \Gamma(K_d)$ *increases (decreases) if inequality decreases (increases).*

Proof: We do it only for access to credit, since the proof for the mass of agents who face no credit restrictions is analogous. If $K_d > E(K_z)$,

$$\frac{\partial CP1}{\partial \lambda} = \underbrace{(\Gamma_0(K_d) - \Gamma_1(K_d))}_{>0} > 0. \qquad \blacksquare$$

This result leads directly to the following proposition, which is the basis for the link between the two graphs in figure 1.

Proposition 5 *Consider a small open economy such that* $K_d > E(K_z)$ *and with an initial wealth distribution* $\Gamma(K_z)$*. Suppose that* $\Gamma_1(K_z)$ *is a similar economy with a wealth distribution which is a Mean Preserving Spread (MPS) of* $\Gamma_0(K_z)$*. Then the economy with more inequality will have higher Gross Output, Total Investment and GDP. If* $K_r, K^* < E(K_z)$*, the economy with less inequality will have higher Gross Output, Total Investment and GDP.*

Proof: See Appendix. $\blacksquare$

To interpret this proposition, note that under the condition $K_d > E(K_z)$, the last agent to receive a loan has more than the average capital in the economy, i.e., only fairly rich agents have access to the credit market. By concentrating wealth, excluded (but still comparatively wealthy) entrepreneurs may now have the capital to obtain a loan and start a firm. Hence, GDP increases.

Contrariwise, if $K_r \leq E(K_z)$, even relatively poor agents can invest and achieve the efficient plant size. This is a wealthy economy. More inequality will reduce the mass of agents with efficient sized firms, and since the resources are shifted away to entrepreneurs that are already wealthy, this reduces the efficiency of the economy. Hence gross output and total investment decrease.

Our theoretical results are consistent with the empirical paper of Brueckner and Lederman (2015), who show that in poor countries, income inequality has a positive effect on GPD per capita. Our results only hold for poor economies ($K_d > E(K_z)$) or wealthy economies ($K_r \leq E(K_z)$), the results are ambiguous for countries with intermediate wealth. Note from lemma 1 that, for a given societal wealth $E(K_z)$, an improvement in the loan recovery rate (i.e., the level of financial development) shifts K_r to the left, so countries with less wealth can be certain to benefit from redistribution.

Next, we examine the effect of the interaction between the distribution of wealth and the institutional development of the financial market, i.e., the loan recovery rate. Our results depends on conditions on the value of the critical K values in relation to the crossings of the distribution. For example, the condition that K_d lies in the fourth interval means that the average wealth in this economy is very low, so that most potential entrepreneurs do not have access to the credit market. In that case, we can interpret the following proposition as showing that if we consider two equally poor economies, with one of them having more concentrated wealth, the positive effect of an improvement in the loan recovery rate is larger in the economy with a better distribution of wealth.

Proposition 6 *Consider two small open economies A and B such that the wealth distribution is an MPS of that in B, and and which have the same credit protection parameter. If in both countries $K_d > K_2$ (or $K_r < K_1$), then the following macroeconomic variables improve relatively more in the more unequal country A when creditor protection improves ($\phi\downarrow$): i) Investment, ii) Gross output, iii) GDP, iv) Total Debt and v) Credit penetration.*

Proof: We prove one case, given that the others are fundamentally the same:

$$\frac{\partial^2 GDP}{\partial\phi\partial\lambda} = \int_{K_d}^{K_r} \underbrace{[f'(K_z + D_z) - (1+r)]}_{>0} \underbrace{\frac{\partial D_z}{\partial\phi}}_{<0} (\partial\Gamma_1 - \partial\Gamma_0) - U(K_d, D_d) \underbrace{\frac{\partial K_d}{\partial\phi}}_{<0} (\gamma_1(K_d) - \gamma_0(K_d)) \quad (19)$$

where the signs under the partials are given by lemma 1. Now, the conditions imply that both K_d and K_r lie within either the first or fourth interval determined by the crossings of the density functions and the expected value of the distribution, see figure 6. In the two cases we have that $\gamma_1(K_z) - \gamma_0(K_z) > 0, \forall K_z \in [K_d, K_r]$. Thus the integral is negative and the second term is positive. Then $\frac{\partial^2 GDP}{\partial\phi\partial\lambda} < 0$. ■

Noting the two terms in (19) helps to interpret the result. The first term measures the change in the contribution to GDP due to the changed size of loans of agents that already had loans (an intensive effect). The second term adds the contribution of the new agents that have access to loans due to the change in the loan recovery rate (a marginal effect).

A better distribution of wealth implies that a larger mass of agents have wealth that is close to the level required for a loan. The improvement in the loan recovery rate allows them to obtain credit. In the economy with more concentrated wealth, a larger fraction of the benefit from improved recovery rates accrues to entrepreneurs which already had credit and can now obtain larger loans. Since the marginal productivity is higher for agents with less capital (or who just got a loan), the effect of improved financial institutions is larger when there is less inequality.

This result is reversed when both K_d and K_r belong to either the second or third intervals in the range of K_z. In that case both the intensive and the extensive components of the change in GDP are negative in the more unequal economy with the increase in ϕ. Hence it is the economy that benefits most from an improvement in the loan recovery rate.

5 Generalizations: A model with bankruptcy and labor

We are interested in studying the political economy consequences of credit restrictions. To do this, we consider a model where firms require labor and capital for production and firms can fail.[9] In this economy, entrepreneurs unable to obtain a loan, lend their services for a wage. Thus all agents have the potential to create enterprises and reap profits or, if they are unable to obtain loan, to work for hire. In this economy, when there are many firms, there is a large demand for labor and there are few available workers, so salaries tend to be relatively high. Any factor that increases the number of firms will benefit both agents that can now form firms as well as agents who continue to work for hire, but may harm wealthy agents whose labor costs increase and receive no other benefit. This means that in this economy there is a potential for conflicts among different groups of agents.

We assume the production function is $f(K, L)$, with $f_K, f_L > 0$ and strictly concave. The profits of a firm are now: $f(K_z + D_z, L_z) - (1 + r_z)D_z - \theta - wL_z$, where wL_z are the total wages. We make the simplification of assuming that from the point of view of the firm, labor is continuous (i.e., it is behaves like capital). On the other hand, when considering the welfare of workers employed by the different types of firms we assume that workers are attached to specific firms.

To make the model interesting for political economy analysis, we assume there is a probability p of success of the project, and that it goes bankrupt with probability $(1 - p)$. In general some assets will survive bankruptcy, and can be used to pay some of the debt owed to creditors. The value of assets that survive bankruptcy depend on the quality of bankruptcy legislation (or alternatively, on the hardness of the assets in a sector). All loans are equally likely to fail, and there is a competitive banking sector. When a firm fails, a fraction η of total investments $(K_z + D_z)$ is recovered. The fraction η depends on the quality of bankruptcy laws –the time it takes to resolution, for example–, and on the hardness of the sector.[10]

The fraction of investment that is recovered in bankruptcy affects agents differently depending on their wealth. This recovery rate depends on the priority of workers in case of bankruptcy. In some countries wages owed to workers are considered normal debt, with no special seniority. In other countries, wages have priority, so workers are paid first from the assets that survive bankruptcy. We simplify the analysis by assuming that the priority obligation to workers can be written as $\Theta wL_z, \Theta \in [0, 1]$, and that any remaining debt to workers is cancelled in bankruptcy. The parameter Θ measures the extent of priority for workers in bankruptcy. Taking all of this into account, we obtain the following expected utility for an entrepreneur with wealth K_z, where we assume that the entrepreneur receives nothing in bankruptcy.

$$U_e(z) = p[f(K_z + D_z, L_z) - (1 + r_z)D_z - \theta - wL_z] \tag{20}$$

[9] Some of the political economy results obtained here can be obtained in a closed economy model with one factor of production, see Shleifer and Wolfenzon (2002); Balmaceda and Fischer (2010). However, most economies nowadays are not closed to financial flows.

[10] See Braun (2005), i.e., sectors in which recovery rates are higher because of sector characteristics, as in property versus intangibles.

Note that now the interest rate is differentiated, and depends on the agent. The reason is that in case of failure, the return to the bank depends on the assets that can be recovered. Apart from parameters that determine the quality of the financial system (ϕ and η) the return in case of failure depends on the original capital invested in the project and on the fraction of wages which have priority over bank debt. The profits of a representative bank are:

$$U_b = p(1 + r_z)D_z + \max\{(1 - p)(\eta(K_z + D_z) - \Theta w L_z), 0\} - (1 + \rho)D_z$$

The zero profit condition on banks determines the interest rate charged to each entrepreneur z:

$$(1 + r_z) = \frac{1 + \rho}{p} - \frac{1}{pD_z} \max\{(1 - p)(\eta(K_z + D_z) - \Theta w L_z), 0\}$$

which we replace to in (20) to obtain:

$$U_e(z) = p[f(K_z + D_z, L_z) - \theta - wL_z] + (1 - p)[\eta(K_z + D_z) - \Theta w L_z] - (1 + \rho)D_z$$

In order to proceed, we define, analogously to the case with no labor, the following auxiliary function:

$$\Psi(K_z, D_z) \equiv p[f(K_z + D_z, L_z) - \theta - wL_z] + (1 - p)[\eta(K_z + D_z) - \Theta w L_z] - (1 + \rho + \phi)D_z \tag{21}$$

Thus, conditions which determine K_d, D_d, L_d are:

$$\begin{aligned} \Psi(K_d, D_d, L_d) &= 0 \\ \Psi_D(K_d, D_d, L_d) &= 0 \\ \partial U_e(K_d, D_d, L_d)/\partial L_d &= 0 \end{aligned} \tag{22}$$

An entrepreneur with access to the credit market solves:

$$\begin{aligned} &\max_{D_z, L_z} U_e(K_z, D_z, L_z) \\ &\text{s.t.} \quad \Psi(K_z, D_z, L_z) \geq 0 \\ &\qquad\; U_e(K_z, L_z, D_z) \geq U_w(K_z, l_z) \end{aligned} \tag{23}$$

where U_w is the agent's utility when working for hire. To simplify the analysis, in what follows we assume that $\eta(K_z + D_z) - \Theta w L_z > 0$, i.e., that after bankruptcy there always remains enough left over to pay workers what they are owed.[11] In that case, it is easy, but cumbersome to show that there is a range $[K_d, K_r]$ where entrepreneurs are credit constrained. In that range firms have more debt than own capital, and leverage increases with the amount of capital, see lemma 4 in the Appendix. Though firms

[11] Otherwise, in their employment decisions, workers would have to evaluate the fraction of Θw that she would obtain in case of the failure of the project. This fraction depends on equilibrium wages and on the investment of the firm, making it a very difficult problem.

are credit constrained, they hire labor efficiently, given their investment. The effect of non-optimal investment is that there is less hiring. It is simple to show that for all firms, labor demand of a firm increases with the capital stock of the entrepreneur in the range K_d, K_r. The reason is that in the credit constrained range, total investment $K_z + D_z$ increases with K_z, thus raising the marginal productivity of labor. All entrepreneurs which are credit unconstrained have the same plant size K^* and hire the same amount of labor. The aggregate labor demand can be defined as:

$$\mathcal{D}(w) \equiv \int_{K_d}^{K_r} L_z(w)\partial\Gamma(K_z) + L^*(w)(1 - \Gamma(K_r)) \tag{24}$$

5.1 Labor supply

We use a simple model of individual labor supply to generate a supply function of labor. We assume if an agent z chooses to be a worker, the cost of providing an amount l_z of labor is $\varsigma(l_z)$, where $\varsigma' > 0, \varsigma'' > 0, \varsigma''' \geq 0$ with $\varsigma(0) = 0, \varsigma(+\infty) = \infty$.[12] To simplify, we assume that agents are expected utility maximizers and can either work or become entrepreneurs. The utility of a worker that provides l_z units of labor to a firm (and deposits his capital K_z) is:

$$U_w = (1 + \rho)K_z + pwl_z + (1 - p)\Theta wl_z - \varsigma(l_z)$$

The solution to this concave problem is an individual labor supply that depends on wages w, but not on the capital stock K_z owned by the agent. Thus all workers offer the same amount of labor $l(w)$.

For an agent to become an entrepreneur, $U_e(K_z, D_z, L_z) \geq U_w(K_z, l_z)$. Without additional conditions on ς, we cannot show that this inequality is always strict. There are then two possible cases. In the first case, society is divided among those that have sufficient wealth to obtain a loan and start their firm, and those that have to work, because they cannot develop their project. Having enough wealth for a loan implies a discrete increase in wellbeing relative to an agent who has to sell his labor. In the second case, some workers have access to loans but choose to work, i.e., there is no jump in utility when an agent obtains a loan. While we work with the strict inequality case, because the analysis is simpler, our results continue to hold in the second case.

We now define labor supply as:

$$\mathcal{S}(w) \equiv l(w) \cdot \Gamma(K_d)$$

The supply is downwards sloping in wages, while labor demand is decreasing in wages (for the proof, see lemma 3 in the appendix). There is a labor market equilibrium that determines a wage such that:

$$\int_{K_d}^{K_r} L_z(w)\partial\Gamma(K_z) + L^*(w)(1 - \Gamma(K_r)) = l(w) \cdot \Gamma(K_d) \tag{25}$$

Given the labor market equilibrium, firm z demand for labor satisfies: $pf_L(K_z+D_z, L_z) = pw+(1-p)\Theta w$.

[12] Imposing nonnegativity conditions on the third derivative is common in these types of problems, see for instance Laffont and Tirole (1993, Sec. 2.3).

5.2 Results with labor

First, we examine the comparative statics effects of changes in the parameters of the model on wages and on the minimum wealth required for a loan K_d. The results resemble those obtained in the model without labor:

Proposition 7 *The equilibrium wage w rises and the minimum capital level K_d decreases after:*

1. *An improvement in* ex-ante *creditor protection* $1-\phi$.
2. *An increase in* ex-post *protection* η.
3. *An decrease in worker protection* Θ.

Proof: See Appendix.

Thus we have that, as the cost of giving workers priority in bankruptcy increases (given by the parameter Θ), the minimum capital level increases, lowering the access to credit of smaller firms. Similarly, if entrepreneurs can appropriate a larger proportion of residual value after the failure of the project, smaller firms also suffer from less access to lending. In addition, increasing the preference of workers in bankruptcy lowers wages. This effect results from the combination of two channels . First, because raising the payment Θ to workers in case of bankruptcy shifts K_d to the right. Some potential employers cannot obtain credit to start a firm and must become workers, thus increasing the supply of labor. A second effect occurs because restricted entrepreneurs ($K_z \in [K_d, K_r)$) obtain smaller loans and therefore hire fewer workers, again lowering wages. Hence total hours supplied also fall and wages decline. Similar effects on wages and labor demand occur if there is less *ex ante* ($\phi\uparrow$) credit protection or worse ($\eta\downarrow$) bankruptcy procedures.

It is easy to show that an increase in worker protection leads to a reduction in the optimal size of a firm. As the generalized cost of labor rises, the firm uses less labor and invests less.

Lemma 2 *If Θ increases, D_z and L_z decrease for all z such that $K_z > K_d$.*

Proof: See Appendix.

5.3 Political economy conflicts

In this section we examine the different interests of the various types of entrepreneurs and of workers towards measures that increase worker protection in bankruptcy and the legal rights of creditors. First we show that improvements in worker protection create a wedge between classes of entrepreneurs, because the adverse effect on smaller firms is relatively larger than the effect on bigger firms.

Proposition 8 *If worker protection Θ increases then:*

1. *All firms experience a decrease in their profits, but there exists a threshold* $K_\Theta \in (K_d, K_r)$ *such that the negative impact of the change on firms with* $K_z \in (K_d, K_\Theta]$ *is relatively larger than on firms with* $K_z \geq K_r$.

2. *On average, workers are better off.*

Proof: See Appendix.

An increase in protection for workers in case of the failure of a firm (Θ) reduces the welfare of those entrepreneurs now unable to form firms, and who go back to being workers. Secondly, firms that are financially constrained will obtain smaller loans, reducing their productive efficiency and their labor demand. Firms that are well capitalized and that continue to use the efficient level of capital after the increase in Θ are less affected, because they do not suffer the effects of the stricter lending constraint. Thus they will tend to be less opposed to proposals to raise Θ, creating a wedge with the interests of SMEs.

There is a further conflict associated to the rise in Θ. While the average worker is better off, the representative worker in a smaller firms is made worse off with the increase in protection in bankruptcy, while those in larger firms are unambiguously better off. To prove this result define the generalized wage $\omega \equiv pw + (1-p)\Theta w$ and the number (mass) of workers in firm z as $n_z(\omega) \equiv \frac{L_z(\omega)}{l(\omega)}$. The total welfare of the workers laboring in firm z is:

$$\tilde{U}_w(L_z) = n_z \cdot (pwl + (1-p)\Theta wl - \varsigma(l)) = pwL_z + (1-p)\Theta wL_z - n_z\varsigma(l) \tag{26}$$

Proposition 9 *Assume* $f_{LL,K} < 0$ *and* $f_{KL,K} < 0$. *If worker protection measured by* Θ *increases, there exists a threshold* $\tilde{K}_\Theta \in (K_d, K_r)$ *such that:*

1. *The representative worker in a firm with* $K_z \in (K_d, \tilde{K}_\Theta)$ *is worse off.*

2. *The representative worker in a firm with* $K_z > \tilde{K}_\Theta$ *is better off.*

Proof: See appendix.

The increase in Θ, which supposedly protects workers in case of the failure of a firm, has ambiguous effects on their welfare, which depend on the type of firm in which they work. While "on average" workers are better off (since total compensation rises), not all workers are better off. Workers in smaller firms are worse off, in some cases the firms close down because the entrepreneur does not obtain financing under the new conditions. SME's that survive have to shrink, because their obtain smaller loans and hire less labor, so workers in those firms can also be made worse off. $\tilde{K}_\Theta$ is the threshold level of capital such that such that the representative worker in SMEs with less capital than the threshold level are worse off with the change in Θ. The result also shows that the representative worker in larger firms is always better off. Thus the model predicts that there will be a conflict of interests regarding Θ between workers in small and those in larger firms.

Similarly, improvements in financial markets, create a wedge between types of entrepreneurs. Credit constrained entrepreneurs are better off, some because they now have access to credit that was denied before, and others who were credit-constrained, because they have access to more credit, leading to more efficient firms. On the other hand, non-constrained entrepreneurs are worse off, because they were unconstrained and operating efficiently before, but now have to pay higher wages to workers due to increased demand for labor by constrained firms.

Proposition 10 *If ex-ante creditor protection* $1 - \phi$ *improves then:*

1. *There exists a threshold* $K_\phi \in (K_d, K_r)$ *such that all firms with* $K_z \in (K_d, K_\phi)$ *are better off, while firms with* $K_z \geq K_r$ *are worse off.*

2. *On average, workers are better off.*

Proof: See appendix.

The opposition to improvements in finance due to, among others, the effects on factor prices (specially labor) is examined by Rajan and Ramcharan (2011, p. 1897) in their study of farming in early twentieth farming in the US. Our result suggests that there might be differences in the position of large and small firms with respect to measures that promote legal improvements protecting creditors. Similarly Rajan and Zingales (2003) in a study of financial development in the twentieth century, propose a factor price explanation for the opposition of incumbents to financial development, see also La Porta et al. (2000). In those models, incumbent –large– firms oppose financial development because it creates competition and raises the cost of finance in their closed economies. In our open economy case, the channel for the conflict among entrepreneurs is through labor.[13]

6 Conclusions

This paper has examined an open economy model with credit constrained entrepreneurs, differentiated by their initial level of wealth. Agents with little initial wealth cannot obtain loans to develop their projects, while those with more wealth can get loans to create SMEs or large enterprises. SMEs are credit constrained and inefficient.

We examine the effect of increased efficiency of financial markets –understood as improvements in the loan recovery rate– on the various types of firms, and we examine the effects of increases in wealth inequality on the performance of the economy. These effects depend on the aggregate wealth of the economy or, alternatively, on the initial quality of the financial system. We find that for countries that have little capital or that have deficient financial systems, a regressive redistribution of resources could improve investment and gross and net output. On the other hand, in wealthy economies or with well functioning financial systems, redistribution will increase investment and gross output.

The model is adapted to incorporate continuous labor as an independent factor of production. We also add the possibility of the failure of the firm, leading to bankruptcy. The main results continue

[13]Shleifer and Wolfenzon (2002) shows that factor price-based opposition to financial market improvements is smaller in open economies.

to hold, and additional effects appear. When labor is present, the interests of SMEs and large firms diverge with respect to improvements in the financial markets, an effect similar to the one studied in Rajan and Ramcharan (2011). Moreover, the interests of workers in small and large firms also diverge with respect to measures to increase labor protection when the firm fails. An interesting extension of this paper would be to examine whether these results can be tested empirically.

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A Appendix: Results in the model without labor

Proposition 2 *The effective debt curve $D_z(K_z)$ satisfies the following properties:*[14]

1. $\frac{\partial D_z(K_z)}{\partial K_z} > 1$ *if* $K_z \in (K_d, K_r]$.

2. $D_z(K_z) > K_z$ *if* $K_z \in (K_d, K_r]$.

3. $D_z(K_z)$ *is concave in* K_z.

Proof:
Differentiation of equation (8) leads to:

$$\frac{\partial \psi(K_z, D_z)}{\partial K_z} + \frac{\partial \psi(K_z, D_z)}{\partial K_z}\frac{\partial D_z}{\partial K_z} = 0$$

$$\Rightarrow \frac{\partial D_z}{\partial K_z} = -\frac{\psi_K}{\psi_D} \tag{27}$$

Using equations (9) and (10), which define K_d and D_d we obtain that:

$$\psi_D(K_d, D_d) = f'(K_d + D_d) - (1 + r + \phi) = 0. \tag{28}$$

Moreover, from the definition of ψ in (8) we have

$$\psi_K(K_z, D_z) = f'(K_z + D_z) > 0 \tag{29}$$

Note that if $K_z > K_d$, $f'(K_z + D_d) < 1 + r + \phi$ (because $f''(\cdot) < 0$). Thus $\psi_D < 0$ to the right of K_d.[15] Using these facts in (27) we conclude that:

$$\frac{\partial D_z}{\partial K_z} = -\frac{f'(K_z + D_z)}{f'(K_z + D_z) - (1 + r + \phi)} > 1$$

For the second item, to show that $D_z(K_z) > 0$, note that differentiating $\psi(K_z, D_z) = 0$ at $K_z = K_d$ and assuming $D_z = 0$ leads to $f'(K_d, D_z) = 0$. On the other hand, if we use (10), one of the two equations that define (K_d, D_d), we have that $f'(K_d) = 1 + r + \phi$, a contradiction. Thus $D_d > 0$.

To show that $D_z(K_z) > K_z \in [K_d, K_r]$, note that we can rewrite the incentive compatibility constraint (5) and the participation constraint respectively as:

$$U(K_z, D_z(K_z)) = \phi D_z$$
$$U(K_z, D_z(K_z)) \geq (1 + r) D_z$$

Comparing, we see that $D_z(K_z) \geq [(1 + r)/\phi]\, K_z$. The result $D_z(K_z) > K_z$ follows, since $0 \leq \phi < 1$.

[14]We do not write the dependence of D_z on K_z when clear.
[15]Recall that this derivative can only be defined to the right of K_d, because there is a discontinuity to the left of K_d.

For the last item, note that differentiating (27) with respect to K_z leads to: $\frac{\partial^2 D_z}{\partial K_z^2} = \frac{f''(K_z+D_z)(1+r+\phi)}{(f'(K_z+D_z)-(1+r+\phi))^2} < 0$. ■

Proposition 5 *Consider two open economies A and B with the same initial wealth distribution, but which differ in their ex-ante protection parameter ϕ_A and ϕ_B respectively. If $\phi_A < \phi_B$ then $GL(K_z, \phi_A) \geq GL(K_z, \phi_B), \forall K_z$.*

Proof:

For $K_z < K_d$ it is straightforward to see that $\frac{\partial GL(K_z)}{\partial \phi} = 0$. Similarly utility does not change with ϕ if $K_z \geq K^*$ (because these agents do not require loans) so $\frac{\partial GL(K_z)}{\partial \phi} = 0$ in that range.

If $K_z \in [K_d, K_r)$ then:

$$\frac{\partial GL(K_z)}{\partial \phi} = \underbrace{[(1+r)K_d - U(K_d, D_d)]}_{<0} \gamma(K_d) \underbrace{\frac{\partial K_d}{\partial \phi}}_{>0} + \int_{K_d}^{K_z} \underbrace{(f'(K_z + D_z) - (1+r))\frac{\partial D_z}{\partial \phi}}_{<0} \partial \Gamma(K_z) < 0$$

Similarly, if $K_z \in [K_r, K^*)$ we obtain that: $\frac{\partial GL(K_z)}{\partial \phi} < 0$. ■

Proposition 6 *Consider a small open economy such that $K_d > E(K_z)$ and with an initial wealth distribution $\Gamma(K_z)$. Suppose that $\Gamma_1(K_z)$ is a similar economy with a wealth distribution which is a Mean Preserving Spread (MPS) of $\Gamma_0(K_z)$. Then the economy with more inequality will have higher Gross Output, Total Investment and GDP. If $K_r, K^* < E(K_z)$, the economy with less inequality will have higher Gross Output, Total Investment and GDP.*

Proof: We only do the case of GDP, proofs are similar for Total Investment and Gross Output. Differentiating GDP with respect to λ and evaluating at $\lambda = 0$:

$$\frac{\partial GO}{\partial \lambda} = \int_{K_d}^{K_r} f(K_z + D_z)\,(\partial\Gamma_1 - \partial\Gamma_0) - f(K^*)(\Gamma_1(K_r) - \Gamma_0(K_r)) \tag{30}$$

$$\frac{\partial I}{\partial \lambda} = \int_{K_d}^{K_r} (K_z + D_z)\,(\partial\Gamma_1 - \partial\Gamma_0) - K^*(\Gamma_1(K_r) - \Gamma_0(K_r)) \tag{31}$$

$$\frac{\partial GDP}{\partial \lambda} = \int_{K_d}^{K_r} U(K_z, D_z)(\partial\Gamma_1 - \partial\Gamma_0) + \int_{K_r}^{K^*} U(K_z, K^* - K_z)(\partial\Gamma_1 - \partial\Gamma_0) - U(K^*, 0)(\Gamma_1(K^*) - \Gamma_0(K^*)) \tag{32}$$

The proof consist on finding upper or lower bounds for the different terms of this expression and simplifying, using the properties of the differences of the distributions and densities in the different intervals. We consider the relevant arrangements and find appropriate bounds.

Case 1: $K_d, K_r, K^* \in [E(K_z), K_2]$. We have that $\gamma_1(K_z) - \gamma_0(K_z) < 0, \forall K_z \in [K_d, K^*]$. Hence $(\partial\Gamma_1 - \partial\Gamma_0) <$

0 in (32) and replacing the integrand by $U(K^*,0)$ we have a lower bound. Simplifying we obtain

$$\frac{\partial GDP}{\partial \lambda} > -U(K^*,0)\underbrace{(\Gamma_1(K_d)-\Gamma_0(K_d))}_{<0} > 0.$$

Case 2 $K_d, K_r \in (E(K_z), K_2)$; $K^* > K_2$ Expression (32) can be written as:

$$\begin{aligned}\frac{\partial GDP}{\partial \lambda} = & \int_{K_d}^{K_r} U(K_z,D_z)\underbrace{(\partial\Gamma_1-\partial\Gamma_0)}_{<0} + \int_{K_r}^{K_2} U(K_z,K^*-K_z)\underbrace{(\partial\Gamma_1-\partial\Gamma_0)}_{<0} \\ & + \int_{K_2}^{K^*} U(K_z,K^*-K_z)\underbrace{(\partial\Gamma_1-\partial\Gamma_0)}_{>0} - U(K^*,0)(\Gamma_1(K^*)-\Gamma_0(K^*))\end{aligned}$$

A lower bound for this expression:

$$\begin{aligned}\frac{\partial GDP}{\partial \lambda} > & \int_{K_d}^{K_r} U(K_2,D_2)(\partial\Gamma_1-\partial\Gamma_0) + \int_{K_r}^{K_2} U(K_2,D_2)(\partial\Gamma_1-\partial\Gamma_0) + \int_{K_2}^{K^*} U(K_2,D_2)(\partial\Gamma_1-\partial\Gamma_0) \\ & - U(K^*,0)(\Gamma_1(K^*)-\Gamma_0(K^*)) \\ = & \underbrace{(U(K_2,D_2)-U(K^*,0))}_{<0}\underbrace{[\Gamma(K^*)-\Gamma_0(K^*)]}_{<0} - U(K_2,D_2)\underbrace{[\Gamma(K_d)-\Gamma_0(K_d)]}_{<0} > 0\end{aligned}$$

Case 3: $K_d \in (E(K_z), K_2)$; $K_r, K^* > K_2$

Using the same trick again we obtain a positive lower bound for this expression:

$$\begin{aligned}\frac{\partial GDP}{\partial \lambda} > & \int_{K_d}^{K_2} U(K_2,D_2)(\partial\Gamma_1-\partial\Gamma_0) + \int_{K_2}^{K_r} U(K_2,D_2)(\partial\Gamma_1-\partial\Gamma_0) + \int_{K_r}^{K^*} U(K_2,D_2)(\partial\Gamma_1-\partial\Gamma_0) \\ & - U(K^*,0)(\Gamma_1(K^*)-\Gamma_0(K^*)) \\ = & \underbrace{(U(K_2,D_2)-U(K^*,0))}_{<0}\underbrace{[\Gamma(K^*)-\Gamma_0(K^*)]}_{<0} - U(K_2,D_2)\underbrace{[\Gamma(K_d)-\Gamma_0(K_d)]}_{<0} > 0\end{aligned}$$

Case 4: Since $K_r > K_d > K_2 > E(K_z)$ we have that $(\partial\Gamma_1 - \partial\Gamma_0) > 0$ in (32). Hence, replacing the integrands by $U(K_d, D_d)$ we have a lower bound. Collecting terms we have

$$\frac{\partial GDP}{\partial \lambda} > \underbrace{(U(K_d,D_d)-U(K^*,0))}_{<0}\underbrace{(\Gamma_1(K^*)-\Gamma_0(K^*))}_{<0} - U(K_d,D_d)\underbrace{(\Gamma_1(K_d)-\Gamma_0(K_d))}_{<0} > 0$$

We conclude that $\frac{\partial GDP}{\partial \lambda} > 0$ if $K_d > E(K_z)$. The other proofs are similar. The proof can be generalized to more than two crossings of the density function. ■

B Appendix: Proof for the results of the model with bankruptcy and labour

Lemma 2 *If Θ increases then D_z and L_z decrease for all z such that $K_z > K_d$.*

Proof: Differentiating condition (38) at (K_z, D_z, L_z) we obtain:

$$\frac{\partial D_z}{\partial \Theta} = \frac{\left(\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w\right)}{\left(f_K - \left[\frac{1+\rho+\phi-(1-p)\eta}{p}\right]\right)} < 0 \tag{33}$$

From the FOC of labour we obtain:

$$f_{KL}\frac{\partial D_z}{\partial \Theta} + f_{LL}\frac{\partial L_z}{\partial \Theta} = \frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w)$$

$$\Rightarrow \frac{\partial L_z}{\partial \Theta} = \frac{\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w - f_{KL}\frac{\partial D_z}{\partial \Theta}}{f_{LL}} < 0 \tag{34}$$

where we have used the fact that $\frac{\partial D_z}{\partial \Theta} < 0$ and $\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w > 0$. ■

Lemma 3 *The level of labour that a firm contracts L_z increases with K_z.*

Proof: The optimal labour demand L_z is defined by:

$$f_L(K_z, L_z) = w\left(1 + \Theta\frac{1-p}{p}\right) \tag{35}$$

Differentiating this condition with respect K_z we obtain that:

$$f_{KL}\frac{\partial D_z}{\partial K_z} + f_{LL}\frac{\partial L_z}{\partial K_z} = 0 \tag{36}$$

$$\Rightarrow \frac{\partial L_z}{\partial K_z} = -\frac{f_{KL}\frac{\partial D_z}{\partial K_z}}{f_{LL}} > 0 \tag{37}$$

■

Lemma 4 *The maximum debt D_z satisfies the following conditions:*

1. $\frac{\partial D_z}{\partial K_z} > 1$ *if $K_z > K_d$.*
2. $D_z > 0$ *and* $D_z > K_z$ *if $K_z \geq K_d$.*

Proof:

Recall the condition which defines $D(K_z)$:

$$\Psi(K_z, D_z, L_z) = 0 \tag{38}$$

Differentiation of this condition leads to:

$$\frac{\partial\Psi(K_z,D_z,L_z)}{\partial K_z}+\frac{\partial\Psi(K_z,D_z,L_z)}{\partial D_z}\frac{\partial D_z}{\partial K_z}+\frac{\partial\Psi(K_z,D_z,L_z)}{\partial L_z}\frac{\partial L_z}{\partial K_z}=0 \tag{39}$$

$$\Rightarrow \frac{\partial D_z}{\partial K_z}=-\frac{\Psi_L\frac{\partial L_z}{\partial K_z}+\Psi_K}{\Psi_D} \tag{40}$$

where:

$$\Psi_K=f_K+\frac{1-p}{p}\eta>0 \tag{41}$$

$$\Psi_L=f_L-w+\frac{1-p}{p}\Theta w=0 \tag{42}$$

$$\Psi_D=f_K-\left(\frac{1+\rho+\phi-(1-p)\eta}{p}\right)\leq 0 \tag{43}$$

Therefore, if $K_z>K_d$, we conclude that:

$$\frac{\partial D_z}{\partial K_z}=-\frac{f_K+\frac{1-p}{p}\eta}{f_K-\left(\frac{1+\rho+\phi-(1-p)\eta}{p}\right)}>0 \tag{44}$$

Moreover, because $f_K(K_z+D_z,L_z)\in\left[\frac{1+\rho+\phi-(1-p)\eta}{p},\frac{1+\rho-(1-p)\eta}{p}\right)$, we have that $\frac{\partial D_z}{\partial K_z}>1$ if $K_z>K_d$. For the second item note that the conditions that define K_d are not satisfied at $D_d=0$. Using the compatibility constraint and the participation constraint jointly:

$$\Psi(K_z,D_z,L_z)=U_e(K_z,D_z,L_z)-\phi D_z=0$$

$$U_e(K_z,D_z,L_z)\geq U_w(K_z,l_z)$$

$$\Rightarrow \phi D_z\geq U_w(K_z,l_z)=(1+\rho)K_z+wl_z[p+(1-p)\Theta]-\varsigma(l)$$

$$\Rightarrow D_z\geq\frac{(1+\rho)}{\phi}K_z>K_z$$

■

Lemma 5 (Equilibrium in the labor market) *The supply of labour is upwards sloping in w, while the demand of labour decreases in w.*

Proof: First note that the optimal amount of labour supplied by each worker $l(w)$ is defined by:

$$w[p+(1-p)\Theta]=\varsigma'(l(w))$$

Differentiating with respect to w leads to:

$$\frac{\partial l(w)}{\partial w} = \frac{p + (1-p)\Theta}{\varsigma''(l)} > 0 \tag{45}$$

From the FOC of labour:

$$\frac{\partial L_z}{\partial w} = \frac{1 + \frac{(1-p)}{p}\Theta - f_{LK}\frac{\partial D_z}{\partial w}}{f_{LL}} < 0 \tag{46}$$

where we have used the fact that $\frac{\partial D_z}{\partial w} = \frac{L_z(1+\Theta)}{f(K_z+D_z,L_z)-\left(\frac{1+\rho+\phi-(1-p)\eta}{p}\right)} < 0$. Total differentiation of condition (38) leads to:

$$\frac{\partial \Psi}{\partial D_d}\frac{\partial D_d}{\partial w} + \frac{\partial \Psi}{\partial L_d}\frac{\partial L_d}{\partial w} + \frac{\partial \Psi}{\partial K_d}\frac{\partial K_d}{\partial w} + \frac{\partial \Psi}{\partial w} = 0 \tag{47}$$

Replacing terms of previous conditions we obtain that:

$$\underbrace{\left(f_K - \left[\frac{1+\rho+\phi-(1-p)\eta}{p}\right]\right)}_{=0}\frac{\partial D_d}{\partial w} + \underbrace{\left(f_L - w - \frac{1-p}{p}\Theta w\right)}_{=0}\frac{\partial L_d}{\partial w} + \frac{\partial K_d}{\partial w}\left(f_K + \frac{(1-p)}{p}\eta\right) = \frac{L_d}{p} + \frac{1-p}{p}L_d\Theta \tag{48}$$

$$\Rightarrow \frac{\partial K_d}{\partial w} = \frac{L_d(1+(1-p)\Theta)}{pf_K + (1-p)\eta} > 0 \tag{49}$$

For notational simplicity we omit the dependence of l, L_z, L^*, K_d and K_r on w.
Now, differentiating the left-hand side of the labour market equilibrium condition we obtain:

$$\frac{\partial \mathcal{S}_L}{\partial w} = \frac{\partial l}{\partial w}\Gamma(K_d) + l \cdot \frac{\partial K_d}{\partial w}\gamma(K_d) > 0 \tag{50}$$

For the demand of labour we have:

$$\frac{\partial \mathcal{D}_L}{\partial w} = \int_{K_d(w)}^{+\infty}\frac{\partial L_z}{\partial w}\partial\Gamma(K_z) - \frac{\partial K_d}{\partial w}L_d\gamma(K_d) < 0 \tag{51}$$

Since $\varsigma'' > 0, \lim_{l\to\infty} w \to \infty$, and thus there is an equilibrium. ■

Proposition 8 *The equilibrium wage w rises and the minimum capital level K_d decreases after:*

1. *An improvement in* ex-ante *creditor protection* $1-\phi$.
2. *An increase in* ex-post *protection* η.
3. *An decrease in worker protection* Θ.
4. *A decrease in fixed costs* θ.

Proof:

We do the case of w first. In order to simplify calculations we define $x = \phi, \eta, \Theta, \theta$. From the equilibrium labour market condition we have[16, 17]

$$\left(\frac{\partial l}{\partial w}\frac{\partial w}{\partial x} + \frac{\partial l}{\partial x}\right)\Gamma(K_d) + l \cdot \gamma(K_d)\left(\frac{\partial K_d}{\partial x} + \frac{\partial K_d}{\partial w}\frac{\partial w}{\partial x}\right) - \left[\int_{K_d(w)}^{+\infty}\left(\frac{\partial L_z}{\partial w}\frac{\partial w}{\partial x} + \frac{\partial L_z}{\partial x}\right)\partial\Gamma(K_z)\right.$$
$$\left. - L_d\gamma(K_d)\left(\frac{\partial K_d}{\partial x} + \frac{\partial K_d}{\partial w}\frac{\partial w}{\partial x}\right)\right] = 0 \quad (52)$$

For $x = \phi$, $x = \Theta$ or $x = \theta$ the direct effect on K_d is: $\frac{\partial K_d}{\partial x} > 0$. If $\frac{\partial w}{\partial x} > 0$ then all terms would be positive and the labour market equilibrium condition would be violated. On the other hand, if $\frac{\partial w}{\partial x} < 0$, we will have terms with opposite signs and the market condition will be satisfied. Therefore, the equilibrium wage decreases after an increase in ϕ, Θ or θ. If $x = \eta$ then we have that: $\frac{\partial K_d}{\partial x} < 0$. Using the same argument we conclude that $\frac{\partial w}{\partial x} > 0$.

Again, for the case of K_d, we define $x = \phi, \eta, \Theta, \theta$. Differentiating condition (22) at (K_d, D_d, L_d) we obtain:

$$\frac{\partial \Psi(K_d, D_d, L_d)}{\partial K_d}\frac{\partial K_d}{\partial x} + \frac{\partial \Psi(K_d, D_d, L_d)}{\partial D_d}\frac{\partial D_d}{\partial x} + \frac{\partial \Psi(K_d, D_d, L_d)}{\partial L_d}\frac{\partial L_d}{\partial x} + \frac{\partial \Psi(K_d, D_d, L_d)}{\partial x} = 0$$
$$\Rightarrow \left(f_K + \frac{(1-p)}{p}\eta\right)\frac{\partial K_d}{\partial x} + \underbrace{\left(f_K - \left[\frac{1+\rho+\phi-(1-p)\eta}{p}\right]\right)}_{=0}\frac{\partial D_d}{\partial x} + \underbrace{\left(f_L - w - \frac{1-p}{p}\Theta\right)}_{=0}\frac{\partial L_d}{\partial x} = -\frac{\partial \Psi(K_d, D_d, L_d)}{\partial x}$$
$$\Rightarrow \frac{\partial K_d}{\partial x} = -\frac{\frac{\partial \Psi(K_d, D_d, L_d)}{\partial x}}{pf_K + (1-p)\eta}$$

Differentiating and replacing terms we obtain that:

$$\frac{\partial K_d}{\partial \phi} = \frac{D_d + \frac{\partial w}{\partial \phi}L_d(p + (1-p)\Theta)}{pf_k + (1-p)\eta} \quad (53)$$
$$\frac{\partial K_d}{\partial \eta} = \frac{-(1-p)(K_d + D_d) + \frac{\partial w}{\partial \eta}L_d(p + (1-p)\Theta)}{pf_k + (1-p)\eta} \quad (54)$$
$$\frac{\partial K_d}{\partial \Theta} = \frac{L_d\left(\frac{\partial w}{\partial \Theta}((1-p)\Theta + p) + w(1-p)\right)}{pf_k + (1-p)\eta} \quad (55)$$
$$\frac{\partial K_d}{\partial \theta} = \frac{p + \frac{\partial w}{\partial \theta}L_d(p + \Theta(1-p))}{pf_k + (1-p)\eta} \quad (56)$$

Note that condition (52) implies that $|\frac{\partial w}{\partial \phi}| < \frac{D_d}{L_d(p+(1-p)\Theta)}$, $|\frac{\partial w}{\partial \Theta}| < \frac{(1-p)w}{p+(1-p)\Theta}$,$|\frac{\partial w}{\partial \eta}| < \frac{(1-p)(K_d+D_d)}{L_d(p+(1-p)\Theta)}$ and

[16] Note that L_d is the amount of labour demanded by a firm which owns K_d.

[17] Notice that total differentiation of $K_d(w)$ with respect any measure x incorporates a direct effect and a indirect effect (given by the change in w): $\frac{\partial K_d(w)}{\partial x} = \left(\frac{\partial K_d}{\partial x} + \frac{\partial K_d}{\partial w}\frac{\partial w}{\partial x}\right)$

$|\frac{\partial w}{\partial \theta}| < \frac{1}{L_d(1+\Theta(1-p))}$, otherwise the equilibrium condition will be violated. Therefore we conclude that: $\frac{\partial K_d}{\partial \phi} > 0$, $\frac{\partial K_d}{\partial \Theta} > 0$, $\frac{\partial K_d}{\partial \eta} > 0$ and $\frac{\partial K_d}{\partial \theta} > 0$. ■

B.1 Proof of the results related to changes in welfare of workers and firms

In order to compare the effects of Θ and ϕ among the different entrepreneurial groups we define the profits of a firm of size $K_z + D_z$ as follows:

$$\Pi(K_z + D_z, L_z) = p[f(K_z + D_z, L_z) - \theta - wL_z] + (1-p)[\eta(K_z + D_z) - \Theta w L_z] - (1+\rho)(D_z + K_z) \quad (57)$$

Proposition 9 *If worker protection Θ increases then:*

1. *All firms experience a decrease in their profits, and there exists a threshold $K_\Theta \in (K_d, K_r)$ such that firms with $K_z \in (K_d, K_\Theta]$ are worse off than firms with $K_z \geq K_r$.*

2. *On average, workers are better off.*

Proof: For firms with $K_z \in [K_d, K_r)$ differentiation of condition (57) with respect Θ leads to:

$$\frac{\partial \Pi(K_z + D_z, L_z)}{\partial \Theta} = \frac{\partial \Pi}{\partial D_z}\frac{\partial D_z}{\partial \Theta} + \underbrace{\frac{\partial \Pi}{\partial L_z}}_{=0}\frac{\partial L_z}{\partial \Theta} + \frac{\partial \Pi}{\partial \Theta} \quad (58)$$

$$\Rightarrow \frac{\partial \Pi(K_z + D_z, L_z)}{\partial \Theta} = \underbrace{\left(f_K - \left(\frac{1+\rho-(1-p)\eta}{p}\right)\right)}_{>0}\underbrace{\frac{\partial D_z}{\partial \Theta}}_{<0} - L_z \underbrace{\left(\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w\right)}_{>0} < 0 \quad (59)$$

For firms which produce optimally ($K_z \geq K_r$) we have that:

$$\frac{\partial \Pi(K^*, L^*)}{\partial \Theta} = \frac{\partial \Pi}{\partial K^*}\frac{\partial K^*}{\partial \Theta} + \underbrace{\frac{\partial \Pi}{\partial L^*}}_{=0} + \frac{\partial \Pi}{\partial \Theta}$$

$$\Rightarrow \frac{\partial \Pi(K^*, L^*)}{\partial \Theta} = p\underbrace{\left(f(K^*, L^*) - \frac{(1+\rho)-(1-p)\eta}{p}\right)}_{=0}\frac{\partial K^*}{\partial \Theta} - L^*\left(\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w\right) < 0$$

Note that $\lim_{K_z \to K_d^+} \frac{\partial \Pi}{\partial \Theta} = -\infty$. Else if $K_z \geq K_r$ then $\frac{\partial \Pi}{\partial \Theta} = -[\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w]L^* > -\infty$. Since $\frac{\partial \Pi}{\partial \Theta}$ is continuous in $(K_d, +\infty]$, there exists an interval $K_z \in (K_d, K_\Theta]$ such that $\frac{\partial \Pi}{\partial \Theta}$ is always more negative than when $K_z \geq K_r$.
For an average worker we have that:

$$\frac{\partial U_w}{\partial \Theta} = \frac{\partial w}{\partial \Theta} l(p + (1-p)\Theta)) + w\frac{\partial l}{\partial w}\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + wl(1-p)\Theta - \varsigma'(l)\frac{\partial l}{\partial w}\frac{\partial w}{\partial \Theta} \tag{60}$$

$$\Rightarrow \frac{\partial l}{\partial w}\frac{\partial w}{\partial \Theta}\underbrace{(p + (1-p)\Theta)w - \varsigma'(l)}_{=0} + l\left(\frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + w(1-p)\right) > 0 \tag{61}$$

■

Proposition 10 *Assume $f_{LL,K} < 0$ and $f_{KL,K} < 0$. If worker protection Θ increases then there exists a cutoff $\tilde{K}_\Theta \in (K_d, K_r)$ such that:*

1. *The representative worker of a firm with $K_z \in (K_d, \tilde{K}_\Theta)$ is worse off.*
2. *The representative worker of a firm with $K_z > \tilde{K}_\Theta$ is better off.*

Proof:

We define the auxiliary function $g(\Theta) \equiv \frac{\partial w}{\partial \Theta}(p + (1-p)\Theta) + (1-p)w$. Differentiating condition (26) with respect Θ:

$$\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta} = g(\Theta)L_z + \frac{\partial L_z}{\partial \Theta}\left(\underbrace{pw + (1-p)\Theta w}_{=\varsigma'(l)} - \frac{\varsigma'(l)}{l}\right) - \frac{L_z}{l}\frac{\partial l}{\partial \Theta}\left(\varsigma'(l) - \frac{\varsigma(l)}{l}\right)$$

$$\Rightarrow \frac{\partial \tilde{U}_w(L_z)}{\partial \Theta} = g(\Theta) \cdot L_z \underbrace{\left[1 - \frac{1}{\varsigma''(l) \cdot l}\left(\varsigma'(l) - \frac{\varsigma(l)}{l}\right)\right]}_{>0} + \underbrace{\frac{\partial L_z}{\partial \Theta}}_{<0}\underbrace{\left(\varsigma'(l) - \frac{\varsigma(l)}{l}\right)}_{>0} \tag{62}$$

where we have used the fact that $\frac{\partial l}{\partial \Theta} = \frac{g(\Theta)}{\varsigma''(l)} > 0$ and lemma 6 . Note that the sign of $\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta}$ is ambiguous and will depend on K_z. For a firm which is operating close enough to K_d we obtain that $\lim_{K_z \to K_d^+} \frac{\partial \tilde{U}_w(L_z)}{\partial \Theta} = -\infty$ (because $\lim_{K_z \to K_d^+} \frac{\partial D_z}{\partial \Theta} = -\infty$ and so, $\lim_{K_z \to K_d^+} \frac{\partial L_z}{\partial \Theta} = -\infty$), so at least in a neighborhood of K_d the representative worker is worse off. In addition, note that the labour market must satisfy the welfare equilibrium condition:

$$[pwl + (1-p)\Theta wl - \varsigma(l)]\,\Gamma(K_d) = \int_{K_d}^{K_r} \tilde{U}_w(L_z)\partial\Gamma(K_z) + \tilde{U}_w(K^*)(1 - \Gamma(K_r)) \tag{63}$$

which leads to:

$$\underbrace{\frac{\partial U_w}{\partial \Theta}\Gamma(K_d) + U_w\gamma(K_d)\frac{\partial K_d}{\partial \Theta}}_{>0} = \int_{K_d}^{K_r} \frac{\partial \tilde{U}_w(L_z)}{\partial \Theta}\partial\Gamma(K_z) - \tilde{U}_w(L_d)\gamma(K_d)\frac{\partial K_d}{\partial \Theta} + \frac{\partial \tilde{U}_w(K^*)}{\partial \Theta}(1 - \Gamma(K_r))$$

we know that $\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta} < 0$ in some neighborhood of K_d and that the second term of the right-hand side is also negative, so $\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta}$ must be positive in some range (otherwise condition (63) is violated). If $\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta}$ is strictly increasing in K_z then we can conclude that there exist some $\tilde{K}_\Theta \in (K_d, K_r)$ such that $\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta} < 0$ if $K_z \in [K_d, \tilde{K}_\Theta)$ and $\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta} > 0$ if $K_z > \tilde{K}_\Theta$.

Now, all we need to show is that $\frac{\partial\left(\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta}\right)}{\partial K_z} > 0$. Differentiation of $\tilde{U}_w$ with respect to K_z leads to:

$$\frac{\partial\left(\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta}\right)}{\partial K_z} = g(\Theta) \cdot \underbrace{\frac{\partial L_z}{\partial K_z}}_{>0} \underbrace{\left[1 - \frac{1}{\varsigma''(l) \cdot l}\left(\varsigma'(l) - \frac{\varsigma(l)}{l}\right)\right]}_{>0} + \frac{\partial\left(\frac{\partial L_z}{\partial \Theta}\right)}{\partial K_z} \underbrace{\left(\varsigma'(l) - \frac{\varsigma(l)}{l}\right)}_{>0}$$

where we have used the result of lemmas 3 and 6. In addition, we have that:

$$\frac{\partial L_z}{\partial \Theta} = g(\Theta)\left(\frac{1}{f_{LL}} - \frac{f_{KL}}{f_{LL}h(K_z, L_z)}\right) \tag{64}$$

where we have defined $h(K_z, L_z) \equiv f_K - \frac{1+\rho+\phi-(1-p)\eta}{p}$.

Differentiating this condition with respect K_z :

$$\frac{\partial\left(\frac{\partial L_z}{\partial \Theta}\right)}{\partial K_z} = g(\Theta)\left(-\underbrace{\frac{f_{LL,K}}{(f_{LL})^2}}_{>0} - \underbrace{\left[\frac{\overbrace{f_{KL,K} f_{LL} h(K_z, L_z)}^{<0} - f_{KL}\overbrace{(f_{LL,K} h(K_z, L_z) + f_{LL} f_{KK})}^{>0}}{(f_{LL} h(K_z, L_z))^2}\right]}_{<0}\right) > 0$$

where we have used the fact that $f_{LL,K} < 0$ and $f_{KL,K} < 0$. Therefore, we conclude that $\frac{\partial\left(\frac{\partial \tilde{U}_w(L_z)}{\partial \Theta}\right)}{\partial K_z} > 0$, which leads to the result of the proposition. ■

Proposition 11 *If ex-ante creditor protection* $1 - \phi$ *improves then:*

1. *There exists a threshold* $K_\phi \in (K_d, K_r)$ *such that all firms with* $K_z \in (K_d, K_\phi)$ *are better off, while firms with* $K_z \geq K_r$ *are worse off.*
2. *On average, workers are better off.*

Proof: Differentiating (57) with respect ϕ we have:

$$\frac{\partial \Pi(K_z + D_z)}{\partial \phi} = \left(f_K - \left(\frac{1 + \rho - (1-p)\eta}{p}\right)\right)\frac{\partial D_z}{\partial \phi} - L_z \frac{\partial w}{\partial \phi}(p + (1-p)\Theta)) \tag{65}$$

where:

$$\frac{\partial D_z}{\partial \phi} = \frac{D_z + L_z \frac{\partial w}{\partial \phi}(p + (1-p)\Theta))}{f_K - \left(\frac{1+\rho+\phi-(1-p)\eta}{p}\right)} \tag{66}$$

Replacing this last expression in (65) we obtain that:

$$\frac{\partial \Pi(K_z + D_z)}{\partial \phi} = \underbrace{\frac{D_z\left(f_k - \frac{(1+\rho)-(1-p)\eta}{p}\right)}{f_k - \frac{(1+\rho+\phi)-(1-p)\eta}{p}}}_{<0} + \underbrace{\frac{\frac{\phi}{p} L_z \frac{\partial w}{\partial \phi}(p + (1-p)\Theta))}{f_k - \frac{(1+\rho+\phi)-(1-p)\eta}{p}}}_{>0} \tag{67}$$

For firms which produce optimally we obtain that:

$$\frac{\partial \Pi(K^*)}{\partial \phi} = p\underbrace{\left(f(K^*) - \frac{(1+\rho)-(1-p)\eta}{p}\right)}_{=0}\frac{\partial K^*}{\partial \phi} - \frac{\partial w}{\partial \phi}L^*(p + (1-p)\Theta) > 0 \tag{68}$$

Notice that the sign of expression (67) is ambiguous.However, note that $\lim_{K_z \to K_d^+} \frac{\partial \Pi}{\partial \phi} = -\infty$. Since $\frac{\partial \Pi}{\partial \phi}$ is continuous in $(K_d, +\infty)$, there exists some cutoff K_ϕ such that $\frac{\partial \Pi}{\partial \phi} < 0$ if $K_z \in (K_d, K_\phi)$.

For an average worker we obtain:

$$\frac{\partial U_w}{\partial \phi} = l \cdot \frac{\partial w}{\partial \phi}(p + (1-p)\Theta) < 0 \tag{69}$$

■

Lemma 6 *The cost function $\varsigma(l_z), l_z > 0$, satisfies:*

1.

$$\varsigma'(l_z)\frac{l_z}{\varsigma(l_z)} \geq 1$$

2.

$$\varsigma(l_z)'' l_z^2 \geq \varsigma'(l_z)l_z - \varsigma(l_z)$$

Proof: Define the auxiliary function:

$$\Upsilon_{l_z}(\bar{l}) \equiv \frac{\varsigma(l_z) - \varsigma(\bar{l})}{l_z - \bar{l}}; \bar{l} < l_z \tag{70}$$

Differentiation with respect $\bar{l}$ leads to:

$$\Upsilon'_{l_z}(\bar{l})(l_z - \bar{l}) = \frac{\varsigma(l_z) - \varsigma(\bar{l})}{l_z - \bar{l}} - \varsigma'(\bar{l}) \tag{71}$$

Note that $\Upsilon'_{l_z}(z) \geq 0$. In fact the convexity of $\varsigma(\cdot)$ implies that:

$$\varsigma(\lambda l_z + (1-\lambda)\bar{l}) \geq \lambda\varsigma(l_z) + (1-\lambda)\varsigma(\bar{l}), \forall \lambda \in [0,1]$$
$$\Rightarrow \frac{\varsigma(\bar{l} + \lambda(l_z - \bar{l})) - \varsigma(\bar{l})}{\lambda} \leq \varsigma(l_z) - \varsigma(\bar{l})$$

Taking the limit $\lim_{\lambda \to 0^+}$ we obtain:

$$\begin{aligned} &\varsigma'(\bar{l})(l_z - \bar{l}) \leq \varsigma(l_z) - \varsigma(\bar{l}) \\ \Rightarrow\ &\Upsilon'_{l_z}(\bar{l})(l_z - \bar{l}) \geq 0 \Rightarrow \Upsilon'_{l_z}(\bar{l}) \geq 0 \end{aligned}$$

where we have used the fact $l_z > \bar{l}$. This last condition implies that $\Upsilon_{l_z}(\bar{l}_1) \leq \Upsilon_{l_z}(\bar{l}_2), \forall \bar{l}_1 \in [0, \bar{l}_2]$. In particular, it is satisfied for $\bar{l}_1 = 0$ and any $\bar{l}_2 \to l_z$ with $l_z \geq 0$. This is,

$$\begin{aligned} &\Upsilon_{l_z}(0) < \lim_{\bar{l}_2 \to l_z \geq 0} \Upsilon_{l_z}(\bar{l}_2) \\ \Leftrightarrow\ &\frac{\varsigma(l_z)}{l_z} \leq \varsigma'(l_z) \end{aligned}$$

which proves the first part of the result. For the last item define the function:

$$\Omega(l_z) \equiv \varsigma(l_z) - \varsigma'(l_z) l_z - \varsigma''(l_z) l_z^2;\ l_z \geq 0 \tag{72}$$

Note that $\Omega(0) = 0$ and that $\Omega'(l_z) = l_z \cdot (\varsigma'''(l_z) l_z + \varsigma''(l_z)) > 0, \forall l_z > 0$ (because $\varsigma'' > 0$ and $\varsigma''' > 0$). Thus $\Omega(l_z) \geq 0, \forall l_z \geq 0 \Leftrightarrow \varsigma(l_z)'' l_z^2 \geq \varsigma'(l_z) l_z - \varsigma(l_z)$.

■

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